FINANCE & LIFE INSURANCE

BY

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ATTORNEY-AT-LAW

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FINANCE
AND
LIFE INSURANCE

A Hand Book of Tables and Formulae, with Rules and Explanations, for the use of Lawyers, Brokers, Bankers Insurance Men and Others

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PREFACE

The object had in view in the preparation of this book has been to place in the hands of my professional brethren an instrument of which I, early in my practice, felt the urgent need, that is, the data and information necessary to readily advise clients as to the value of securities, incomes, insurances, estates and other instruments of finance and investment. The tendency is stronger now than ever for lawyers to serve in the capacity of business experts and in every office, there is frequent occasion to pass upon the value and soundness of some investment, security or estate. The forty-three tables and explanatory text matter contained in the succeeding pages, it is believed, afford all of the facts and information necessary in order to answer most of the practical questions that arise on the face of the instruments of finance and life insurance. A second object sought has been to provide the banker, broker and financier an easy means of making the calculations called for in the conduct of his business and in advising his customers. And finally, it is believed that the practical insurance man will find in these pages much that will be helpful. No effort has been made to develop the scientific principles involved in finance and insurance. That has been left to the actuaries whose business it is to teach those subjects, but the endeavor has been to apply the principles and rules demonstrated by the actuaries to the solution of the practical problems arising in these great and ever widening fields. The purpose has been to make the subjects simple and the scientific principles have only been discussed when the attainment of this object seemed to require it. The writer takes this occasion to acknowledge his obligations to the Acturial Society of America for permission to republish its regraduated American Experience table of mortality and force of mortality, and publish annuity tables derived therefrom; to the Connecticut Mutual Life Insurance Company, for permission to republish its three per cent American Experience Commutation columns and annuities, and to Mr. Abb Landis, actuary, for permission to republish the National Fraternal Congress table, commutation tables and annuities computed by him. The preparation of the materials and work of reducing them to the form here presented has covered a period of several years, during which time, the writer has been actively engaged in the practice of law, and it would be remarkable indeed if no errors should be found; but it is believed that both the rules and the tables given may be safely employed for the purposes for which they are intended, and it is hoped that they may be found useful.

Troy, Mo., June, 1916.

CHAPTER I

Of Short Methods

Facility in computation is a thing to be acquired by practice intelligently followed, and not many of the so-called short methods possess any practical value. Grouping in addition, like phrasing in stenography, gives speed, but this will come from intelligently directed practice. In fact, an active computer soon finds himself employing combinations of numbers which greatly facilitates his work but rules would have been of little use to him in forming them or in employing them in practice. In nearly all of the operations involved in the matters treated of in this book, decimal fractions are employed and in many of these, figures must be discarded beyond a limited number of decimal places. Usually, not more than six decimal places are retained. What has long been known as the **contracted methods of multiplication and of division** have been found helpful and will be noticed here.

1. Multiplication of decimals by the contracted method consists in disregarding all figures in the product beyond the number of decimal places intended to be retained except that they are considered in carrying to the right hand figures of the partial products. To use this method, first determine from the nature of the factors how many integral figures will occur in the product, then include as many decimals as will be required, dropping those falling to the right.

Let it be required to multiply 4.549383 by 2.854339 correct to six places of decimals. We know there will be two whole numbers and six decimals in the product and eight places will be required. The multiplication begins with the left hand figure of the multiplier and it is generally more convenient to reverse the multiplier. In this case, they appear as follows:

```
4.549383
9 334582
---------
9 098766
3 639506
  227469
   18198
    1365
     136
      41
---------
12.985481
```

First, the entire multiplicand is multiplied by 2. Next, we begin by multiplying 3 by 8, but the product is not entered, then, we have 8×8, to which we add 2 from the product of 8×3, put down the right figure under 2 and carry 6 as in ordinary multiplication. The

only other thing to be noticed is that the first and sometimes the second figure to the right of the figure above the partial multiplier is to be multiplied in order to determine how many to carry. Thus we say $5\times8=40$ and carry 4, then $5\times3=15$, and add 4, putting down 9 in the first column. Next, $4\times8=32$, then $4\times3+3=15$, and finally, $4\times9+2=38$, and we put 8 in the first column. The rule being, in carrying tens from the nearest rejected figure of the multiplicand, to carry 1 additional for any number in units place over five.

2. The **contracted method of division** of decimals also saves time and space and should be employed.

It consists in cutting off those figures extending beyond the number of places intended to be retained, carrying into the partial products the proper numbers to avoid loss by the omission of figures from the right of the divisor in multiplying. Thus, let it be required to divide 11.46992 by 36.78559, correct to six places of decimals. We know 36 will go into 11.4, .3 times and hence there will be no integers in the quotient. Arrange the dividend and divisor in the usual way and divide:

```
36.78559)11.46992(.311533
         11 03568
         -------
           43424
           36786
           -----
            5638
            3679
            ----
            1959
            1839
            ----
             120
             110
             ---
              10
              11
              --
```

In dividing, we reject the figure nine but carry 3 into the product of 3×5, putting down 8. For the next divisor, we reject 5 and find 1 as the quotient figure then to 1×5 we add 1 for the rejected 59 and put down 6 in the partial product, the rule for carrying being the same as in contracted multiplication.

3. But the most valuable instrument yet devised for shortening the processes of multiplication, division, involution and evolution is the table of logarithms. It not being the object of this book to teach mathematics, but rather to apply the mathematics in the solution of problems in finance, and life insurance, a discussion of the principles of the subject will not be undertaken. But such an explanation of the tables and the methods of using them will be given as will enable the reader to apply them in the solution of problems.

CHAPTER II

Of Logarithms

1. Logarithms are exponents of the powers of some number which is taken as a base. In the tables of logarithms given in this book, the number 10 is taken as the base and all numbers are considered as powers of ten. And, since

$10^0 = 1$ the logarithm of 1 is 0
$10^1 = 10$ the logarithm of 10 is 1
$10^2 = 100$ the logarithm of 100 is 2
$10^3 = 1000$ the logarithm of 1000 is 3

and so on, the logarithm of any number between 1 and 10 is between 0 and 1, and hence, is a fraction; the logarithm of any number between 10 and 100 is between 1 and 2 and hence is 1 plus a fraction; and the logarithm of any number between 100 and 1000 is between 2 and 3 and hence is 2 plus a fraction and so on.

Again, in the case of fractional numbers, if we take the logarithm of the denominators, we have the same rule applying except that the sign, by the theory of exponents, becomes negative when the denominator is transferred to the numerator, thus:

$$\frac{1}{\text{Log. } 10} = \frac{1}{10^1} = -1, \quad \frac{1}{\text{Log. } 100} = \frac{1}{10^2} = -2, \text{ and so on,}$$

so that the logarithm of a number between 1 and .1 is between 0 and -1 plus a fraction; the logarithm of any number between .1 and .01 is between -1 and -2 and hence is -2 plus a fraction. The integral part of the logarithm, whether positive or negative, is called the **characteristic** and the fractional part which is always positive is called the **Mantissa.**

2. The determination of the characteristic of a logarithm will be better understood by considering the following exhibit:

The number 7436 is between the third and fourth powers of ten and hence is 3 plus a fraction. In fact, $7436 = 10^{3.87134}$. Hence, the logarithm of 7436 is 3.87134. Also 743.6 is between the second and third powers of 10, hence is 2 plus a fraction. In fact, log. 743.6 is $10^{2.87134}$.

We may arrange the following scheme:

$10^{3.87134} = 7436$ or log. $7436 = 3.87134$
$10^{2.87134} = 743.6$ or log. $743.6 = 2.87134$
$10^{1.87134} = 74.36$ or log. $74.36 = 1.87134$
$10^{0.87134} = 7.436$ or log. $7.436 = .87134$
$10^{-1.87134} = .7436$ or log. $.7436 = \bar{1}.87134$
$10^{-2.87134} = .07436$ or log. $.07436 = \bar{2}.87134$
and so on.

3. From the foregoing, we may derive the rule for finding the characteristic of a logarithm.

The characteristic of an integral or mixed number is one less than the number of integral places. When the number is entirely decimal, the characteristic is negative and numerically one greater than the number of 0's between the decimal point and first significant figure.

4. In practice, two methods are used in writing the characteristic of a decimal number. These may be understood from the following illustration:

Log. 7.436 = 10.87134 − 10 or .87134
Log. .7436 = 9.87134 − 10 or $\bar{1}$.87134
Log. .07436 = 8.87134 − 10 or $\bar{2}$.87134
Log. .007436 = 7.87134 − 10 or $\bar{3}$.87134
and so on, the minus sign over the characteristic indicating that it is negative.

5. It will be noticed from the above illustration that the same sequence of figures has the same mantissa and that the characteristic alone is changed by moving the decimal point; that is, by multiplying or dividing the number by ten.

6. It was first purposed by the writer to arrange and publish a table of common logarithms to six places, but it was concluded that the space it would occupy would be out of proportion to the benefit it would confer in view of the fact that such tables are generally available. There is included, however, a table which will meet most of the requirements of the work and occupies little space. The first part, Table 1a, may be used to find the logarithms of numbers up to say, four or five significant figures, by interpolating, true to five decimal places by using the first five left hand figures of the group. Thus, the mantissa of log of 264 would be found in column 4 opposite 2.6 under N. and to five places is .42160. The mantissa of 269 would be found in column 9 opposite 26 and to five places is .42975. The characteristic according to principles explained in the preceding sections is in each case 2 and the logs are, respectively, 2.42160 and 2.42975. Had the problem required the log of 26472, we would first have to find the log of 264 as above and add to it seventy-two hundredths of the difference between the logs of 264 and 265. We find log of 265 on the same line in column 5. It is .42325 and subtracting .42160, we have for the difference, .00165. Multiplying the latter by .72, we have .00118.80. We call it .00119 and add it to .42160. This with the characteristic 4, gives 4.42279 as the logarithm. By the seven figure table, the log of the same number is given as 4.4227868 so that the result is correct to five places, which is sufficient for most purposes. The tables published may also be used to obtain the logarithms of any number to eight places, by methods which will be explained further on.

7. From the last section we may, for finding the mantissa, derive the following:

Rule

For a number consisting of three figures the logarithm will be found in the column headed with the right hand figure of the number and in the line opposite the two left hand figures of the number where they appear in the left hand column of the table under the letter N. To find the logarithm of a number consisting of four or more significant figures, first find the log of the three left hand figures in the manner just described. Then multiply the difference between the log thus found and the log of the number in the next column at the right, by the remaining figures of the number at the right and add the product as a correction to the original logarithm found. This will give the mantissa. For the characteristic of the number, if the number is integral, or partly integral, write for the characteristic a number one less than the number of whole numbers in the given number. If the number is a decimal with one or more ciphers between the decimal point and the significant figures, write the characteristic negative and for one cipher write $\bar{0}$, for two ciphers $\bar{1}$, for three ciphers $\bar{2}$, and so on. Thus the logarithm of .0004724 is $\bar{2}.67431$, and of .000004724 is $\bar{4}.67431$ (see Section 4).

8. To find the number corresponding to a given logarithm, often called the anti log, we suggest the following: Neglecting the characteristic for the time being, find, if possible, the logarithm in the table. The two left hand figures of the number will be found in the left hand column of the table under N and opposite the logarithm found and the units figure will be found at the top of the column in which the log was found. Thus the number corresponding to the log 2.68395 is 483. If the exact mantissa can not be found in the table, find the one less and nearest to the given log. The three left hand figures will be found as already explained. The next figure, or figures, at the right of the number will be the quotient found by dividing the difference between the given log and the log first found by the difference between the first log found and the log of a number one unit greater. Thus take the logarithm 2.57702. The nearest mantissa in the table is .57634 found under 7 opposite 37. The last three figures of the number sought are therefore, 377. The difference between the given log and the log of 377 is .00068. The difference between the log of 377 and the log of 378 (57749) is .00115 and $68 \div 115 = .591$. The right hand figure being less than five is rejected and 59 affixed to the original number and we have 37759. The characteristic being 2, three of the figures are integral and two decimal and the number is 377.59. The table 1a may be used with confidence to five places as a rule, but sometimes the quotient should be carried one or two places further so that if the rejected figure is greater than five, the last retained figure must be increased by one. If less than five, the rejected figures are disregarded.

9. We know by the theory of exponents that the exponent of the product of two or more numbers may be expressed as the sum of

the exponents of the numbers, hence **for multiplication,** we have the **Rule:**

The sum of the logarithms of two or more numbers is the logarithm of their product. The anti log of this sum is the product of the numbers. Thus to find the product of 15.13 by 1.348, we take the logs:

Log of 15.13 =1.17984
Log of 1.348 = .13001

Their sum is 1.30985 =20.4104, answer.

10. **For division,** we have the **Rule:**

Subtract the logarithm of the divisor from the logarithm of the dividend, the remainder will be the logarithm of the quotient. The anti log or corresponding natural number will be the quotient.

Thus to divide 3.390 by 84.51, we have

Log 3.390 = .53020 =10.53020 − 10
Log 84.51 =1.92691 = 1.92691

8.60329 − 10 = .040114, answer.

11. Since a common fraction is merely an indicated division, **the logarithm of a fraction** is the difference found by subtracting the log of the denominator from the log of the numerator. Thus the log of $^3/_4$ =log 3 − Log 4.

12. **The logarithm of a power** of any number may be found by multiplying the log of the number by the exponent of the power to which the number is to be raised.

13. As an example, let it be required to raise 1.065 to the 20th power. By the rule (Art. 12) $(1.065)^{20}$ =20 log 1.065. Now, log 1.065 = .02735 and 20 × .02735 = .54700 =3.5237.

14. **The logarithm of the root** of a given number is the quotient found by dividing its logarithm by the index of the root.

15. As an example, let it be required to find the cube root of 625.

By the rule (Art. 14) log 625 =2.79588. The latter sum divided by three = .93196 =8.55, Answer.

16. **The co logarithm** of a number is found by subtracting the given logarithm from the log of 1, that is, from 0, written as 10 − 10. To add the co log is equivalent to deducting the log of the given number, that is, to dividing by that number.

17. The principle of Article 12 enables us to find readily the **value of an exponential equation,** that is, an equation in which the exponent of one of the numbers is an unknown quantity.

As an example, let it be assumed that the amount of a loan for a certain time at 5 per cent, is 2.65330. What is the term, the principal being 1?

Here, $(1.05)^x = 2.6533$ and $x \log 1.05 = \log 2.6533$.
And finally $x = \log 2.6533 \div \log 1.05 = .42379 \div .02119 = 20$.
Answer, 20 years.

18. Tables 1a and 1b may be used to find the logarithms and anti logs of numbers to eight places. Their use is based upon the following principles, already noticed, viz., the mantissa of the log of a number depends on the sequence of the figures and not upon the location of the decimal point. The logarithm of a number is equal to the sum of the logs of its factors. Also the log of any number less than 10 is wholly decimal and may be resolved into two factors, one of which will be between 9.99 and 1.00, and the other between 1.10 and 1.0099900. Table 1a gives the first set of factors mentioned and Table 1b the second set, carried to eight decimal places.

19. As illustrating the use of these factors to find the log of a number, let it be required to find the logarithm of the number 56947848 true to eight places. We may suppose for the purposes of the solution that the decimal point is placed after 5 and we may then find the log of 5.69 in part 1a of the table. It is on the last page of part 1a and is .75511227. Now dividing 5.6947848 by 5.69, we obtain 10008409 as the other factor. Its logarithm appears on page 1 of Table 1b, and is .00036465 for the number 1.00084. We require then the log of .0000009. The difference between 6465 under 4 and 6899 under 5 is 434, which multiplied by .09, gives .039 as the correction to the log of 1.00084. We now have as the log of both factors, the following values:

Of 5.69	= .75511227
Of 100084	= .00036465
Correction to latter	039
Total	.75547731, which is the mantissa.

The characteristic on principles already explained is 7, so that the required log is 7.75547731.

20. To find the natural number corresponding to a given logarithm, we in effect, reverse the process just employed. Thus, we first find in Table 1a the largest logarithm that can be taken from the given log. This is the log of one factor. Deduct it from the given log and the remainder is the other factor. We then "take out" the corresponding natural numbers. These are the factors. Their product is the number.

As an example, let it be required to find the number corresponding to log 4.48368759. Disregarding the characteristic for the moment, and looking into table 1a we find .48287358, the anti log of which is 3.04. Then $48368759 - 48287358 = .00081401$. Looking into Table 1b, we find as the nearest log to the latter sum, .00081137 which is the log of 1.00187. But 137 from 401 gives .00000264 as the dif-

ference between the factor log and the nearest log thereto. The difference between the logs of 1.00187 and 1.00188 is .00000434, dividing 264 by 434 gives 61 which is affixed to the number found and we have as the second factor 1.0018761, multiplying this by the other factor, 304, we have 30457033. Recurring now to the characteristic, we know that there must be five figures in the integral part so that the number sought is 30457.033.

21. From the foregoing, it is believed a working knowledge of logarithms may be realized without much difficulty by those not already familiar with their use, though the treatment of the subject has necessarily been very imperfect indeed. The use of logs is so helpful, and sometimes necessary, in the computation of the values dealt with thruout the work, that it was deemed proper to devote a little space to the subject.

CHAPTER III

Of Series

In the solution of many of the problems of finance and insurance, a knowledge of the method of developing a function into a series is helpful or necessary, but only a few of the more important principles can be noticed here.

1. An infinite series may be developed in some cases by division, by the principle of indeterminate co-efficients or by the bi-nominal theorem. Taylor's and MacLauren's theorems are also available, the former in particular being very general in its applicability to the expansion of functions.

2. The method of finite differences may be employed to extend a series or to insert intermediate terms and also to compute the sum of a series, where a sufficient number of terms are given to represent the law governing the development of the series. The subject will be briefly noticed in the following Articles:

3. Let it be required to extend the series 1, 8, 27, 64, 125, to 8 terms.

Arrange the given terms in a column and difference them indicating first, second, and so forth differences by the symbols, $\triangle$, $\triangle^2$, $\triangle^3$, and so on. We then have the following scheme:

Terms	Series	Differences		
		$\triangle$	$\triangle^2$	$\triangle^3$
1	1 +	7 +	12 +	6 + 0
2	8 +	19 +	18	6
3	27 +	37 +	24 +	6
4	64 +	61 +	30	
5	125 +	91 +	36	
6	216 +	127 +	42	
7	343 +	169		
8	512			

In the process, we first deducted each of the numbers from the one below it forming the first order of differences, 7, 19, 37 &c. Then each of these was deducted from the one below it forming the second order of differences. The numbers 7, 12, and 6 are called the leading differences and the entire series may be formed from these. Here we have the series to the 5th term. We add 6 to 24 putting 30 in the second order, then add 30 to 61, putting 91 in the first order, and then add 91 to the series, making the 6th term. The others were obtained in the same manner. The third order of differences consists of 6 and 6, that is, are constant, giving 0 as the 4th leading difference. Such

a series may be summed exactly by the methods of finite differences. And also one in which one of the higher orders of differences may be *assumed* to be constant without material error may be summed approximately.

4. Suppose it were desired to find the 15th term of the series of the last article directly. In such case, we may use the formula:

$$U_n = a + (n-1)\triangle + \frac{(n-1)\,(n-2)}{1.2}\triangle^2 + \frac{(n-1)\cdot(n-2)\,(n-3)}{1.2.3}\triangle^3 + \text{etc.}$$

in which a is used for the first term, 1. $n = 15$, the number of terms, and, as we have seen $\triangle = 7$, $\triangle^2 = 12$ and $\triangle^3 = 6$. The 4th difference we have seen is 0. Inserting these values in the formula, we have

$$1 + 14 \cdot 7 + \frac{14 \cdot 13 \cdot 12}{2} + \frac{14 \cdot 13 \cdot 12}{2 \cdot 3} \cdot 6 = 1 + 98 + 1092 + 2184 = 3375.$$

The foregoing formula is called the *formula for the nth term*, and is an important one, and should become familiar to the reader.

5. A method which is adapted to the development of a series of terms which substantially follow some law and which do not change rapidly is to obtain the values of a number of equidistant terms, such as the 5th, 10th, 15th and so forth, or the 5th, 15th, 25th, 35th and etc., then assume.

$ux = A + Bx + Cx^2 + Dx^3 + \text{etc.}$ Then, if the intervals be five, that is, $x = 5$, we have:

$u = A.$
$u5 = A + 5B + 25C + 125D$
$u10 = A + 10B + 100C + 1000D$
$u15 = A + 15B + 225C + 3375D$

By inserting the given values of u0, u5 and etc. in the several equations, we may compute the values of the co-efficients, A, B, C and etc. by differencing.

6. An instructive example of this method is given, in the Institute of Actuaries Text Book at page 435, illustrating the construction of a mortality table.

Having given $l_{10} = 2844$ $l_{30} = 2501$
$l_{20} = 2705$ $l_{40} = 2236$ to

find the intermediate values of l. Here we may assume $l_{10} = u_0$, $l_{20} = u_{10}$, $l_{30} = u_{20}$ and $l_{40} = u_{30}$ and the formula appears as follows:

$2844 = u_0 = A$
$2705 = u_{10} = A + 10B + 100C + 1000D$
$2501 = u_{20} = A + 20B + 400C + 8000D$
$2236 = u_{30} = A + 30B + 900C + 27000D$

Differencing both sides of the equations we have:

$-139 = 10B + 100C + 1000D$
$-204 = 10B + 300C + 7000D$
$-265 = 10B + 500C + 19000D$

Differencing these equations we have:

$-65 = 200C + 6000D$
$-61 = 200C + 12000D$

Differencing these equations we have:

$4 = 6000D$
$D = 4 \div 6000 = .0006\dot{6}$
$-65 = 200C + 4$
$C = -69 \div 200 = -.3450$
$-139 = 10B + -34.50 + .\dot{6}$
$B = -105.166\dot{6} \div 10 = -10.5166\dot{6}$
$A = 2844$
$1_{11} = A + B + C + D = 2844 + (-10.5166) + (-.3450) + .0006 = 2833$
$1_{12} = A + 2B + 4C + 8D = 2844 + (-21.0333) + (-1.3800) + .0053 = 2822$
$1_{13} = A + 3B + 9C + 27D = 2844 + (-31.550) + (3.1050) + .0180 = 2809$

The method of procedure is sufficiently disclosed to enable the reader to continue the series.

7. The method of Article 4 may be used to insert terms in a series by taking n **fractional** and finding the nth term in the manner illustrated in that article. The process is called **interpolation.**

8. As an example, let it be required to find the value of an annuity at $4^1/_4\%$ computed by the American Experience table of mortality, the age of the nominee being 30 years, having given the values at 3, $3^1/_2$, 4 and $4^1/_2$ per cent, respectively, as follows:

20.093, 18.605, 17.291 and 16.124.
Here $n = {}^7/_2$ and we arrange the functions and difference as follows:

Rate	Annuity	Differences		
		Δ	Δ^2	Δ^3
3	20.093	−1.488	+.174	−.027
$3^1/_2$	18.605	−1.314	+147	
4	17.291	−1.167		
$4^1/_2$	16.124			

Inserting the values found in the formula, we have:

$A30 = 20.093 + {}^5/_2 \cdot (-1.488) + {}^5/_2 \cdot {}^3/_2 \cdot {}^1/_2 \cdot 174 + {}^5/_2 \cdot {}^3/_2 \cdot {}^1/_2 \cdot {}^1/_6 \cdot (-.027)$
$20.093 - 3.720 + .326 - .008 = 16{,}691$, Answer.

9. The last method is available only where the given terms represent equidistant values of the function. The distance between the rates in the given case is $^1/_2\%$ and is counted as 1 in finding n. The nth term on this theory would be $3 + {}^1/_2 = {}^7/_2$ and $(n-1) = {}^5/_2$.

10. The formula for finding the nth term, Article 4, may sometimes be used where only a few consecutive terms and a distant term are given, to find intermediate terms. Thus, to take another example from the Text Book, we have:

$u_0 = 96779$, $u_1 = 97245$, $u_2 = 97624$, $u_9 = 100000$, to find the terms from u_3 to u_8, inclusive.

Here we have three equidistant terms from which first and second differences may be obtained, thus:

$$\begin{array}{lll} & \triangle & \triangle^2 \\ u_0 = 96779 & +466 & -87 \\ u_1 = 97245 & +379 & \\ u_2 = 97624 & & \end{array}$$

The third leading difference is required. By the formula

$$u_9 = u_0 + 9\triangle u_0 + \frac{9\cdot 8}{1\cdot 2}\triangle^2 u_0 + \frac{9\cdot 8\cdot 7}{1\cdot 2\cdot 3}\triangle^3 u_0 = \text{When simplified } u_0 + 9\triangle u_0 + 36\triangle^2 u_0 + 84\triangle^3 u_8.$$

Transposing terms, we have

$$-84\triangle^3 = u_0 + 9\triangle + 36\triangle^2 - u_9.$$

Dividing through by 84

$$\triangle^3 = \frac{u_9 - (u_0 + 9\triangle u_0 + 36\triangle^2 u_0}{84} = \frac{100000 - (96779 + 4194 - 3132)}{84} = 25.7$$

In the problem u_9 is the 10th term.

Proceeding now, as in Article 4, we have three leading differences:

$\triangle$ $\triangle^2$ $\triangle^3$

466, -87 and 25.7 and the first term 96779 from which to find the others. For the 9th term, u_8, we have:

$$96779 + 8\cdot 466 + \frac{8\cdot 7\cdot(-87)}{1\cdot 2} + \frac{8\cdot 7\cdot 6\cdot 25.7}{1\cdot 2\cdot 3} =$$

$96779 + 3728 - 2436 + 1439.2 = 99510.$

For u_7, we have $96779 + 7\cdot 466 + \frac{7\cdot 6\cdot(-87)}{2} +$

$\frac{7\cdot 6\cdot 5\cdot 25.7}{6} = 96779 + 3262 - 1827 + 899 = 99113$

For u_6, we have $96779 + 2796 - 1305 + 514 = 98784$.

For u_3, we have $96779 + .466\cdot 3 + \frac{3\cdot 2\cdot(-87)}{2} +$

$\frac{3\cdot 2\cdot 1\cdot 25.7}{1\cdot 2\cdot 3} = 97942$. And so on, and thus we may find the first ten

values of the mortality table as follows:

$l_{10} = 100000$ $\quad$ $l_{15} = 98224$
$l_{11} = 99510$ $\quad$ $l_{16} = 97942$
$l_{12} = 99113$ $\quad$ $l_{17} = 97624$
$l_{13} = 98784$ $\quad$ $l_{18} = 97245$
$l_{14} = 98496$ $\quad$ $l_{19} = 96779$

11. **Lagrange's Theorem,** may be conveniently employed where only one or two terms are required and the data to be used consists of terms which are not equidistant. The theorem is sometimes expressed as follows:

$$u_x = A(x-b)(x-c)(x-d) \ldots (x-n)$$
$$+B(x-a)(x-c)(x-d) \ldots (x-n)$$
$$+C(x-a)(x-b)(x-d) \ldots (x-n) \text{ and so on to n terms.}$$

The values of A, B, C and etc. are determined by substituting a, b, c and etc. respectively in the successive members, and simplifying the result, by which means the formula is expressed as follows:

$$\frac{u_x}{(x-a)(x-b)(x-c) \ldots (x-n)} = \frac{u_a}{(x-a)(a-b)(a-c)(a-d) \ldots (a-n)}$$
$$+\frac{u_b}{(b-a)(x-b)(b-c)(b-d) \ldots (b-n)}$$
$$+\frac{u_c}{(c-a)(c-b)(x-c)(c-d)(c-n)}$$
$$+\frac{u_d}{(d-a)(d-b)(d-c)x-d)(d-n) \& \text{etc.}}$$

12. As an example, let it be required to find the logarithm of 326, having given log 323 = 2.5092, log 325 = 2.5119, log 328 = 2.5159 and log 329 = 2.5172. We may write these values:

Log 323 = u_a = 2.5092 and a = 0
Log 325 = u_b = 2.5119 and b = 2
Log 328 = u_c = 2.5159 and c = 5
Log 329 = u_d = 2.5172 and d = 6

x, the required term, is 3. We may now place these values in the formula and solve for the required value, u_x. We have:

$$\frac{ux}{(3-0)(3-2)(3-5)(3-6)} = \frac{ux}{(3)(1)(-2)(-3)} = \frac{ux}{18}$$

$$= \begin{cases} \dfrac{ua}{(3-0)(0-2)(0-5)(0-6)} = \dfrac{ua}{(3)(-2)(-5)(-6)} = \dfrac{-ua}{180} \\ \dfrac{ub}{(2-0)(3-2)(2-5)(2-6)} = \dfrac{ub}{(2)(1)(-3)(-4)} = \dfrac{ub}{24} \\ \dfrac{uc}{(5-0)(5-2)(3-5)(5-6)} = \dfrac{uc}{(5)(3)(-2)(-1)} = \dfrac{uc}{30} \\ \dfrac{ud}{(6-0)(6-2)(6-5)(3-6)} = \dfrac{ud}{(6)(4)(1)(-3)} = \dfrac{-ud}{72} \end{cases}$$

We may now write the derived values as an equation, thus:

$$\frac{ux}{18} = \frac{-ua}{180} + \frac{ub}{24} + \frac{uc}{30} + \frac{-ud}{72}$$

Reducing these fractions to their least common denominator, which is 360, and clearing of fractions, we have:

$20ux = -2ua + 15ub + 12uc(-5ud)$ and solving

$-2ua = 2.5092 \times 2 = -\ 5.0184$

$-5ud = 2.5172 \times 5 = -12.5860$

-17.6044

$15ub = 2.5119 \times 15 = 37.6785$

$12uc = 2.5159 \times 12 = 30.1908$

67.8693

Uniting these sums algebraically, we have:

$20ux = 67.8693 - 17.6044 = 50.2649.$

Dividing through by 20, we have:

$ux = 2.5132$, which is the required logarithm.

13. In this formula, x is regarded as the unknown term, and in the example given is the 3rd term from 323 which is called ua. If the log of 327 had been required, x would be 4 and the solution would have had the same form, 4 taking the place of 3 throughout. In writing the denominator of ux in the formula each of the given values is deducted from x in succession. Thus (x−a), (x−b) and so on. In writing the denominators of the given factors ua, ub & etc., the same course is followed, that is, each given factor is deducted. Thus, to begin with a, we have (a−b), (a−c), (a−d) and (a−a). The latter is changed in producing the formula to (x−a) and is so written. Also,

with b, we have (b−a), (b−b), written (x−b), (b−c) and (b−d) as the factors of the denominators of ub, and so with c and d. The peculiar value of this formula lies in the fact that equidistant terms are not essential in order to find the unknown term.

14. The method of finite differences may be used also to find the **sum of a series** where the series follows some law of development and the differences vanish after a few orders or when the series is convergent. In the first case, the sum is an exactly, and in the last, εn approximately, correct summation, sufficiently exact for some purposes. The formula for this method is,

$$\text{Sum or } S = na + \frac{n(n-1)}{2}\triangle + \frac{n(n-1)(n-2)}{\lfloor 3}\triangle^2 + \text{etc.}$$

15. Let it be required to find the sum of the series:

1+5+15+35+70+126+ to 30 terms by the method of differences.

Arrange the terms and differences as follows:

Series	$\triangle^1$	$\triangle^2$	$\triangle^3$	$\triangle^4$
1	+ 4	+ 6	+ 4	+ 1
5	+10	+10	+ 5	+ 1
15	+20	+15	+ 6	
35	+35	+21		
70	+56			
126				

Inserting these differences in the formula, we have:

$$30.1 + \frac{30\cdot29\cdot4}{2} + \frac{30\cdot29\cdot28\cdot6}{1\cdot2\cdot3} + \frac{30\cdot29\cdot28\cdot27\cdot4}{1\cdot2\cdot3\cdot4} + \frac{30\cdot29\cdot28\cdot27\cdot26}{1\cdot2\cdot3\cdot4\cdot5} =$$

278256, Answer.

In this example, the 4th differences are constant and the summation exact.

16. **Lubbock's Formula,** for the approximate summation of a series of equidistant terms, may be employed when a single annuity value, ax, or insurance premium, Ax, is required.

For Insurances, this formula may be conveniently written.

$$Ax = n(u_0+u_1+u_2+\text{etc.}) - \frac{n-1}{2}u_0 + \frac{n^2-1}{12n}\triangle - \frac{n^2-1}{24n}\triangle^2 + \frac{(n^2-1)(19n^2-1)}{720n^3}\triangle^3 - \frac{(n^2-1)(9n^2-1)}{480n^3}$$

in which Ax is the present value of the insurance.

For Annuities, it may be written

$$ax = n(u_0+u_1+u_2+\text{etc}) \frac{n+1}{2} + \frac{n^2-1}{12n}\triangle - \frac{n^2-1}{24n} + \text{terms as above.}$$

17. The values u_0, u_1 and etc., in the case of insurances are represented by the general formula, $ux+n=\frac{v^{n+1}\cdot dx+n}{1x}$ For annuities, they are represented by the general formula $un=\frac{v^n 1x+n.}{1x}$ The formula is an approximation in any event and third or fourth differences are sufficient for practical purposes. The co-efficients of the first, second, third and fourth differences are $(-.4)$, $(.2)$, $(-.1264)$ and .0896, respectively, when $n=5$, and are $(-.825)$, .4125, (-2611) and .1854 respectively when $n=10$. Usually the result obtained by computing the value of u at intervals of ten years will be close enough.

18. Let it be required to compute the approximate value, single premium, for an insurance of \$1000 at age 35 on the American Experience table and $4^1/_2\%$ interest. Here there may be six periods of ten years each from age 35 to the end of the table. First compute the values of an insurance of 1 payable at death during one year at each of these periods. First, we have $d_{35}v=732\times v=.956938\div(l_{35}=81822)=.008560$. Second, we have $(d_{45}=828)\times(v^{11}=.616199)\div(l_{35}=81822)=.006326$, and so on to six terms. These results may then be arranged for differencing and summation as follows:

	△	△²	△³	△⁴	
$u_0=$	.008561	−002325	+001903	−001112	−001252
$u_1=$	.006236	−000422	+000791	−002364	
$u_2=$	.005814	+000369	−001573		
$u_3=$	.006183	−001204			
			$\frac{-n-1}{2}\times .008561=$		$-.038525$
$u_4=$	.004979		$\frac{n^2-1}{12n}\times -.002325=$		$-.001918$
$u_5=$	.001673		$\frac{-n^2-1}{24n}\times .001903=$		$-.000785$
$u_6=$	.000003		$\frac{(n^2-1)(19n^2-1)}{720n^3}\times -.001112=$		$-.000290$
	.033449		$\frac{-(n^2-1)(9n^2-1)}{480n^3}\times -.001252=$		.+000232
	10				$-.041286$
	.334490				
	−.041285				

$.293204\times 1000=\$293.20$, Answer.

19. A number of formulas for the computation of special values of insurances or annuities are available, but they are not as a rule adapted to computations on the tables legally adopted in this country and the ones given here may be employed by the use of a little care and are deemed sufficient for this work. More occasions will be found for the use of Lubbock's formula by those for whom this book is designed, in the computation of annuities in connection with joint lives and this application of the formula will be further elucidated in treating that subject. Those unfamiliar with insurance notation might omit Articles 17 and 18 until after having read the chapters on annuities and insurance.

CHAPTER IV

Probability

The doctrines of probability have frequent application in the solution of problems depending upon life contingencies, and this chapter will be devoted to explaining and illustrating some of the principles of probability.

1. The **Mathematical Probability** of the happening of an event has been defined as the number of favorable opportunities divided by the whole number of opportunities, favorable and unfavorable.

2. Suppose, in common language, the odds are 3 to 2 against the happening of an event. Here the event may occur in two ways, the sum of the ways being 5. The probability of the happening of the event is two of these, or, $^2/_5$, and the chance of failure is $^3/_5$. The sum of these probabilities is $^2/_5 + ^3/_5 = 1$, unity or certainty, since the event must either happen or fail.

3. The principle may be extended to include any number of ways in which a single event may happen when one and only one can happen. Thus, suppose there are three applicants for a position and that A may obtain it in 2 ways, B in 3 ways and C in 4 ways, all of which are equally likely to occur. Here, there are 9 ways in which the event may be brought to pass. A's chance is therefore $^2/_9$ and his probability of failure $^7/_9$. That is, the odds are 7 to 2 against him. B's chance is $^3/_9$ and C's $^4/_9$. The sum of the chances is 1.

4. A familiar example illustrative of this subject is to suppose, say, 3 white balls, 4 black balls and 5 red balls to be mixed in a bag from which balls are to be drawn at random and the chances determined. There being twelve balls in all and three possibilities that the first ball drawn may be white, the probability of drawing a white ball is $^3/_{12}$. After the first ball is drawn, if it be a white one, the probability of again drawing a white ball is $^2/_{11}$. In the same way, it appears that the probability that the first ball drawn will be black is $^4/_{12}$, and that it will be red, $^5/_{12}$.

5. Suppose it is desired to draw out the balls in pairs; what would be the probabilities in the several cases?

On the principles of permutations and combinations, it may be shown that the number of combinations of m things taken n at a time is $\frac{\lfloor m}{\lfloor m-n \ \lfloor n}$. Therefore, the number of pairs which may be formed from 12 things taken 2 at a time is $\frac{12 \cdot 11}{1 \cdot 2} = 66$. The number of

pairs that can be formed of the three white balls taken 2 and 2 is $\frac{3 \cdot 2}{1 \cdot 2} = 3$;

of four black balls $\frac{4 \cdot 3}{1 \cdot 2} = 6$, and of five red balls, $\frac{5 \cdot 4}{1 \cdot 2} = 10$. The

probability of drawing two white balls is therefore $^3/_{66}$, which is also the product of the probabilities of drawing two white balls in succession which we have seen is $^3/_{12}$ and $^2/_{11}$. The probability of drawing two red balls is $^{10}/_{66}$, which is likewise the product of drawing them in succession. That is $^5/_{12} \cdot {}^4/_{11} = {}^{10}/_{66}$.

6. Again, as the three white balls may each be paired with a black or a red ball, there may be formed 12 pairs consisting of white and black balls, 15 pairs consisting of white and red balls and 20 pairs consisting of black and red balls. From these may be derived the probabilities of drawing pairs, consisting of balls of the different colors. Thus, the probability of drawing a white and black pair is $^{12}/_{66}$, a white and red pair, $^{15}/_{66}$, and a black and red pair, $^{20}/_{66}$.

7. If we know the respective probabilities of the happening of two or more independent events, we may find the probability that *all* will happen **by multiplying together the several probabilities.**

8. When the events are dependent, the probability that *all* will happen is the product of the probability of the first event and the probability that when the first has happened, the second will follow.

9. Thus, in the example, the probability in the first instance of drawing a white ball was $^3/_{12}$. Supposing the ball to be replaced, the probability of drawing a white ball would again be $^3/_{12}$. The probability that a white ball would be drawn in both instances is $(^3/_{12})^2$, or $^1/_{16}$. But we have seen that the probability of drawing the first is $^3/_{12}$ and the second, without replacing the first, $^2/_{11}$. The probability of drawing a white ball in both cases is $^3/_{12} \cdot {}^2/_{11}$, or $^3/_{66}$. In the first instance, we deal with independent, and in the second, dependent, probabilities.

10. The probability of throwing an ace with a single dice is $^1/_6$; the probability of failing, $1 - {}^1/_6 = {}^5/_6$. The probability of throwing an ace twice in two throws is $(^1/_6)^2 = {}^1/_{36}$. The probability of failing twice in two throws is $(^5/_6)^2 = {}^{25}/_{36}$, and the probability of not failing twice, that is, of succeeding at least once is $1 - {}^{25}/_{36} = {}^{11}/_{36}$. The probability of throwing exactly n aces in a given number of throws will be noticed in the next article.

11. Let p be the probability of succeeding, q the probability of failing. Now let it be required to find the probability of throwing an ace exactly three times in, say, 4 throws. By the principles of Articles 4, 5 and 10, we may derive the following formula:

$P = \frac{n(n-1)\,(n-r+1)}{\lfloor r} p^r q^{n-r}$ which is general. In the present case, $n=4$ and $r=3$, so that the formula becomes numerically $\frac{4\cdot3\cdot2}{1\cdot2\cdot3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)=\frac{5}{324}$

12. Suppose it is desired to find in what number of trials the chance of throwing an ace becomes $^1/_2$, that is, an even wager. Let $x =$ the number of trials. The probability of failing on each trial is $^5/_6$ and that of failing x times, that is, every time $(^5/_6)^x$ and the probability of succeeding is $1-(^5/_6)^x$. This, by the conditions of the problem, must equal $^1/_2$, that is, $1-(^5/_6)^x = {}^1/_2$. Transposing and uniting terms, we have $(^5/_6)^x = {}^1/_2$. Also $x \log {}^5/_6 = \log {}^1/_2$.

And $x = \frac{\log {}^1/_2}{\log {}^5/_6} = \frac{\log 1 - \log 2}{\log 5 - \log 6} = \frac{.3010}{.6999 - .7781} = \frac{.3010}{.0782} = 3.85,$

which means that the chances of winning are a little better than one half in four throws.

13. By a certain mortality table, 89685 persons are living at age 30; at age 40, 82277 of them survive. The probability of one of the original number living ten years, supposing their chances of living to be equal, is the quotient found by dividing the latter number by the first, thus, $82277 \div .89685 = .9397$. At age 40, the probability of living ten years would be $72795 \div 82277 = .8848$.

14. The probability of dying in ten years is the quotient found by dividing the number dying during the term by the number living at the beginning, thus, $7408 \div 89685 = .0837$.

15. The probability of x, aged 30, and y, aged 40, living ten years is the product of the probabilities that each will live ten years, thus, $.9397 \times .8848 = .83144$.

16. The probability that y will live and x die in ten years is likewise the product of the separate probabilities, thus, $.0837 \times .8448 = .074057$.

17. The probability that x will live ten years and die within the next ten years is the product of the probability of living the first ten years into the probability of his dying within the second ten years, thus, $.9397 \times .11512 = .10816$.

18. The following notation will be extensively used in the succeeding pages and should be remembered. (x) denotes a person aged x years, (y) a person aged y years; (l) denotes the number living according to a given mortality table, lx is therefore the number of persons living of a given age. So of ly, lz, & etc. (p) denotes the probability of living and unless modified by the use of other symbols, means the probability

of living one year. (q) denotes the probability of dying and likewise involves only one year unless modified by other symbols. If n years are involved, the symbols are written npx or nqx, respectively. (p) also is to denote the probability of the occurrence of any event and (q) the probability of its not occurring.

19. On principles already noticed, $1-p=q$, and $1-q=p$, and $p+q=1$. The probability of two independent events happening is $p_1 \times p_2$ or $p_1 p_2$. The probability of failure is $q_1 \times q_2$ or $q_1 q_2$. The probability of one person living and another dying is $p \cdot q$. If the term be made n years, the probabilities are $npx \times nqy$. (Art. 16.) The same principles apply to larger numbers of persons.

20. The joint probability of living is $\overline{pxy}$, meaning one year. If the term be n years, the form for the probability is $n\overline{pxy}$, which is equal to $npx \times npy$ or in terms of a mortality table, $\frac{lx+n}{lx} \times \frac{ly+n}{ly}$.

21. The probability that both will die in n years is $|n\overline{qxy}$ ($nqx \times nqy$) or in terms of the mortality table, $\frac{lx-lx+n}{lx} \times \frac{ly-ly+n}{ly}$

22. The probability that one, but not both, **exactly one,** of two persons will die in n years, is the sum of the probability that x will die and y will live and the probability that y will die and x live. That is, $(nqx \times npy)+(nqy \times npx)$. In terms of the mortality table, this is

$$\left(\frac{lx-lx+n}{lx} \times \frac{ly+n}{ly}\right)+\left(\frac{ly-ly+n}{ly} \times \frac{lx+n}{lx}\right)$$

23. The probability that **at least one** of two persons will survive n years is the probability that *both* will not die in that time. That is, $1-|n\overline{qxy}=npx+npy-npxy$.

24. The probability that x will die in any certain year is the product of the probability that he will survive the intervening term and the probability that he will die in the succeeding year. Thus the probability that x, aged 30, will die in his 40th year is in terms of the mortality table, n being 10.

$$\frac{l_{39}}{l_{30}} \times \frac{l_{39}-l_{40}}{l_{30}}$$

25. The probability that one of two lives will fail in the nth year is the probability that both will survive $n-1$ years, less the probability that both will survive n years. In terms of the mortality table

$$\frac{l_{39} \times l_{49}}{l_{30} \times l_{40}} - \frac{l_{40} \times l_{50}}{l_{30} \times l_{40}}$$

The foregoing is sufficient to illustrate most of the processes employed in this book in the solution of problems involving the element of chance or probability.

CHAPTER V

Compound Interest

1. When interest is added to the principal and interest computed upon the amount as a new principal, the interest is called **Compound Interest.** Most of the operations in finance involve compound interest.

2. Tables of **Compound Interest** computed on a principal of one for a term of sixty periods at rates ranging from $^{6}/_{8}$ of one per cent to ten per cent and for one hundred and twenty periods at rates ranging from one-fourth of one per cent to one-half of one per cent are published in this book, the results given being the amounts of one improved at the given rates.

3. The Compound Interest upon a given principal for a given rate is found by multiplying the given principal by one plus the rate for one period and this product by the same multiplier, and so continue for each period in the term. From the last product, deduct the principal. The remainder will be the compound interest.

4. If we represent the rate by the letter i, the principal by P, the term or number of periods by n, the amount of principal and interest by S and the compound interest by I, we may from Section 3, develop some useful formulae.

5. Thus, we notice that the amount for the first period is $1+i$; that for the second period is $(1+i)(1+i)=(1+i)^2$; that for the third is $(1+i)^2(1+i)$ or $(1+i)^3$ and generally the law of the accumulation being apparent, $S=(1+i)^n$. (1)

6. From Article 5, it will be apparent that logarithms afford a very rapid and accurate means of producing a table of amounts at compound interest. Thus, in computing, for instance, the compound amounts of 1 at one-fourth of one per cent, it was only necessary to put on the adding machine the log of 1.0025 to eight places of decimals and add continuously to 120 places, recording the amount at each pull of the handle. The anti logs to six places is the table desired.

7. Transformations of the equation $S=(1+i)^n$ will enable us to find the other factors involved in the computation, thus:

$S=(1+i)^n$, $\text{Log } S=n \log (1+i)$ and

$$n=\frac{\log S}{\log (1+i)} \qquad (2)$$

Again, $S=(1+i)^n$, $\sqrt[n]{S}=1+i$ and

$$i=\sqrt[n]{S}-1=\frac{\text{Log } S}{n}-1 \qquad (3)$$

8. If instead of 1, we introduce the principal P, formulas 1, 2 and 3 become respectively, $S = P(1+i)^n$ (1)

$$n = \frac{\log S - \log P}{\log (1+i)} \quad (2)$$

$$i = \sqrt[n]{\frac{S}{P}} - 1 = \frac{\text{Log } S - \text{Log } P}{n} - 1 \quad (3)$$

9. From the equation $S = P(1+i)^n$, the value of P is derived as follows:

$$S = P(1+i)^n \text{ and } P = S \div (1+i)^n = S \frac{1}{(1+i)^n} \text{ also } P = S\, v^n \quad (4)$$

10. A numerical example of the applications of each of the four equations, just derived will sufficiently illustrate their meaning. Let $P = 500$, $n = 8$, $i = .05$. First, to find S, we have $1+i = 1.05$

$1 \times (1+i) = 1.05$ $\quad 1.05 = (1+i)$ (1)

(1) multiplied by 1.05 = .1525

$\quad 1.1025 = (1+i)^2$ (2)

(2) multiplied by 1.05 = 55125

$\quad 1.157625 = (1+i)^3$ (3)

(3) multiplied by 1.05 = 57881

$\quad 1.215506 = (1+i)^4$ (4)

(4) multiplied by 1.05 = 60775

$\quad 1.276281 = (1+i)^5$ (5)

(5) multiplied by 1.05 = 63814

$\quad 1.340096 = (1+i)^6$ (6)

(6) multiplied by 1.05 = 67005

$\quad 1.407101 = (1+i)^7$ (7)

(7) multiplied by 1.05 = 70355

$\quad 1.477456 = (1+i)^8$ (8)

(8) multiplied by 500 $= 738.728 = P(1+i)^8$

Again from Art. 8, $n = \frac{\text{Log } S - \text{Log } P}{\text{Log } (1+i)}$ and from the table of logs we have Log 738.73 = 2.868486, Log 500 = 2.698970, Log 1.05 = .021189

and $\frac{2.868486 - 2.698970}{.021189} = 8$. Hence $n = 8$ years or periods.

Also $i = \frac{\log S - \log P}{n} - 1$ and from the figures just found we have log

P − log S = .169516, which divided by 8, gives .021189, the log of 1.05. Subtracting 1, we have i = .05, which we know to be correct.

Finally, $p = S\frac{1}{(1+i)^n}$ or $S\ v^n$. From Art. 10, we already have $(1+i)^n = 1.477456$. Dividing 1 by the letter sum using the contracted method, thus:

1.477456)1.0000000(.676839
8864736
1135264
1034219
101045
88647
12398
11819
579
443
136
132

This quotient is the function v^n and is the present value of 1 due in 8 years, money being worth five per cent.

Multiplying S or 738.728 by .676839, using the contracted method, we have P, thus:

738.728
938.676
4432 368
517 110
44 324
5 910
222
66
500.0000

11. While the unit of time in quoting rates of interest is usually considered to be one year, it is convenient to regard the tabulated values as referring simply to periods. By this means, the tables may be used to compute the amounts where the interest is payable semi-annually, quarterly or at other fractional periods of a year. Thus, had the problem of Article 10 prescribed interest payable semi-annually a term of 8 years would have comprised 16 periods and the table of compound amounts at $2^1/_2$ per cent should be used in the computation. Or if direct computation be resorted to, the formula used would be

$$S = P(1+i/_2)^{2n} \qquad (5)$$

12. It is proper to explain that the results reached in the two cases are not identical for the reason that by formula (5), one-half of the annual interest is paid each year six months before the end of the

year and is compounded at one-half the annual rate so that the lender actually receives 5.063 per cent interest on his principal. It is believed to be the general practice, however, to compute the interest as stated in Article 11 and if the transaction is understood by the parties, no injustice is done and the great inconvenience which would be occasioned by having to ascertain and base all calculations upon the effective rates is thereby avoided.

13. The following short tables of nominal and effective rates are taken from Mr. George King's book, Theory of Finance. In the tables this effective rate is designated as i and the nominal rate as j, in accordance with the notation used by Mr. King.

1 Nominal Rate j	2 Effective Rate i Semi-Annual	3 Effective Rate Quarterly	4 Effective Rate i	5 Nominal Rate Semi-Annual	6 Nominal Rate Quarterly
.025	.025156	.125236	.025	.024846	.024369
.03	.030225	.030339	.03	.029778	.029668
.035	.035306	.035462	.035	.034699	.034550
.04	.040400	.040604	.04	.039608	.039414
.045	.045506	.045765	.045	.044505	.044260
.05	.050625	.050945	.05	.049390	.049089
.055	.055756	.056145	.055	.054264	.053901
.06	.060900	.061364	.06	.059126	.058695
.065	.066056	.066602	.065	.063976	.063473
.07	.071225	.071859	.07	.068816	.068234
.08	.081600	.082432	.08	.078461	.077706
.09	.092025	.093083	.09	.088061	.087113
.10	.102500	.103813	.10	.097618	.096455

14. Illustrating the use that may be made of the tables of Article 13, let us suppose that the rate quoted is 4 per cent. If interest is made payable semi-annually (by Column 2), the principal will earn 4.04 per cent per annum, and, if payable quarterly (by Column 3) it will earn 4.0604 per cent per annum. Again, if it is desired to earn exactly four per cent per annum, the rate which must be quoted payable semi-annually is (by Column 5) 3.9608 per cent and the rate which must be quoted payable quarterly is (by Column 6) 3.9414 per cent.

15. The effective rate may be derived from the nominal rate by formula (5), n being taken as one year and the principal as 1, thus

$$i = (1 + {}^{i}/_{p})^{p} \qquad (5a)$$

Where p is the number of times interest is convertable in one year.

16. The nominal rate, or rate to be quoted, payable semi-annually, quarterly or at some other fractional part of a year may be computed by the following formula:

$$j = p(1+i)^{1}/_{p} - 1 \text{ or } p\sqrt[p]{1+i} - 1 \qquad (6)$$

17. Thus, suppose it is desired to realize 5 per cent per annum upon a rate to be quoted payable semi-annually. Applying formula 6, we have:

$2(\sqrt{1.05}-1)=j$. The square root of 1.05 is 1.024695. Subtracting 1 and multiplying by 2 we obtain .04939, which is the rate given in the table. If a rate payable quarterly is required, the 4th root of one plus the effective rate is taken, one subtracted and the remainder multiplied by four. The fourth root may be found by taking the square root of the square root, or logarithms may be employed.

18. It is sometimes desirable to know what rate of interest per annum will be realized upon a bond or note bearing interest payable annually, semi-annually or quarterly, where it is bought at a discount or premium. In such case, the amount on the principal named in the instrument is found at the prescribed rate for the term. Then the price paid for the bond or note may be taken as the principal for the purpose of finding the rate, and formula (3) is used in the solution, thus:

19. Let it be required to find what rate will be realized upon a bond of $1000.00, bearing six per cent interest payable semi-annually and running ten years, for which $1100.00 is paid. First, compute the amount on the principal, $1000.00, for 20 periods at 3 per cent, or from the table, No. 2, we find it to be $1806.00. We now assume the latter sum to be the amount of an investment of $1100.00 at the end of ten years, and for the purposes of the formula $S=1806.11$, $P=1100$ and $n=10$. Log $S=3.2567442$, Log $P=3.0413827$ and Log $S-\text{Log } P\div 10=.0215352=1.05087$. Deducting 1, we have the rate 5.09 per cent.

20. Conversely, it is often desirable to know what sum may be paid for a bond or note to realize a given rate per cent on the investment. In this case, the problem is to find the principal, the amount being known, or ascertainable, from the terms of the bond. Formulas 1 and 4 may be employed, but the factor v^n is computed at the rate intended to be realized, thus:

21. Let it be required to find what price may be offered for the bond described in Article 19 so that seven per cent per annum may be realized on the investment. First, the amount of the bond at the end of the ten years we have found is $1806.11. v^{10} computed at the rate of 7 per cent in the manner illustrated in Article 10, or also as given in the table, is .508349. In the formula $P=Sv^n$, the first number 1806.11 corresponds to S and the latter, .508349 to v^n and their product $918.13 is P. The latter sum therefore is the price which may be paid.

22. Formula 2 provides a method by means of which we may find in what time a principal will double or otherwise multiply itself at a given rate per cent, thus to find in what time money will double itself in the formula let $2P=S$; to quadruple itself, $4P=S$, and so on.

23. Let it be required to find in what time a debt of one dollar will amount to two dollars at three and one-half per cent compound interest. Let $S=2$, $P=1$ and $i=.035$. Log $S=.30103$, Log $P=0$, Log $1.035=.014940$. Then

$$\frac{\text{Log } S - \text{Log } P}{\text{Log } (1+i)} = 20.1 \text{ years.}$$

The quotient is taken as a natural number, not a logarithm.

24. It may be easily shown that $\frac{69}{i}+.35$ is a reliable approximate formula for finding the time in which money doubles itself at compound interest. That is, divide 69 by the given rate and add .35 to the quotient.

25. If S be taken as 1, then P, according to formula 4, will be $\frac{1}{(1+i)^n}=v^n$, or if the term be one year, the first number becomes $\frac{1}{1+i}$ which is called simply v. This is the present value of 1 for one year. v^n is the present value of 1 for n years. The process of finding the present value of 1 is called **discounting**. This present value corresponds to P representing the principal in the operations of compound interest and its value is the reciprocal of S, that is, of the amount of the principal and interest.

26. A few numerical examples of the principles of Article 25 will help to elucidate the subject. If we assume $S=1$, $i=5$ and $n=1$, then $v=1\div 1.05=.952381$. If interest be computed on .952381 at 5%, the amount will be 1. Hence, .952381 is the present value of 1 at 5%. If the present value be deducted from 1, we have .047619. This is called **discount** and is usually designated by the symbol d. If we change n in the first example to say, 20, then by formula 4, we have the equation $P\frac{1}{(1+i)^{20}}=1\div(1.05)^{20}$. The latter quantity will be recognized as the amount of 1 in 20 years at 5 per cent, and by the table is 2.653298. Performing the indicated division, we have $v^{20}=.376889$. Subtracting the latter sum from 1, we have .623111 which may be appropriately called the **compound discount** on 1 in 20 years at 5 per cent. From the foregoing, the remark is justified that v and P are equivalents in that they are the present values of an amount. The following additional formulas may be drawn from the foregoing discussion:

$$v = \frac{1}{1+i} \tag{7}$$

$$d = 1 - v \tag{8}$$

$$v^n = \frac{1}{(1+i)^n} \text{ or } (1+i)^{-n} \tag{9}$$

27. It has been noticed that the denominators in the right numbers of equations 7 and 9 are compound interest amounts. They are therefore powers of $1+i$ and since they increase as the ratio, $1+i$, is raised to higher powers, the numerator remaining the same, the function v^n decreases, thus:

$\frac{1}{(1+i)^n} \times \frac{1}{(1+i)} = \frac{1}{(1+i)^{n+1}}$, or, $v^n \times v = v^{n+1}$. On the other hand,

$\frac{1}{(1+i)^n} \times (1+i) = \frac{1}{(1+i)^{n-1}}$ or v^{n-1}.

We may therefore make a table of v^n by either of the two methods but the latter would be simpler, since $1+i$ is an easier multiplier than v. As in the case of compound interest, logarithms afford the best means of producing a table of v^n and such was the method used in computing the tables of this book. First, find $(1+i)^{120}$ at $^1/_4\%$. Take its log and deduct from the log of 1, which is 0 or $10-10$, using eight places of decimals. Put this remainder on the adding machine and add successively the log of 1.0025, recording the sum at each pull of the handle. The anti logs to six places will be the result sought.

28. For short terms, it is practicable to make the table by actual multiplication. Thus for v^{20} at say, 5%, we may begin with $v = .952381$ and multiply successively by .952381 to 20 periods. The contracted method will be best. We may compute the amount of 1 at compound interest at 5 per cent, to twenty periods, divide it into 1 and use 1.05 as a multiplier. We have $1 \div 2.653298 = .376889 = v^{20}$. Then,

$$\begin{array}{rcl} & .376889 & = v^{20} \\ & 18845 & \\ \hline v^{20} \times 1.05 = & .395734 & = v^{19} \\ & 19787 & \\ \hline v^{19} \times 1.05 = & .415521 & = v^{18} \\ & 20776 & \\ \hline v^{18} \times 1.05 = & .436297 & = v^{17} \end{array}$$

and so on, it being only necessary to use 5 in the actual multiplication being careful to carry into the right figure of the product the proper number from the two rejected figures. Of course, if a compound interest table is available, the v^n table can be made directly from that by formula (9).

29. Conversely, a compound interest table may be derived directly from a table of v^n. Thus, from the equation, $v^n = \frac{1}{(1+i)^n}$ we have

$v^n(1+i)^n = 1$ and finally,

$$(1+i)^n = \frac{1}{v^n} \tag{10}$$

By formula 10, therefore, we may find at once the compound amount on any number by simply taking the reciprocal of the present value of the same number for the same time and rate of interest.

30. A compound interest table may be used to find the compound interest for a term extending beyond the term of the table on the following principle: The amount of 1 for n periods, by equation (1), is $(1+i)^n$. Likewise, the amount of 1 for m periods is $(1+i)^m$, the exponents being considered general. And by the theory of exponents, $(1+i)^{m+n} = (1+i)^m \times (1+i)^n$. Therefore, if it be desired to find the compound interest on 1 at say 3% for, say 110 years, we have simply to multiply the amounts for fifty and sixty years together, thus:

```
  4.383906
  3 061985
-----------
 21 919530
  3 507125
    394551
      4384
      2630
        13
-----------
 25.28233
```

In the multiplication, for convenience, the figures of the multiplier are reversed. To find the interest, deduct 1 from the product. If the principal be other than one, multiply the result for 1 by the principal.

31. The table of v^n is governed by the same principles and may be extended in exactly the same way. An example is not necessary.

32. We have seen, Article 11, that $S = P\,(1+i/m)^{mn}$ represents the compound amount of 1 for n years payable fractionally. If the term be made one year and the principal 1, then the interest for one

year would be represented by the expression $(1+i/m)^m-1$. The quantity, m, may be supposed to increase without limit, in which case, the interest would be treated as accruing continuously or momently. Now, if we suppose m, in the expression $(1+i/m)^m$ to increase without limit, the expression by the theory of logarithms has e^i for its limit, e being the base of the Napierian system of logarithms and i the nominal rate of interest. The effective rate then becomes e^i-1 from which we have the formula $\bar{i}=e^i-1$. (11)

Thus, to find the effective rate of interest convertible momently the nominal rate being, say $4^1/_2\%$, we have $e^i=4^1/_2$ log. $2.71828=(.045\times.43294)=.0195432$. The anti log of the last quantity is 1.0460277, from which we deduct 1 according to formula 11, leaving as the effective rate 4.6028 per cent.

33. Again, by simple transformations of formula 10 we derive a formula for the nominal rate, sometimes called the force of interest represented by the symbol δ or d′, thus:

$\bar{i}=e^i-1$. $e^i=\bar{i}+1$ or i into log e $=\log \bar{i}+1$ and finally,

$$d' \text{ or } \delta=\frac{\log (i+1)}{\log\ ^e}=\frac{\log (1+\bar{i})}{.43294} \qquad (12)$$

It may be proper to remind the reader that as used in the last two articles, 2.71828 is "e" of the Napierian System of logs and .43294 is the modulus of the common system.

34. In the same way the value of v and d′ may be found when the discounting is regarded as progressing continuously or momently and we have: $v=e^{-d'}$ (13)

35. An approximate rule for finding the **force of discount** or force of interest is to take one-half the sum of the effective rates of interest and discount, the result will be the continuous or momently rate. Thus,

$$d'=\frac{i+d}{2} \qquad (14)$$

This formula is accurate enough at all usual rates and dispenses with the use of logarithms where interest and discount tables are available. For instance, to find d′ at the rate, $4^1/_2\%$, we have from the v^n table $v=.956938$ and $d=1-.956938=.043062$ and $(.045+.043062)\div 2=.04403$.

CHAPTER VI

Of Annuities Certain

1. An **annuity certain** is a fixed sum payable at the end of each period of a fixed term. It is distinguished from a life annuity by the fact that the term of the latter is fixed or affected by the duration of a given life or certain lives. Ordinarily, the payments of an annuity are due at the end of the periods, usually one year, but they may be contracted to fall due at the beginning of the year or period. Such an annuity is called an **annuity due,** and its symbol is the ordinary letter a.

2. If 1 be made payable at the end of one year its value now, as we saw in last chapter, is v. If it be payable in two years, its present value is v^2; in three years, v^3, and so on. That is, the values are

$$\frac{1}{1+i}, \quad \frac{1}{(1+i)^2}, \quad \frac{1}{(1+i)^3} \text{ and so on.}$$

From this, we see that the present value of 1 a year for 2, 3 or n, that is, any number of years, is $v+v^2+v^3+ \ * \ * \ +v^n$. Such a sum is the present value of an annuity of one for n years, and is often represented by the symbol $a\overline{n}|$, the character $\overline{n}|$ indicating any fixed or certain term of years or periods, and the letter *a* being printed in italics.

3. On the principles of Article 2, a table of the present values of 1 payable in a series of years or periods may be made by summing the values of v, v^2, v^3 and so forth, beginning with the value v and recording the sums at each step. Thus to produce a table $a\overline{n}|$ at six per cent, we may first write a line of v^n and then sum across the page thus:

	v^n	$a\overline{n}|$
(1)	.943396	.943396
(2)	.889996	1.833392
(3)	.839619	2.673011
(4)	.792094	3.465105

adding (1) of the second line to (2) of the first and so on.

4. The value of any particular annuity may be found directly by the formula

$$a\ \overline{n}| = \frac{1-v^n}{i} \tag{15}$$

Referring to Article 2, it will be noticed that $a\ \overline{n}|$ is the sum of a geometrical series of which $\frac{1}{1+i}$ is the first term, $\frac{1}{(1+i)^2}$ the second and $\frac{1}{(1+i)^n}$ the last. Also that the ratio is $\frac{1}{1+i}$.

The value $\frac{1-\frac{1}{(1+i)^n}}{i}$ or $\frac{1-v^n}{i}$, its equivalent, will at once be recognized as the sum of this geometrical series. Its reasonableness will also be appreciated if we consider that if 1 be invested for a term of n years it will produce i interest each year and the original capital, 1, will remain to be returned to the investor, the two amounting to $(1+i)^n$. Now it is apparent that 1 is the present value of the amount $(1+i)^n$ because it is the value that produced it. Also, we know that v^n is the present value of 1 in n years and if we deduct v^n from 1, the remainder will be the present value of an annual payment of i for n years.

Then, by proportion, i:1 as $1-v^n$: $\frac{1-v^n}{i}$

5. We saw in the last chapter, Articles 25 and 26, that v^n is the reciprocal of $(1+i)^n$. This fact gives us a hint as to the proper method of dealing with annuities having fractional payments and based on interest convertible at fractional periods of a year. Thus, if $(1+i)^{-n}=v^n$, $(1+i/p)^{-np}$, also equals v^{np} or $v^{n}_{(p)}$ as it is more correctly written.

6. We may now write down formulae expressing in a concise way the value of annuities payable fractionally and based on interest compounded at fractional periods of the year. Let m be the payment periods, p the number of times money is convertible each year and n the term of the annuity in years. We then have the general expression:

$$a\,\overline{n}| = \frac{1-(1+i/p)^{-np}}{m(1+\frac{i}{p})^{p/m}} \qquad (16)$$

7. The operations indicated in the right member of equation 16 may be stated in the form of a rule as follows: First, find the value of the numerator by raising 1 plus the fraction of the rate prescribed to a power equal to the number of periods in the whole term. Divide 1 by this amount and subtract the quotient from 1. Second, find the effective annual rate for one year at the given rate of interest convertible as prescribed in the problem; then from this find the nominal rate for one year with payments as prescribed in the problem. Third, divide the value first found by this rate. A few examples will illustrate the application of the formula and rule and also indicate the effect of differences in the interest periods and payment periods.

8. Let it be required to find the value of an annuity of 1 for ten

years at six per cent interest convertible quarterly with payments one-half each half year. Applying the formula, we have:

$$1 - \frac{(1+{}^1/_4)^{-4n}}{2((1+i/4)^4/_2 - 1)}$$

From the table, we obtain the value of $(1.015)^{40} = 1.8140$. Dividing 1 by this value, we have .551267. Subtracting the latter amount from 1, we have for the numerator .448732. Then $((1.015)^2 - 1) =$ .030225. Multiplying by 2, we have for the rate .06045. Finally dividing .551267 by .06045, we have the value of the annuity, 7.4232.

9. Let it be required to find the present value of an annuity for 25 years at 4 per cent convertible four times a year, with payments $^1/_4$ each quarter. First, computing the compound interest for 100 periods at one per cent, we have 2.704814. Taking its reciprocal as before, we obtain .369711. Subtracting the latter sum from 1, we have .630289. The fourth root of the fourth power of 1.04, we know, is 1.04, and the rate of interest is therefore 4. Dividing .630289 by .04, we have the answer 15.7572.

10. Let the conditions of the problem of Article 8 be so changed that the interest shall be compounded semi-annually, and the payments be due $^1/_4$ each quarter. $\frac{1}{(1.03)^{20}} = .553676$. And $1 - .553676 = .446324$. This is the numerator. Also $4((1+3)^2/_4 - 1) = 4(\sqrt{1.03} - 1) = .059556$ This is the denominator of the formula. Performing the division as indicated in the formula, we have the answer 7.4942.

11. The value found by formula 16 is that of an annuity of 1, the value of any other annuity may of course be found by multiplying the value of $a\,\overline{n}|$ by the given annuity.

12. The value of an annuity due may be found by computing the value of an annuity for a term one less than the given term and adding 1 to the result. Or take from the table an annuity for a term 1 less than the given term and add 1. Thus an annuity due of 1 for 20 years at 4% is $13.133939 + 1$. $14.133939 = a\overline{19}| + 1$. From this we get the equation, $a\,\overline{n}| = a\,\overline{n-1}| + 1$. (17)

13. An annuity to begin at some future date is called a **deferred annuity** and is often represented by the symbol $m|a\overline{n}| = v^m a\overline{n}|$. (18)

14. Let it be required to find the value of a ten-year annuity to be entered upon five years hence, at 5%. From the table, we find the value of a ten-year annuity at 5%, to be 7.721735. But this value is not due for five years and must be discounted five years at 5%. That is v^5, that is by .783526. Then $.783526 \times 7.721735 = 6.050180$.

15. When an annuity table is available, the value of a deferred annuity may be found more directly by finding an annuity for the

sum of the two terms and deducting therefrom the value of an annuity for the intervening term. Thus, to solve the problem of Article 14, from $a\overline{15}| = 10.379658$, deduct $a\overline{5}| = 4.329477 = 6.050181$. From this we have the equation $m|a\overline{n}| = a\overline{m+n}| - a\overline{m}|$ (19)

16. Let us suppose that instead of receiving the annual payments of 1, the annuitant allows them to accumulate for n years at the prescribed rate of interest. What is its value? First, if we have the present value of the annuity, $a\overline{n}|$, we may treat it as the principal of an investment and solve by equation 1 of the preceding chapter, $S = P(1+i)^n$, and substituting $a\overline{n}|$ for P, we have the formula:

$$S\overline{n}| = a\overline{n}|\ (1+i)^n \tag{20}$$

17. The amount $(1+i)^n$ consists of the 1 together with an annuity of i, the annual interest, and its accumulations for n years. If the 1 be deducted, the remainder, $(1+i)^n - 1$ will be the amount of the annuity i. Then by proportion, i: 1 as $(1+i)^n - 1$: $\frac{(1+i)^n - 1}{i}$, from which we derive the formula:

$$S\overline{n}| = \frac{(1+i)^n - 1}{i} \tag{21}$$

This may be stated in the form of a **Rule**: Divide the compound interest for the given term by the interest for one period of the term.

18. In cases where the annuity is payable, a certain part at the end of each corresponding part of a year, it is necessary to modify the denominator in equation (21) to correspond with the nominal rate, so that the formula for the amount of an annuity payable 1/m each 1/mth part of a year becomes:

$$S\overline{n}|^{(m)} = \frac{(1+i)^n - 1}{m(1+i)^{1/m} - 1} \tag{22}$$

19. If the problem involves also interest convertible at fractional periods, 1/p th, of the year, a corresponding modification of the last equation must be made, thus,

$$S\overline{n}| = \frac{(1+{}^{i}/_{p})^{p/m} - 1}{p(1+{}^{i}/_{p})^{p/m} - 1} \tag{23}$$

20. A few examples illustrative of the last four equations will be helpful. First, let there be an annuity of 1 per year for ten years, what will be its amount at the end of the term at five per cent interest?

From the table, we have $a\overline{10}| = 7.721735$ and $(1.05)^{10} = 1.628895$ and by formula 20, the product of these $= S\overline{10}|$. Performing the multiplication, we have the answer, 12.577894.

Again, the amount of $(1.05)^{10}$ is 1.628895. Subtracting 1 and dividing by .05, as indicated by equation 21, we have $S.\overline{10}| = .628895 \div .05 = 12.5779$ as before.

Again, suppose the payments are to be made $^1/_2$ each half year. by formula 22, we have

$$\frac{(1.05)^{10}-1}{2(\sqrt{1.05}-1)} = \frac{.628895}{.04939} = 12.73326.$$

Suppose, also, that to the problem as last stated, the condition is added that interest shall be convertible quarterly. By equation 23,

$$\frac{(1.0125)^{40}-1}{2(1.0125)^{4}/_{2}-1} = \frac{.643619}{.050212} = 12.8181.$$

Finally, let the conditions be that the interest shall be convertible quarterly, the payments being due annually. By equation 23, we have

$$\frac{(1+{}^1/_4)^{40}-1}{(1+{}^1/_4)^{4}/^{1}-1} = \frac{(1.0125)^{40}-1}{(1.0125)^{4}-1} = \frac{.643619}{.050945} = 12.6336.$$

21. We have seen, Article 20, that $S\overline{n}| = a\overline{n}|\ (1+i)^n$. By a simple transformation of this equation, we have the formula

$$a\overline{n}| = \frac{S\overline{n}|}{(1+i)^n} \qquad (24)$$

22. Let us suppose $S\overline{n}| = 1$. Let it be required to find what annual payment will produce it. Obviously, it is that part of 1 obtained by dividing it by $S\overline{n}|$. Thus, if $S\overline{10}|$ at $5\% = 12.5779$ being the accumulations of 1 a year for ten years, then $1 \div 12.577894 = .079505$ is the annuity which will amount to 1 in ten years at 5 per cent. This function is called a *Sinking Fund.*

It may be computed by the formula, Sinking Fund $= \frac{1}{S\overline{n}|}$ (25)

23. A table of sinking funds may be readily computed from a table of $S\overline{n}|$ by means of logarithms. Take the logs of a column of $S\overline{n}|$. Take the arithmetical complement or co log of the latter and the anti log will be the sinking fund.

24. The sinking fund table may also be conveniently formed by dividing the interest for a single year or period by the compound interest for the given term and rate, thus

$i \div ((1+i)^n - 1) =$ sinking fund. (26)

25. We have seen, Article 21, that $S\overline{n}|$ is composed of 1 plus the annuity i and its accumulations. Hence, if we add the annuity i to the annuity which will produce 1 in n years, that is, to the sinking fund,

we will have an annuity the present value of which is 1. Thus, we obtain the **annuity which 1 will purchase.**

26. We may also find the annuity which 1 will purchase by converting 1 into an annuity. This is, by dividing it by $a\overline{n}| = \frac{1}{an}$

27. An annuity represents an investment repayable in equal annual installments for a given term. The function $a\overline{n}|$ is the principal invested and the annuity contains not only the principal but also the interest on the same. That is, at the end of the term both the principal and the interest will have been repaid by the annual payments. It is sometimes important to know how much of the installments of a loan repayable by equal installments is principal and how much is interest. A hint as to the method of doing this will be found in Article 25, where it is noticed that an annuity is composed of two elements, consisting of a sinking fund which will produce its present value and an annuity of the interest on the present value. The present value of each of these as the payments progress will represent the amount of each remaining unpaid. Naturally as the debt is reduced the portion required as interest decreases and the part going to discharge the principal increases.

28. As an example, let it be required to separate the installments of a debt of $1000.00 into principal and interest contributions, supposing ten equal installments in ten years, interest at five per cent.

First, dividing 1000 by $a\overline{10}|$ as found in the 5 per cent column of the table, that is, 1000 by 7.721735, we have $129.505 as the annual installment. From the sinking fund table we find .079505 per annum will produce 1 in ten years at 5%, hence, 79.505 is required to produce 1000 in the same time. This strikes one as being proper because we know that 50.00 is the interest for the first year. This is also in accordance with Article 25. Hence, the first payment is composed of 50.00 interest and 79.505 for sinking fund. The sinking fund for the next year with 5 per cent on the last year deposit is 83.47. Hence, for the second year, we have $129.50 - $83.47 = $46.03 for interest and $83.47 for sinking fund. We may see that the interest $46.03 is correct, by computing the interest on $920.50, the principal less the sinking fund. Again $(920.50 - 83.47) \times .05 = 41.85$ is the interest contribution for the third year and $129.57 - 41.85 = 87.66$, the sinking fund. And thus a schedule may be made of payments, interest and sinking fund, for an installment loan.

29. The amount in a sinking fund may be found at any time by treating the annual contributions to the sinking fund as an annuity accumulated for a term equal to the number of payments made. Thus, at the end of the 6th year, the sinking fund amounts to $S\overline{6}| = 6.8019 \times 79.51 = 540.785. The amount of the debt remaining unpaid at the same date is, of course, $1000 - 540.79 = 459.21.

30. If the conditions of the problem of Article 28 be so changed that an equal installment of the loan plus the interest shall be paid each year, the schedule may be made up as well by direct computation as otherwise. The annual installment of principal is of course, $100.00 and the work may be arranged as follows:

Years	Principal	Installment	Interest	Total Payment
1	1000	100.00	50.00	150.00
2	900	100.00	45.00	145.00
3	800	100.00	40.00	140.00

and so on.

31. A case may arise where the sinking fund bears a less rate of interest than the bond calls for. This presents a different situation from that discussed in Articles 27-29. Thus we may suppose the sinking fund in the problem of Article 28 to be deposited in a bank or trust company at, say, $3^1/_2$ per cent until the bond matures, the annual interest to be paid the bond holder. If we keep in mind the fact that the purpose of the sinking fund is to accumulate a sum sufficient to pay the principal of the debt at the end of the loan period, the solution is simple. In that case, we need only to take the annual payment from the $3^1/_2\%$ sinking fund column, and in the present case, it will be $.087231 \times 1000$ or 87.231 per annum. Adding $50.00, interest, the annual sum to be paid by the debtor is $137.23 instead of $129.51 as in the problem of Article 28.

32. The annuity tables enable us to solve problems similar to the one given in Article 21 of last chapter, providing the interest used in valuation is convertible at the same periods as the rate named in the bond. Thus, let it be required to find the price at which a ten year bond for $1000.00 bearing four per cent payable semi-annually may be sold to earn six per cent, payable semi-annually. The present value of $1000.00 payable in ten years at 6% semi-annually as given by the table, v^{20} of the 3% column, is, $553.676. The present value of a semi-annual annuity of $20.00 for 20 periods, found in 3% $a\overline{n}|$ table is 14.87748 for 1, which multiplied by $20 = 297.50$. The sum of these, $553.68 + 297.50 = \$851.18$ which is the value of the bond or price that may be paid for it to earn 6% semi-annual interest.

33. **The rate at which an annuity has been accumulated** may be found directly from a table of annuities by running along the line opposite the given term until its value is found, and taking the rate given at the top. Thus, we may find the rate of a 20-year annuity, the present value of which is 13.00794 by following the horizontal line opposite 20 until we strike the value under the rate $4^1/_2\%$.

34. In cases where the annuity has not been computed upon any of the usually tabulated rates, a more difficult problem is presented. Two formulas will be given, one such as is usually found in books on interest and one from Mr. King's book, attributed to Mr. G. F. Hardy.

35. If we let $a\overline{n}|$ be the value of the given annuity, $a'\overline{n}|$ the value of an annuity for the same term taken from the same table and being as near as possible to the value of the given annuity, j the rate of $a'\overline{n}|$ and $p+j$ the required rate. The problem then is to find the value of p. For this the formula has been given:

$$p = \frac{j(a'-a)}{a'-nv^{n+1}} \tag{27}$$

36. Let us take an annuity of 1 for twenty years, the present value of which is 12.1261. What is the rate per cent? Looking into the table of $a\overline{n}|$ opposite 20, we find 12.4622 as produced by the rate of 5%. From the table v^n we get v^{21} at 5%, .358942. Substituting these values in equation, (27) we have the following:

$$p = \frac{.05(12.4622-12.1261)}{12.4622-20\times.358942}$$

Solving the equation, we have $.016805 \div 5.2834 = p = .003181$. Finally $.05+003181 = .053181$ which is the approximate rate required.

37. The formula of Hardy is as follows:

$$p = \frac{h(\triangle - {}^1/_2\triangle^2}{a-a_1} + \frac{{}^1/_2\triangle^2)^{-1}}{\triangle - {}^1/_2\triangle^2} \tag{28}$$

The formula involves the processes of differencing and will be best understood by following its application in the solution of a problem. Let the example of Article 36 be taken. Take the values from the $a\overline{n}|$ table opposite 20 and of value below and above 12.1261. Call the latter a_1, a_2 and a_3, and arrange them and difference to second differences as follows:

a_1	12.4622	−.9923	.1164
a_2	11.4699	−.8759	
a_3	10.5940		

Here $\triangle$ is the first, and $\triangle^2$ the second, difference of a_1 and a, a_2 and a_3, it will be noticed are rates 5, 6 and 7 per cent, respectively, and hence, h = .01. Inserting these values in the formula, we have:

$$p = .01\left(\frac{.9923-.0582}{12.1261-12.4622} + \frac{.0582}{-9923-0582}\right)^{-1} = \frac{.01}{3.0706} = .003257.$$

And $p+(j=5) = .053257$.

A closer approximation could be had by taking a_1, a_2, and a_3 from a table in which h is $^1/_2$ or a smaller fraction.

38. In cases where a value is desired which is not included in the tables, it may sometimes be conveniently found by interpolation, from the tabulated values. Examples of this method are given at Articles 4 and 8 of Chapter 3 on Series. Some further elucidation will be given here.

39. Given the amounts of 1 at compound interest for 20 years, as follows: At 4%, 2.1911; at 5%, 2.6533; at 6%, 3.2071 and at 7%, .86973, to find the compound amount of 1 at $5^1/_2$% for the same term.

1. Arrange the given values and differences thus:

$$\begin{array}{lllll} & \triangle & \triangle^2 & \triangle^3 & \\ a_1 = 2.1911 & +.4622 & +.0916 & +.0172. & \\ a_2 = 2.6533 & +5538 & +.1088. & & \\ a_3 = 3.2071 & +6626 & & & \\ a_4 = 3.8697. & & & & \end{array}$$

Then we have:

$n = {}^5/_2$, $\triangle = .4622$, $\triangle^2 = .0916$ and $\triangle^3 = .0172$.

Restating the formula for the nth term:

$$a\overline{n}| = a + (n-1)\triangle + \frac{(n-1)(n-2)}{\underline{|2}}\triangle^2 + \frac{(n-1)(n-2)(n-3)}{\underline{|3}}\triangle^3 + .$$

Substituting the values found, in this equation, we have:

$$a\overline{n}| = 2.1911 + ({}^3/_2 \times .4622) + ({}^3/_2 \times {}^1/_2 \times -{}^1/_2 \times .0916) +$$

$$({}^3/_2 \times {}^1/_2 \times -{}^1/_2 \times \frac{1}{2\cdot 3} \times .0172) = 2.1911 + .6933 + .0344 - .0011 =$$

2.9177, which is a very close approximation, the value being usually written, 2.9178.

40. In further elucidation of the principles discussed in this and the preceding chapter and for convenience of reference a series of examples with rules and solutions will now be given.

41. **To compute the compound interest on a given principal.** Rule: Multiply the principal by the rate per cent, expressed decimally. Add the product to the principal and take the sum for a new principal and proceed as before. Repeat these operations to the end of the term. Subtract the principal from the last sum found and the remainder will the the compound interest. Example: Compute the compound interest for three years at 5%, on 1000.

$1000 \times .05 = 50$ and
$1000 + 50 = 1050$.
$1050 \times .05 = 52.50$ and
$1050 + 52.50 = 1102.50$
$1102.50 \times .05 = 55.125$ and
$1102.50 + 55.125 = 1157.625$ and $1157.625 - 1000 = 157.625$
which is the compound interest. Answer, $157.63.

42. **To compute compound interest by the use of logarithms.** Rule: First find the logarithm of 1 plus the rate per cent expressed decimally. Multiply this value by the term and to the product add the log of the principal. The sum will be the log of the compound amount, and its anti log the compound amount. Deduct the principal from the latter and the remainder will be the compound interest.

Example: Compute by logs the compound interest on $9675.00 for 19 years at $4\frac{1}{2}\%$, the interest to be compounded semi-annually.

Here there are 38 half-year periods or rests in the payments of interest and the rate for each period is one-half of $4\frac{1}{2}\%$ or $2\frac{1}{4}\%$. 1 plus this rate is 1.0225 and its log is .009664. And, $.009664 \times 38 = .367213$. The log of 9675 is 3.985651. Also $.367213 + 3.985651 = 4.352864 = 22536.34$ and subtracting 9675, we have $12,860.34, Answer.

43. **To compute compound interest at the nominal rate.**

Suppose that in the last problem the condition is imposed that the interest earned shall be only $4\frac{1}{2}\%$ per annum. That is, that the effective rate shall be $4\frac{1}{2}\%$ per annum. The computer must first find the nominal rate payable semi-annually which will produce $4\frac{1}{2}\%$ per annum. This we have seen, Chapter 5, Article 16, is $2\sqrt{1.045} - 1$. That is twice the square root of 1, plus the rate, -1, which in this case is .044505. Computing by the latter rate, we have log 1.0222525 $= .009558$, which multiplied by 38 gives .363208. And $.363208 + 3.985651 = 4.348859$ which is 22328.46. Subtracting the principal 9675, we have $12653.46, Answer.

44. **To find the time in which a given principal will amount to a given sum at compound interest.**

Rule: When tables of compound amounts are available, the term may be found approximately by dividing the given amount by the given principal. The quotient will be the amount of 1. Then follow down the line of the table under the given rate until the said amount of 1 is found. The term will be found under n in the column at the left.

Example: In what time will $1000 accumulate to $1500.00 at 5% compound interest?

Dividing 1500 by 1000, we have 1.50000 as the compound amount of 1. Looking down the 5% column of the compound amounts, $s\overline{n}|$ table, we find the value 1.477455 opposite n 8, and 1.5511328 opposite n 9 from which we know that the time is less than 9 years and only slightly exceeds 8 years. The term can be still more closely approximated by interpolating between these terms and amounts. Thus a difference of one year in the term makes a difference of $1.551328 - 1.477455$ or .073873 in the amount. Then a difference of $1.500000 - 1.477455$ or .022545 makes a difference of $\frac{.022545}{.073873}$ or .3 in the term.

So that the term is approximately 8.3 years.

45. **To find the term of a loan by the use of logarithms.**

RULE: Subtract the log of the given principal from the log of the given amount, and divide the remainder by the log of 1 plus the given rate expressed decimally.

EXAMPLE: Find the term of the loan of the preceding example.

The log of 1500 is 3.176091.

The log of 1000 is 3.000000 and their difference is .176091. The log of 1.05 is .021189 and .176091÷.021189=8.3577, 8 years, 4 months, 8 days, Answer.

46. **To find the rate per cent of a loan, the principal, amount and term being given.**

RULE: When tables of compound amounts are available for minutely divided rates of interest a close approximation may be had by dividing the given amount by the given rate, thus finding the amount of 1 for the given time. Then looking across the page opposite the given term in the n column until an amount corresponding, or nearly corresponding to the amount of 1 is found. The rate or approximate rate will be found at the top of that column.

EXAMPLE: Given $1327.85 as the amount of $1000 in 13 years, what is the rate?

Dividing the amount by 1000, we have 1.32785 as the amount of 1 in 13 years. Running across the page opposite n 13, we find 1.293607 under 2% and 1.335436 under $2^1/_4$ from which we know the rate is nearly $2^1/_4$ per cent. We might get a closer approximation by interpolating between the two rates in the usual way.

47. **To find the rate of a loan by the use of logarithms.**

RULE: From the log of the given amount subtract the log of the given principal; divide the remainder by the given term and subtract 1 from the anti log of the quotient, the remainder will be the rate.

EXAMPLE: Find the rate of the problem of Article 46 by logarithms.

The log of 1327.84 is 3.123147.

The log of 1000 is 3.000000.

Their difference is .123147. Dividing the latter by 13, we have .009473=1.02205. Deducting 1, we have .002205 or $2^1/_5$ per cent. Answer, $2^1/_5$ per cent.

48. **To find the rate which a given note or bond will pay when sold at a discount or premium.**

RULE: First find the amount of the bond at maturity at the rate given in the bond. Then treating the price paid as the principal of the loan, find the rate per cent by the rule of Article 47.

EXAMPLE: A 20 year drainage bond for $1000, bearing six per cent interest payable semi-annually sells at .96. What rate per cent on the price does it pay the purchaser?

The amount of the loan for 40 periods is, by the table, 3262.04. The investment is 960. The log of 3262.04 is 3.513489. The log of 960 is 2.982271 and their difference is .531218. Dividing by 40 we get the rate for one period, six months, plus 1. That is .013280 = 1.031 or .062 for a year. Answer, $6^1/_5$ per cent. In the above solution the distinction between effective and nominal rates is disregarded and the computation made as though the nominal rate was three per cent for six-month periods which we know is in excess of six per cent per annum, but this is in accordance with the general practice in the commercial world as we understand it. Also the excess or bonus of $40.00 paid at maturity is treated as interest which is correct enough if so understood by the parties to the transaction.

49. **To find the price which may be paid for a given bond or note in order to realize a specified rate of interest.**

RULE: Treat the principal sum as a deferred payment and the annual interest prescribed in the bond as an annuity for the term of the loan. Then find the present value of each at the rate to be realized on the purchase price to be paid. Their sum will be the price to be paid.

EXAMPLE: What price can be paid for a 20-year city bond for $1000.00 bearing $4^1/_2$ per cent per annum, to realize 6 per cent on the investment?

The present value of 1000 due in 20 years is $1000 \times v^{20}$: Taking v^{20} from the 6% column of the v^n table, we have .311805 and $.311805 \times 1000 = 311.81$, the value of the principal. Also the value of an annuity of 1 for 20 years taken from the $a\overline{n}|$ table at 6 per cent is 11.47 and 11.47×45 gives 516.15 as the present value of the annual interest payment of 45.00. The sum of 311.81 and 516.15 is 827.96. Answer $827.96.

The foregoing rule does not apply when the rate to be realized is convertible at periods different from those named in the bond. See Article 53 for a rule of general application.

50. **To find the amount to which a forborne annuity will accumulate in a given time at a given rate of interest.**

RULE: Compute the compound interest on 1 for the given term at the given rate of interest. Divide the compound interest found by the rate of interest used and multiply the quotient by the annual payment of the annuity.

EXAMPLE: What would be the amount of an uncollected pension of $288 per annum, payable $72 per quarter, after ten years at $4^1/_2$ per cent interest per annum?

The compound interest on 1 for 40 periods at $1^1/_8$ per cent is $(1.01125)^{40}-1=.564376$. The nominal rate to produce $4^1/_2$ per cent per annum is $4(1.045)^{\frac{1}{4}}-1=.04426$ and $.564376 \div .04426 = 12.751$ for the term of ten years. Multiplying this by the payment 288, we have as the whole accumulation, 3672. Answer, $3672.

51. **To find the payments of a Sinking Fund. That is, to find the annual payments which will discharge a given sum payable at a future date at a given rate of interest.**

RULE: Divide the interest on 1 for one year by the compound interest on 1 for the given term, both at the given rate per cent and multiply the quotient by the amount to be paid off, the product will be the annual payment.

EXAMPLE: What annual payment will be required to pay off $1000.00 due in ten years, the payments to bear 4 per cent interest?

The interest on 1 for one year is .04. The compound interest on 1 for ten years at 4 per cent is .423312. And $.04 \div .423312 = .094493$. And $.094493 \times 1000 = 94.493$. Answer, $94.493.

EXAMPLE 2: What monthly payment will mature a $1000.00 share of Building and Loan stock in six years at 5 per cent?

There will be 72 periods at $^5/_{12}$ of 1 per cent which will amount to 1.349018 when computed on a principal of 1. Of this sum, .349018 is interest. Dividing $.00^5/_{12}$ by .349018 gives .0119379. And $.0119379 \times 1000 = 11.9379$. Answer, $11.94.

RULE 2: Multiply the sinking fund as given in the tables by the sum to be raised, matured or paid off, the product will be the amount of the periodical payment.

EXAMPLE 3: What semi-annual deposit must be made to replace electric light machinery to cost $12000.00 in ten years, the deposits to bear four per cent?

There are 20 deposits to be made and the sinking fund will therefore be computed at 2 per cent. By the table, we find this to be .041157. And $.041157 \times 12000 = 493.884$. Answer, $493.88.

52. **To find the time in which a given periodical payment will amount to a given sum at a given rate per cent.**

RULE: From the logarithm of the annual payment subtract the logarithm of the difference between the annual payment and the annual interest on the debt and divide the remainder by the logarithm of 1 plus the rate of interest borne by the debt.

EXAMPLE: How long a term will be required in which to pay off a city debt of $4,000,000.00 if an annual deposit of $400,000.00 is placed in the sinking fund, the debt bearing $4^1/_2$ per cent interest?

The annual deposit being 400000, the annual interest 180000, leaves a net deposit of 220000. The log of 400000 is 5.602060. The log of 220000 is 5.342423. The log of 1.045 is .019116. Then

$5.602060 - 5.342423 = .259637$. And $.259637 \div .019116 = 13.58$. Answer, 13 years, 7 months.

EXAMPLE 2: In what time will a monthly payment of $10.00 mature a $1000.00 share of building and loan stock at 6 per cent interest?

Here the monthly interest rate is $^6/_{12}$ or .005. The term is n. Using the formula:

$$S\overline{n}| = \frac{(1+i)^n - 1}{i} \text{ we have } 10\,\frac{(1+i)^n - 1}{i} = 1000 \text{ and}$$

$$\frac{(1+i)^n - 1}{i} = 100 \text{ and } (1+i)^n = 100i + 1 \text{ and}$$

$$n \log (1+i) = \log (100i+1) \text{ and}$$

$$n = \log \frac{(100i+1)}{\log (1+i)} = \frac{.176091}{.002166} = 81.3 \text{ months}$$

Answer, 6 years, 9 months and 9 days.

Another method of finding the term would be to look down the $S\overline{n}|$ column under the rate $^1/_2\%$ until a value is found which multiplied by 10 will produce 1000. Following this suggestion, we find 99.558046 opposite 81 and 101.055836 opposite 82, and we know therefore that the term is between 81 and 82 months. That is, in 81 periods or $6\,^9/_{12}$ years, the monthly payments will amount to 995.56. The term may be still more closely approximated by interpolating between the 81 and 82 periods in the way already familiar to the reader.

53. **To find the price at which a bond payable in equal periodical installments or otherwise may be purchased so as to realize a given rate of interest.**

RULE: First find the present value of the capital at the rate intended to be realized on the purchase price. Subtract this value from the sum to be repaid. Multiply the remainder by the rate of the bond, divide the product by the rate to be realized and add the result to the present value first found. The sum will be the purchase price sought.

EXAMPLE: A 4 per cent bond for $1000.00 payable in ten years by equal annual installments is offered. What price can be paid for it to realize 6 per cent on the investment?

The present value of the capital to be included in the installments is: $\frac{1000 \times a\overline{10}|}{10}$ and at 6 per cent, this is $\frac{(1000 \times 7.36)}{10} = 736.00$.

The present value of the interest included in the installments at 6% is $(1000-736)$ and $(1000-736)\times{}^4/_6=88$. Finally, $736+88=824$. Answer, $824.00.

EXAMPLE 2: Public improvement bonds for $36000.00 bearing 5 per cent interest to be repaid in fifteen equal annual installments beginning in six years, are offered. What price can be paid to realize 6 per cent on the series?

The problem differs from the first only in that the first payment is deferred six years, making it necessary to discount the values five years. The present value of the payments of principal is therefore, $36000\times.747258\times9.712249\div15=17418.12$, the second and third factors being respectively v^5 and $a\overline{15}|$ taken from the 6 per cent tables Then $36000-17418.12\times{}^5/_6=15484.90$. And finally, $17418.12+15484.90=32903.02$. Answer, $32903.02 or $913.97 per 1000.

EXAMPLE 3: Suppose the conditions in example 1 are so changed that a premium or bonus of 25 per cent shall be included with the principal. What is the solution?

Here we may treat the 1250 to be paid on the bond as the principal in which case the rate is ${}^4/_{125}$ or $3{}^1/_5$ per cent instead of 4% as specified in the bond. That is, $3{}^1/_5$% on 1250 is equivalent to 4% on 1000.0

Solving on this theory, we have:

$\frac{1250\times7.36}{10}=920$. Also $(1250-920)\times\frac{3.2}{6}=176.50$. And $920+176.50=1126.50$. Answer, $1126.50.

The rule given and illustrated in this Article has sometimes been expressed as a formula, thus:

$$A=K+\frac{j}{i}(C-K) \qquad (29)$$

A. is the value of the bond at the rate to be realized by the purchaser, K the present value of the capital at the rate to be realized, C, the capital to be repaid by the borrower, j, the nominal rate paid on the capital and i, the rate to be realized by the purchaser.

In the last problem $A=1126.50$, $K=920$, $C=1250$, $j=3.2$ and $i=6$.

In applying the rule or formula, the only difficulty which will be encountered will be found in computing the present value of the capital included in the payments, that is, in finding K. This is because of the various forms which the contract may assume with respect to the payments. But whatever the form, if K be correctly computed and inserted in the formula, the result will be the value sought. The formula is, therefore, of very general application and may be used in valuing all sorts of contracts for the repayment of loans.

Thus to solve the problem of Article 49, we have for K, $1000 \times v^{20}$ at 6% or 311.81. The value of the interest at 6% is $1000 - 311.81 \times \frac{4.5}{6} = 516.14$ and the purchase price, A $827.95, as before.

54. **To find the rate which an installment loan will pay when purchased at a given price.**

Rule: Compute the present value of the payments of capital at a rate which by inspection appears to be near the rate which the loan will earn at the purchase price, that is, find K of formula (29). Subtract this value from the capital to be repaid by the borrower, that is, from C. Divide this remainder by the difference between the value first found and the purchase money, A, and multiply the quotient by the rate of the loan. The result will be the approximate rate of interest earned on the investment. In the notation of Article 53, this rule may be expressed as follows:

$$i = \frac{j(C-K)}{A-K} \qquad (30)$$

If the approximation is not close enough, use the approximate rate first found in again computing K and solve for i as before.

Example: A railroad improvement loan of $800000.00 is authorized, payable in ten years at 5 per cent by equal semi-annual installments. What rate will be realized by a purchaser who pays .92 for the bonds? Cutting off ciphers for convenience, we have C = 80, A = 73.60 and $j = 2^1/_2$. Also assume 6 per cent as a trial rate or 3 per cent semi-annually. By the 3 per cent table, we have $a\overline{20}| = 14.8775$ and $\frac{80 \times 14.8775}{20} = 59.51$ which is K of the formula. Then $\frac{80 - 59.51}{73.60 - 59.51} \times 2^1/_2 = 1.454 \times .025 = .03635$, which is evidently too large. Again finding K by the rate, .03635, we have $\frac{80 \times 13.4381}{20} = 53.7524$ and $\frac{80 - 53.7524}{73.60 - 53.7524} \times .02^1/_2 = .03306$ or .0661 per annum.

Answer, 6.61 per cent.

Example 2: What rate will be earned on a six per cent bond for $1000.00 payable at par in 20 years, bought at 105?

Here, the principal sum is not paid until maturity. The value of the principal then is $1000 \times v^n$ at the rate to be realized. Assume 5 per cent as a trial rate. v^{20} at that rate by the table is .37689. Hence, K = 376.89. Also C = 1000 and A = 1050. Inserting these values in the formula, we have:

$$\frac{1000 - 376.89}{1050 - 376.89} \times .06 = .05554.$$

Answer, 5.55 per cent.

Example 3: What rate will be earned on a 5 per cent bond payable in 16 years interest payable semi-annually, the bond being purchased at 95?

Let it be assumed that the rate earned will be 6 per cent, or 3 per cent for half year periods. v^{32} at 3 per cent is .3883 and K is therefoer 38.83.

$$\frac{100 - 38.83}{95 - 38.83} \times .025 = .02723, \text{ which double for the yearly rate.}$$

Answer, 5.446 per cent.

The following formula which is said to give a closer approximate rate will be given and illustrated:

$$h = I\frac{(C - A)}{C - K} \qquad (31)$$

I, being a trial rate and h, a correction to be added to the rate prescribed in the bond.

Solving the last problem by way of illustration, we have:

$$\frac{100 - 95}{100 - 38.83} \times .03 = .00245. \quad \text{Then}$$

$(.025 + .00245) \times 2 = .0549$. Answer, 5.49 per cent.

5. Suppose a farm loan of $12000, bearing 4 per cent interest, the principal of which is payable in three installments of $4000 each, at the end of four, eight and twelve years, is offered at $11500, what rate of interest will it pay the purchaser?

Let us take a trial rate $4^1/_2$ per cent, then by the table the present values of the three payments of capital are:

$$.838561 \times 4000 = 3354.24$$
$$.703185 \times 4000 = 2812.74$$
$$.589664 \times 4000 = 2358.67$$
$$8525.65$$

And by formula 27, and the conditions of the problem, we have

$$\frac{12000 - 11500 \times .045}{12000 - 8525.65} = \frac{22.5}{3474.35} = .0064 \text{ and } .04 + .0064 = .0464.$$

Answer, 4.64 per cent.

If 5 per cent be used as a trial rate, the result is 4.66 and we may regard 4.65 as a close approximation.

56. Greater accuracy may be attained by finding two approximate rates from two trial rates and interpolating for a new approximate rate. For this purpose an excellent formula has been suggested by Mr. King. This, with his notation, will be given and illustrated as a conclusion to this branch of our subject. Let I_1 be the first trial rate, I_2 the second trial rate, J_1 the first approximate rate, that is, the one derived from I_1 and J_2 the second approximate rate. Also let $\triangle$ be the difference between the trial rates, d the difference between the approximate rates, J_1 and J_2 and i, the new approximate rate. Then we have the formula:

$$i = I_1 + \frac{\triangle (J_1 - I_1)}{\triangle - d} \qquad (32)$$

57. Drainage district bonds for $1000 each payable in 15 years and bearing 4% interest per annum, are sold at 90. What per cent do they pay the purchaser?

Solution: He gets a bonus of $100 at maturity and 4 per cent per annum on $1000. Let us suppose he earns $4^1/_2$ per cent, we then have $I_1 = .045$ and by formula (31) we have:

$$\frac{(1000 - 900) \times .045}{1000 - 516.72} = .00934,$$

so that J_1 is .04934. Again trying 5% or $I_2 = .05$, we find $J_2 = .04963$. From which we derive $\triangle = .5$, and $d = .029$. Inserting the proper values in formula (32) we have:

$$i = 4.5 + \frac{.5(.04934 - .04500)}{.5 - .029} = 4.5 + .0461 = 4.96 \text{ per cent.}$$

59. A little practice with the aid of the tables given will enable the student or investor to make very accurate calculations of the values required in the operations of finance. The tables are nearly all computed true to six places of decimals giving correct values up to 100000. If greater accuracy is required, which will rarely occur, the short eight figure table of logarithms may be used and the formula and rules given will enable the computor to obtain all the values his problems will require and as a rule, with little labor.

CHAPTER VII

Of The Tables

There are forty-three tables published in this book which have been prepared with the view of enabling the computer to solve with little labor nearly all of the problems that arise in transactions relating to finance and life insurance, after reading the brief texts given in the body of the work.

Table 1. Is a short table of logarithms adapted to computing by logarithms to five and to eight places of decimals. Its use is explained in Chapter 2.

Table 2. Gives the amount of one at compound interest at rates of interest ranging from 1/4 of one per cent to ten per cent and for periods of from one to sixty. By deducting one from these amounts, the compound interest is obtained.

Table 3. Gives the present value of one, discounted at compound interest for the same periods and at the same rates included in Table 2. It is of frequent application in computations relating to all of the subjects treated in this book. Its symbol is v^n. The superscript, n, relating to the number of years or periods of discount. v deducted from 1 gives the discount of one for one period. Its symbol is d.

Table 4. Gives the amount of an annuity of one computed at rates of interest from one-fourth of one per cent to ten per cent for periods of from one to sixty. Its symbol is $s\overline{n}|$, n referring to the number of periods in the term.

Table 5. Is an annuity table. It gives the present value of an annuity certain of a payment of one computed for the terms and rates of compound interest mentioned in the three preceding tables. It and Table 3 are used in computing the values of bonds and loans and is otherwise of frequent application. Its symbol is $a\overline{n}|$.

Table 6. Gives sinking funds accumulating at the rates of interest and for the terms mentioned in the last mentioned table. By dividing the sum to be discharged by the sinking fund for the term and rate to be realized on the deposits, we get at once the amount of the periodical deposits to be made. By adding the rate of the interest to the sinking fund, the annuity which a unit will purchase is obtained.

Tables 7 to 11, inclusive, present the five mortality tables which may be regarded as standard tables in this country. These tables give the number living of the assumed group of lives for each year of the human life, the number of the group dying each year, the yearly probability of living and of dying. They are called **life tables** and are the basis upon which all forms of life insurances and life annuities are computed. The American Experience table is the standard table for life insurance valuations in probably more than one-half the states

in the Union. The Actuaries or Combined Experience Table—also known as the Seventeen Office table, is standard in the rest of the states. The Carlisle table is employed in many states in computing annuities for the valuation of life estates, life incomes and reversions. The National Fraternal Congress table is used in Fraternal insurance circles in computing rates of contribution or premiums, valuations, etc. All four of the tables have had frequent legislative and judicial recognition thruout the country and may be relied on as fairly representing the rate of mortality. The Northampton table was formerly much used in computing annuities and may still be recognized as authoritative in some of the courts.

Tables 12 to 15, inclusive, give commutation columns for the computation of annuities on single lives at the rates of five and six per cent according to the Actuaries, Carlisle, Northampton and American Experience Tables of Mortality.

Table 16. Gives joint life commutation columns on two and three lives of equal ages at five per cent by the regraduated Carlisle table by means of which joint and contingent life annuities on lives of different ages may be readily computed.

Table 17. Differs from 16 only in that it is computed at 6 per cent. It being necessary to use the one based on the established rate in the state where the valuation is required.

Tables 18, 19 and 20. are based on the regraduated American Experience table at rates of $3^1/_2$, 5 and 6 per cent. The latter two are adapted to the computing of joint and contingent life annuities on two and three lives in the same manner as the Carlisle table. The $3^1/_2$ per cent columns may be used to compute annuities from which joint life insurances may be derived.

Tables 21 to 26, inclusive, are based upon the regraduated Actuaries table and may be used in computing joint life annuities at 4, 5 and 6 per cent on two and three lives. The four per cent table is also adapted to computing annuities from which joint life insurances may be derived. One or the other of the foregoing tables may be used for valuing annuities, incomes, reversions and insurances where joint lives are involved in all the states.

Table 27. Gives the force of mortality by the three regraded tables last mentioned and is used in connection with those tables in making calculations as explained in the text.

Table 28 to 30. Give joint life annuities at the rates and by the tables mentioned in Articles 16 to 26.

Table 31. Gives the expectation of life as derived from the Northampton, Carlisle, Actuaries and American Experience tables of mortality. They represent the average number of years lived by a life of the ages given. They do not furnish a proper basis for computing insurance or annuity values, but are sometimes used in courts where values depending upon the duration of human life are involved.

Table 31, to 39, inclusive, are Commutation tables based on the Actuaries, American Experience and Fraternal Congress tables at from 3 to $4^1/_2$ per cent interest. They are computed at rates appropriate for the computation of insurance values.

Tables 40 to 42, inclusive, give annuities on single lives at from 3 to 6 per cent based on Actuaries, American Experience and Carlisle Mortality Tables.

Table 43. Gives the net single premiums for an insurance of 1 at the rates of 3, $3^1/_2$, 4 and $4^1/_2$ per cent on the American Experience table and at the rates 3, $3^1/_2$ and 4 per cent on the Actuaries or Combined Experience table. They are only given, on ages from 10 to 70 years, for the reason that little occasion will, in practice, be found for the values above that age and space is thereby saved. These premiums may be used in computing annual premiums for whole life or limited payment policies and in valuations. Single premiums on the Carlisle and Northampton tables are omitted for the reason that they are no longer used, to any extent, as standards for life insurance contracts.

TABLE NO. Ia

Logarithms to Eight Places

N	0	1	2	3	4
1.0	.00000000	.00432137	.00860172	.01283722	.01703334
1.1	.04139269	.04532298	.04928023	.05307844	.05690485
1.2	.07918125	.08278537	.08635983	.08990511	.09342169
1.3	.11394335	.11727130	.12057328	.12385164	.12710480
1.4	.14612804	.14921911	.15228834	.15533604	.15836249
1.5	.17609126	.17897695	.18184359	.18469143	.18752072
1.6	.20411998	.20682588	.20951501	.21218760	.21484385
1.7	.23044892	.23299611	.23552845	.23804610	.24054925
1.8	.25527251	.25767857	.26007139	.26245109	.26481782
1.9	.27875360	.28103337	.28330123	.28555831	.28780173
2.0	.30103000	.30319606	.30535137	.30749604	.30963017
2.1	.32221929	.32428246	.32633586	.32837960	.33041377
2.2	.34242268	.34439227	.34635297	.34830486	.35024802
2.3	.36172784	.36361198	.36548798	.36735592	.36921586
2.4	.38021124	.38201704	.38381537	.38560627	.38738983
2.5	.39794001	.39967372	.40140054	.40312052	.40483372
2.6	.41497335	.41664051	.41830129	.41995575	.42160393
2.7	.43136376	.43296929	.43456890	.43616265	.43775056
2.8	.44715803	.44870632	.45024911	.45178644	.45331834
2.9	.46239800	.46389299	.46538285	.46686762	.46834733
3.0	.47712125	.47856650	.48000694	.48144263	.48287358
3.1	.49136169	.49276039	.49415459	.49554434	.49692965
3.2	.50514998	.50650503	.50785587	.50920252	.51054501
3.3	.51851394	.51982799	.52113808	.52244423	.52374647
3.4	.53147892	.53275438	.53402611	.53529412	.53655844
3.5	.54406804	.54530712	.54654266	.54777471	.54900326
3.6	.55630250	.55750720	.55870857	.55990663	.56110138
3.7	.56820172	.56937391	.57054294	.57170883	.57287160
3.8	.57978360	.58092498	.58206336	.58319877	.58433122
3.9	.59106461	.59217676	.59328607	.59439255	.59549622
4.0	.60205999	.60314437	.60422605	.60530505	.60638137
4.1	.61278386	.61384182	.61489722	.61595005	.61700034
4.2	.62324929	.62428210	.62531245	.62634037	.62736586
4.3	.63346846	.63447727	.63548375	.63648789	.63748973
4.4	.64345268	.64443859	.64542227	.64640373	.64738297
4.5	.65321251	.65417654	.65513843	.65609820	.65705585
4.6	.66275783	.66370093	.66464198	.66558099	.66651798
4.7	.67209786	.67302091	.67394200	.67486114	.67577834
4.8	.68124124	.68214508	.68304704	.68394713	.68484536
4.9	.69019608	.69108149	.69196510	.69284692	.69372695
5.0	.69897000	.69983773	.70070372	.70156799	.70243054
5.1	.70757018	.70842090	.70926996	.71011737	.71096312
5.2	.71600334	.71683772	.71767050	.71850169	.71933129
5.3	.72427587	.72509452	.72591163	.72672721	.72754126
5.4	.73239376	.73319727	.73399929	.73479983	.73559890

TABLE NO. Ia.

Logarithms to Eight Places

N	5	6	7	8	9
1.0	.02118930	.02530581	.02938378	.03342376	.03742650
1.1	.06069784	.06445799	.06818586	.07188201	.07554696
1.2	.09691001	.10037055	.10380372	.10720997	.11058971
1.3	.13033377	.13353891	.13672057	.13987909	.14301480
1.4	.16136800	.16435286	.16731733	.17026172	.17318627
1.5	.19033170	.19312460	.19589965	.19865709	.20139712
1.6	.21748394	.22010809	.22271647	.22530928	.22788670
1.7	.24303805	.24551267	.24797327	.25042000	.25285303
1.8	.26717173	.26951294	.27184161	.27415785	.29646180
1.9	.29003461	.29225607	.29446623	.29666519	.29885308
2.0	.31175386	.31386722	.31597035	.31800633	.32014629
2.1	.33243846	.33445375	.33645973	.33845649	.34044411
2.2	.35218252	.35410844	.35602586	.35793485	.35983548
2.3	.37106786	.37291200	.37474835	.37657696	.37839790
2.4	.38916608	.39093511	.39269695	.39445168	.39619935
2.5	.40654018	.40823997	.40993312	.41161971	.41329976
2.6	.42324587	.42488164	.42651126	.42813479	.42975228
2.7	.43933269	.44090908	.44247977	.44404480	.44560420
2.8	.45484486	.45636603	.45788190	.45939249	.46089784
2.9	.46982202	.47129171	.47275645	.47421626	.47567119
3.0	.48429984	.48572143	.48713838	.48855072	.48995848
3.1	.49831055	.49968708	.50105926	.50242712	.50379068
3.2	.51188336	.51321760	.51454775	.51587384	.51719590
3.3	.52504481	.52633928	.52762990	.52891670	.53019970
3.4	.53781910	.53907610	.54032947	.54157924	.54282543
3.5	.55022835	.55145000	.55266822	.55388303	.55509445
3.6	.56229286	.56348109	.56466606	.56584782	.56702637
3.7	.57403127	.57518784	.57634135	.57749180	.57863921
3.8	.58546073	.58658730	.58771097	.58883173	.58994960
3.9	.59659710	.59769519	.59879051	.59988307	.60097290
4.0	.60745502	.60852603	.60959441	.61066016	.61172331
4.1	.61804810	.61909333	.62013606	.62117628	.62221402
4.2	.62838893	.62940960	.63042788	.63144377	.63245729
4.3	.63848926	.63948649	.64048144	.64147411	.64246452
4.4	.64836001	.64933486	.65030752	.65127801	.65224634
4.5	.65801140	.65896484	.65991620	.66086548	.66181269
4.6	.66745295	.66838592	.66931688	.67024585	.67117284
4.7	.67669361	.67760696	.67851838	.67942790	.68033551
4.8	.68574174	.68663627	.68752896	.68841982	.68930886
4.9	.69460520	.69548168	.69635639	.69722934	.69810055
5.0	.70329138	.70415052	.70500796	.70586371	.70671778
5.1	.71180723	.71264970	.71349054	.71432976	.71516736
5.2	.72015930	.72098574	.72181062	.72263392	.72345567
5.3	.72835378	.72916479	.72997429	.73078228	.73158876
5.4	.73639650	.73719264	.73798733	.73878056	.73957234

TABLE NO. Ia.

Logarithms to Eight Places

N	0	1	2	3	4
5.5	.74032690	.74115160	.74193908	.74272513	.74350976
5.6	.74818803	.74896286	.74973632	.75050839	.75127910
5.7	.75587486	.75663611	.75739603	.75815462	.75891189
5.8	.76342799	.76417613	.76492298	.76566855	.76641285
5.9	.77085201	.77158748	.77232171	.77305469	.77378645
6.0	.77815125	.77887447	.77959649	.78031731	.78103694
6.1	.78532984	.78604121	.78675142	.78746047	.78816837
6.2	.79239169	.79309160	.79379038	.79448805	.79518459
6.3	.79934055	.80002936	.80071708	.80140371	.80208926
6.4	.80617997	.80685803	.80753503	.80821097	.80888587
6.5	.81291336	.81358099	.81424760	.81491318	.81557775
6.6	.81954394	.82020146	.82085799	.82151353	.82216808
6.7	.82607480	.82672252	.82736927	.82801506	.82865990
6.8	.83250891	.83314711	.83378437	.83442070	.83505610
6.9	.83884909	.83947805	.84010609	.84073323	.84135947
7.0	.84509804	.84571802	.84633711	.84695532	.84757266
7.1	.85125835	.85186960	.85247999	.85308953	.85369821
7.2	.85733250	.85793526	.85853720	.85913830	.85973857
7.3	.86332286	.86391738	.86451108	.86510397	.86569605
7.4	.86923172	.86981821	.87040391	.87098881	.87157294
7.5	.87506126	.87563994	.87621784	.87679498	.87737135
7.6	.88081359	.88138466	.88195497	.88252454	.88309336
7.7	.88649073	.88705438	.88761730	.88817949	.88874096
7.8	.89209460	.89265103	.89320675	.89376176	.89431606
7.9	.89762709	.89817648	.89872518	.89927319	.89982050
8.0	.90308999	.90363252	.90417437	.90471555	.90525605
8.1	.90848502	.90902085	.90955603	.91009055	.91062440
8.2	.91381385	.91434316	.91487182	.91539984	.91592721
8.3	.91907809	.91960102	.92012333	.92064500	.92116605
8.4	.92427929	.92479600	.92531209	.92582757	.92634245
8.5	.92941893	.92992956	.93043959	.93094903	.93145787
8.6	.93449845	.93500315	.93550727	.93601080	.93651374
8.7	.93951925	.94001816	.94051648	.94101424	.94151143
8.8	.94448267	.94497591	.94546859	.94596070	.94645227
8.9	.94939001	.94987770	.95036485	.95085146	.95133752
9.0	.95424251	.95472479	.95520654	.95568775	.95616843
9.1	.95904139	.95951888	.95999484	.96047078	.96094620
9.2	.96378783	.96425963	.96473092	.96520170	.96567197
9.3	.96848295	.96894968	.96941591	.96988164	.97034688
9.4	.97312785	.97358962	.97405090	.97451169	.97497199
9.5	.97772361	.97818052	.97863695	.97909290	.97954837
9.6	.98227123	.98272339	.98317507	.98362629	.98407703
9.7	.98677173	.98721923	.98766626	.98811284	.98855896
9.8	.99122608	.99166901	.99211149	.99255352	.99299510
9.9	.99563519	.99607365	.99651167	.99694925	.99738638

TABLE NO. Ia.

Logarithms to Eight Places

	5	6	7	8	9
5.5	.74429298	.74507479	.74585520	.74663420	.74741181
5.6	.75204845	.75281643	.75358306	.75434834	.75511227
5.7	.75966784	.76042248	.76117581	.76192784	.76267856
5.8	.76715587	.76789762	.76863810	.76937733	.77011529
5.9	.77451697	.77524626	.77597433	.77670118	.77742682
6.0	.78175375	.78247262	.78318869	.78390358	.78461729
6.1	.78887512	.78958071	.79028516	.79098848	.79169065
6.2	.79588002	.79657433	.79726754	.79795964	.79865065
6.3	.80277337	.80345712	.80413943	.80482068	.80550086
6.4	.80955971	.81023252	.81090428	.81157501	.81224470
6.5	.81624130	.81690383	.81756537	.81822589	.81888541
6.6	.82282165	.82347423	.82412583	.82477646	.82542612
6.7	.82930377	.82994670	.83058867	.83122969	.83186977
6.8	.83569057	.83632412	.83695674	.83758844	.83821922
6.9	.84198480	.84260924	.84323278	.84385542	.84447718
7.0	.84818912	.84880470	.84941941	.85003326	.85064624
7.1	.85430604	.85491302	.85551916	.85612444	.85672889
7.2	.86033801	.86093662	.86153441	.86213138	.86272753
7.3	.86628734	.86687781	.86746749	.86805636	.86864444
7.4	.87215627	.87273883	.87332061	.87390160	.87448182
7.5	.87794695	.87852180	.87909588	.87966921	.88024178
7.6	.88366144	.88422877	.88479536	.88536122	.88592634
7.7	.88930170	.88986172	.89042102	.89097960	.89153746
7.8	.89486966	.89542255	.89597473	.89652622	.89707700
7.9	.90036713	.90091307	.90145832	.90200289	.90254678
8.0	.90579588	.90633504	.90687353	.90741136	.90794852
8.1	.91115761	.91169016	.91222206	.91275330	.91328390
8.2	.91645395	.91698005	.91750551	.91803034	.91855453
8.3	.92168648	.92220628	.92272546	.92324402	.92376196
8.4	.92685671	.92737036	.92788341	.92839585	.92890769
8.5	.93196611	.93247376	.93298082	.93348729	93399316
8.6	.93701611	.93751789	.93801910	.93851973	.93901978
8.7	.94200805	.94250411	.94299959	.94349452	.94398887
8.8	.94694327	.94743372	.94792362	.94841297	.94890176
8.9	.95182304	.95230801	.95279244	.95327634	.95375969
9.0	.95664858	.95712820	.95760728	.95808585	.95856388
9.1	.96142109	.96189547	.96236934	.96284268	.96331551
9.2	.96614173	.96661099	.96707973	96754798	.96801571
9.3	.97081162	.97127585	.97173959	.97220284	.97266559
9.4	.97543181	.97589114	.97634998	.97680834	.97726621
9.5	.98000337	.98045789	.98091194	.98136551	.98181861
9.6	.98452731	.98497713	.98542647	.98587536	.98632378
9.7	.98900462	.98944982	.98989456	.99033885	.99078269
9.8	.99343623	.99387691	.99431715	.99475694	.99519629
9.9	.99782308	.99825934	.99869516	.99913054	.99956549

TABLE NO. Ib.

Logarithms to Eight Places

N.		0	1	2	3	4	5	6	7	8	9
1.0000	0.0000	0000	0434	0869	1303	1737	2171	2606	3040	3474	3908
1.0001		4343	4777	5211	5645	6080	6514	6948	7382	7817	8251
1.0002		8685	9119	9553	9988	0422	0856	1290	1724	2159	2593
1.0003	.0001	3227	3461	3895	4329	4764	5198	5632	6066	6500	6934
1.0004		7368	7802	8237	8671	9105	9539	9973	0407	0841	1275
1.0005	.0002	1709	2143	2577	3012	3446	3880	4314	4748	5182	5616
1.0006		6050	6484	6918	7352	7786	8220	8654	9088	9522	9956
1.0007	.0003	0390	0824	1258	1692	2126	2560	2994	3428	3862	4296
1.0008		4730	5164	5598	6031	6465	6899	7333	7767	8201	8635
1.0009		9069	9503	9937	0371	0805	1228	1672	2106	2540	2974
1.0010	.0004	3408	3842	4275	4709	5143	5577	6011	6445	6878	7312
1.0011		7746	8180	8614	9048	9481	9915	0349	0783	1217	1650
1.0012	.0005	2084	2518	2952	3385	3819	4253	4687	5120	5554	5988
1.0013		6422	6855	7289	7723	8187	8590	9024	9458	9891	0325
1.0014	.0006	0759	1192	1626	2060	2493	2927	3361	3794	4228	4662
1.0015		5095	5529	5963	6396	6830	7264	7697	8131	8564	8998
1.0016		9432	9865	0299	0732	1166	1600	2033	2467	2900	3334
1.0017	.0007	3767	4201	4634	5068	5502	5935	6369	6802	7236	7669
1.0018		8103	8536	8970	9403	9837	0270	0704	1137	1571	2004
1.0019	.0008	2438	2871	3305	3738	4172	4605	5038	5472	5905	6339
1.0020		6772	7206	7639	8022	8506	8930	9373	9806	0239	0673
1.0021	.0009	1106	1540	1937	2406	2840	3273	3706	4140	4573	5006
1.0022		5440	5873	6307	6740	7173	7606	8040	8473	8906	9340
1.0023		9773	0206	0640	1073	1506	1939	2373	2806	3239	3673
1.0024	.0010	4106	4539	4972	5406	5839	6272	6705	7138	7572	8005
1.0025		8438	8871	9305	9738	0171	0604	1037	1471	1904	2337
1.0026	.0011	2770	3203	3636	4070	4503	4936	5369	5802	6235	6668
1.0027		7101	7535	7968	8401	8834	9267	9700	0133	0566	0999
1.0028	.0012	1433	1866	2299	2732	3165	3598	4031	4464	4897	5330
1.0029		5763	6196	6629	7062	7495	7928	8361	8794	9227	9660
1.0030	.0013	0093	0526	0959	1392	1825	2258	2691	3124	3557	3990
1.0031		4423	4856	5289	5722	6155	6588	7021	7454	7887	8319
1.0032		8752	9185	9618	0051	0484	0917	1350	1783	2215	2648
1.0033	.0014	3081	3514	3947	4380	4813	5246	5678	6111	6544	6977
1.0034		7410	7842	8275	8708	9141	9574	0007	0439	0872	1305
1.0035	.0015	1738	2170	2603	3036	3469	3902	4334	4767	5200	5633
1.0036		6065	6498	6931	7363	7796	8229	8662	9094	9527	9960
1.0037	.0016	0392	0825	1258	1690	2123	2556	2988	3421	3854	4286
1.0038		4719	5152	5584	6017	6450	6832	7315	7748	8180	8613
1.0039		9045	9478	9911	0343	0776	1208	1641	3074	2506	2939
1.0040	.0017	3371	3804	4236	4669	5102	5534	5967	6399	6832	7264
1.0041		7697	8129	8562	8994	9427	9859	0292	0724	1157	1589
1.0042	.0018	2022	2454	2887	3319	3752	4184	4616	5049	5481	5914
1.0043		6346	6779	7211	7644	8076	8508	8941	9373	9806	0238
1.0044	.0019	0670	1103	1535	1968	2400	2830	3265	3697	4129	4562
1.0045		4994	5426	5859	6291	6723	7156	7588	8020	8453	8885
1.0046		9317	9750	0182	0614	1047	1479	1911	2343	2776	3208
1.0047	.0020	3640	4072	4505	4937	5369	5801	6234	6666	7098	7530
1.0048		7963	8395	8827	9259	9691	0124	0556	0988	1420	1852
1.0049	.0021	2285	2717	3149	3581	4013	4445	4878	5310	5742	6174

TABLE NO. Ib.

Logarithms to Eight Places

N.		0	1	2	3	4	5	6	7	8	9
1.0050	.0021	6606	7038	7470	7903	8335	8767	9199	9631	0063	0495
1.0051	.0022	0927	1359	1791	2224	2656	3088	3521	3952	4384	4816
1.0052		5248	5680	6112	6544	6976	7408	7840	8272	8704	9136
1.0053		9568	0000	0432	0864	1296	1728	2160	2592	3024	3456
1.0054	.0023	3888	4320	4752	5184	5616	6048	6480	6912	7344	7776
1.0055		8207	8639	9071	9503	9935	0367	0799	1231	1663	2095
1.0056	.0024	2526	2958	3390	3822	4254	4686	5118	5549	5981	6413
1.0057		6845	7277	7709	8140	8572	9004	9436	9868	0300	0713
1.0058	.0025	1163	1595	2027	2458	2890	3322	3754	4186	4617	5049
1.0059		5481	5913	6344	6776	7208	7639	8071	8503	8935	9366
1.0060		9798	0230	0661	1093	1525	1957	2388	2820	3252	3683
1.0061	.0026	4115	4547	4978	5410	5842	6273	6705	7136	7568	8000
1.0062		8431	8863	9295	9726	0158	0589	1021	1453	1884	2316
1.0063	.0027	2747	3179	3610	4042	4474	4905	5337	5768	6200	6631
1.0064		7063	7494	7926	8357	8799	9220	9650	0083	0515	0946
1.0065	.0028	1378	1809	2241	2672	3104	3525	3967	4398	4830	5261
1.0066		5693	6124	6555	6987	7418	7850	8281	8713	9144	9575
1.0067	.0029	0007	0438	0870	1301	1732	2164	2595	3027	3458	3889
1.0068		4321	4752	5183	5615	6046	6477	6909	7340	7771	8203
1.0069		8634	9065	9497	9928	0359	0791	1222	1653	2084	2516
1.0070	.0030	2947	3378	3810	4241	4672	5103	5535	5966	6397	6828
1.0071		7260	7691	8122	8553	8984	9416	9847	0278	0709	1141
1.0072	.0031	1572	2003	2434	2865	3296	3728	4159	4590	5021	5452
1.0073		5883	6315	6746	7177	7608	8039	8470	8901	9332	9764
1.0074	.0032	0195	0625	1057	1488	1919	2350	2781	3212	3643	4074
1.0075		4505	4937	5368	5799	6230	6661	7092	7523	7954	8385
1.0076		8816	9247	9678	0109	0540	0971	1402	1833	2264	2695
1.0077	.0033	3126	3557	3988	4419	4850	5281	5712	6143	6574	7004
1.0078		7435	7866	8297	8728	9159	9590	0021	0452	0883	1314
1.0079	.0034	1745	2175	2606	3037	3468	3899	4330	4761	5192	5622
1.0080		6053	6484	6915	7346	7777	8207	8638	9069	9500	9931
1.0081	0035	0361	0792	1223	1654	2085	2515	2946	3377	3808	4239
1.0082		4669	5100	5531	5962	6392	6823	7254	7685	8115	8546
1.0083		8977	9407	9838	0269	0700	1130	1561	1992	2422	2853
1.0084	.0036	3284	3714	4145	4576	5006	5437	5868	6298	6729	7160
1.0085		7590	8021	8452	8882	9313	9743	0174	0605	1035	1466
1.0086	.0037	1896	2327	2758	3188	3619	4049	4480	4910	5341	5772
1.0087		6202	6633	7063	7494	7924	8355	8785	9216	9646	0077
1.0088	.0038	0507	0938	1368	1799	2229	2660	3090	3521	3951	4382
1.0089		4812	5243	5673	6104	6534	6964	7395	7825	8256	8686
1.0090		9117	9547	9977	0408	0838	1269	1699	2129	2560	2990
1.0091	.0039	3421	3851	4281	4712	5142	5572	6003	6433	6864	7294
1.0092		7724	8155	8585	9015	9445	9876	0306	0736	1167	1597
1.0093	.0040	2027	2458	2888	3318	3748	4179	4609	4639	5470	5900
1.0094		6330	6760	7191	7621	8051	8481	8911	9342	9772	0202
1.0095	.0041	0632	1063	1493	1923	2353	2783	3213	3644	4074	4504
1.0096		4934	5364	5795	6225	6655	7085	7515	7945	8375	8806
1.0097		9236	9666	0096	0526	0956	1386	1816	2246	2676	3107
1.0098	.0042	3537	3967	4397	4827	5257	5687	6117	6547	6977	7407
1.0099		7837	8267	8697	9127	9557	9987	0417	0847	1277	1707

TABLE NO. II.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
1	1.002500	1.003333	1.003750	1.004167	1.005000
2	1.005006	1.006678	1.007514	1.008350	1.010025
3	1.007519	1.010033	1.011292	1.012552	1.015075
4	1.010038	1.013401	1.015082	1.016771	1.020151
5	1.012563	1.016878	1.018891	1.021008	1.025251
6	1.015094	1.020167	1.022712	1.025262	1.030378
7	1.017632	1.023568	1.026547	1.029536	1.035529
8	1.020176	1.026980	1.030396	1.033824	1.040707
9	1.022726	1.030403	1.034260	1.038131	1.045906
10	1.025283	1.033833	1.038136	1.042478	1.051140
1	1.027846	1.037284	1.042032	1.046800	1.056396
2	1.030416	1.040742	1.045939	1.051162	1.061678
3	1.032992	1.044211	1.049862	1.055543	1.066986
4	1.035574	1.047691	1.053799	1.059940	1.072321
5	1.038163	1.051184	1.057748	1.064356	1.077683
6	1.040757	1.054688	1.061717	1.068789	1.083071
7	1.043361	1.058205	1.065698	1.073244	1.088487
8	1.045969	1.061731	1.069695	1.077716	1.093928
9	1.048584	1.065268	1.073706	1.082267	1.099398
20	1.051206	1.068821	1.077733	1.086716	1.104896
1	1.053834	1.072383	1.081774	1.091244	1.110420
2	1.056468	1.075958	1.085831	1.095590	1.115972
3	1.059109	1.079544	1.089904	1.100356	1.121552
4	1.061757	1.083143	1.093990	1.104941	1.127160
5	1.064411	1.086754	1.098092	1.109545	1.132796
6	1.067072	1.090379	1.102240	1.114169	1.138459
7	1.069741	1.094011	1.106343	1.118808	1.144152
8	1.072414	1.097657	1.110492	1.123472	1.149873
9	1.075095	1.101316	1.114656	1.128153	1.155622
30	1.077784	1.104987	1.118836	1.132741	1.161400
1	1.080478	1.108670	1.123032	1.137574	1.167207
2	1.083178	1.112366	1.127243	1.142314	1.173043
3	1.085887	1.116074	1.131495	1.147074	1.178908
4	1.088602	1.119794	1.135715	1.151850	1.184803
5	1.091323	1.123527	1.139992	1.156653	1.190727
6	1.094051	1.127272	1.144248	1.161472	1.196681
7	1.096787	1.131029	1.148538	1.166312	1.202664
8	1.099529	1.134800	1.152846	1.171171	1.208677
9	1.102277	1.138582	1.157168	1.176051	1.214721
40	1.105033	1.142377	1.161508	1.180951	1.220794
1	1.107796	1.146185	1.165864	1.185872	1.226898
2	1.110570	1.150006	1.170236	1.190813	1.233033
3	1.113342	1.153839	1.174624	1.195775	1.239198
4	1.117625	1.157686	1.179029	1.200757	1.245394
5	1.118918	1.161544	1.183450	1.205760	1.251621
6	1.121713	1.165416	1.187888	1.210784	1.257879
7	1.124517	1.169301	1.192343	1.215830	1.264168
8	1.127328	1.173198	1.196814	1.220895	1.270494
9	1.130146	1.177109	1.201302	1.225982	1.276847
50	1.132972	1.181036	1.205807	1.231090	1.283226
1	1.135904	1.184969	1.210329	1.236220	1.289639
2	1.138641	1.188920	1.214868	1.241371	1.296090
3	1.141490	1.192883	1.219423	1.246544	1.302571
4	1.144344	1.196859	1.223996	1.251737	1.309084
5	1.147205	1.200849	1.228586	1.256953	1.315629
6	1.150073	1.204851	1.233194	1.262190	1.322207
7	1.152948	1.208868	1.237818	1.267450	1.328818
8	1.155830	1.212894	1.242460	1.272730	1.335462
9	1.158720	1.216940	1.247119	1.278033	1.342133
60	1.161617	1.220996	1.251796	1.283359	1.348850

TABLE NO. II. Continued.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated.

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
61	1.164521	1.225067	1.256461	1.288706	1.355594
2	1.167432	1.229150	1.261202	1.294075	1.362372
3	1.170351	1.233247	1.265902	1.299467	1.369184
4	1.173276	1.237358	1.270678	1.304882	1.376030
5	1.176210	1.241483	1.275443	1.310319	1.382910
6	1.179150	1.245621	1.280227	1.315772	1.389825
7	1.182098	1.249973	1.285027	1.321230	1.396772
8	1.185058	1.253942	1.289816	1.326766	1.403758
9	1.188016	1.258119	1.294683	1.332294	1.410776
70	1.190986	1.262312	1.299538	1.337846	1.417830
1	1.193991	1.266520	1.304412	1.343420	1.424920
2	1.196948	1.270742	1.309303	1.349018	1.432044
3	1.199941	1.274978	1.314213	1.354640	1.439204
4	1.202941	1.279228	1.319141	1.360283	1.446401
5	1.205948	1.283491	1.324088	1.365951	1.453632
6	1.208963	1.287770	1.329050	1.371638	1.460901
7	1.211985	1.292063	1.334038	1.377357	1.468206
8	1.215015	1.296369	1.339040	1.383096	1.475546
9	1.218053	1.300691	1.344030	1.388859	1.482924
80	1.221098	1.305026	1.349102	1.394645	1.490339
1	1.224151	1.309376	1.354161	1.400457	1.497790
2	1.227211	1.313741	1.359239	1.406292	1.505279
3	1.230279	1.318120	1.364336	1.412151	1.512806
4	1.233355	1.322514	1.369452	1.418035	1.520369
5	1.236438	1.326922	1.374585	1.423944	1.527971
6	1.239529	1.331345	1.379742	1.429877	1.535611
7	1.242628	1.335783	1.384916	1.435835	1.543289
8	1.245738	1.340236	1.390110	1.441817	1.551006
9	1.248850	1.344703	1.395323	1.447825	1.558761
90	1.251971	1.349185	1.400555	1.453858	1.566555
1	1.255101	1.353683	1.405808	1.459915	1.574387
2	1.258239	1.358198	1.411079	1.465998	1.582259
3	1.261385	1.362722	1.416771	1.472107	1.590171
4	1.264538	1.367265	1.421882	1.478240	1.598118
5	1.267699	1.371822	1.427010	1.484400	1.606112
6	1.270868	1.376395	1.432365	1.490585	1.614142
7	1.274046	1.380983	1.437736	1.496792	1.622213
8	1.277231	1.385587	1.443128	1.503032	1.630325
9	1.280424	1.390305	1.448539	1.509295	1.638476
100	1.283624	1.394839	1.453971	1.515588	1.646668
1	1.286834	1.399488	1.459424	1.521899	1.654902
2	1.290051	1.404153	1.464894	1.528237	1.663176
3	1.293276	1.408834	1.470390	1.534608	1.671492
4	1.296509	1.413530	1.475904	1.541002	1.679850
5	1.299751	1.418242	1.481439	1.547423	1.688249
6	1.303000	1.422970	1.486994	1.553871	1.696690
7	1.306260	1.427712	1.492570	1.560345	1.705174
8	1.309523	1.432471	1.498167	1.566847	1.713699
9	1.312797	1.437247	1.503787	1.573375	1.722268
110	1.316079	1.442037	1.509418	1.579931	1.730879
1	1.319369	1.446844	1.515085	1.586514	1.739534
2	1.322667	1.451665	1.520767	1.593122	1.748232
3	1.325974	1.456502	1.526469	1.599862	1.756973
4	1.329289	1.461361	1.532193	1.606428	1.765758
5	1.332612	1.466235	1.537939	1.613121	1.774586
6	1.335944	1.471119	1.543707	1.619843	1.783459
7	1.339284	1.476023	1.549495	1.626592	1.792377
8	1.342632	1.480943	1.555306	1.633370	1.801338
9	1.345988	1.485880	1.561139	1.640175	1.810345
120	1.349354	1.490833	1.566992	1.646630	1.819397

TABLE NO. II. Continued.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	5-8 %	3-4 %	7-8 %	1 %	1 1-8 %
1	1.006250	1.007500	1.008750	1.010000	1.011250
2	1.012539	1.015056	1.017596	1.020100	1.022663
3	1.018865	1.022690	1.026510	1.030301	1.034131
4	1.025235	1.030339	1.035503	1.040604	1.045765
5	1.031643	1.038067	1.044571	1.051010	1.057531
6	1.038091	1.045852	1.053724	1.061520	1.069427
7	1.044576	1.043698	1.062955	1.072135	1.081458
8	1.051106	1.061599	1.072266	1.082857	1.093624
9	1.057679	1.069536	1.081659	1.093685	1.105927
10	1.064287	1.077582	1.091134	1.104622	1.118369
1	1.070939	1.085666	1.100693	1.115668	1.130951
2	1.077633	1.093807	1.110335	1.126825	1.143674
3	1.084368	1.102010	1.120062	1.138093	1.156540
4	1.091145	1.110275	1.129873	1.149474	1.169524
5	1.097965	1.118603	1.139771	1.160969	1.182709
6	1.104826	1.126992	1.149755	1.172579	1.196014
7	1.111732	1.135444	1.159827	1.184304	1.209467
8	1.118583	1.143960	1.169987	1.196174	1.223076
9	1.125672	1.152540	1.180236	1.208109	1.236835
20	1.132708	1.161182	1.190575	1.220190	1.250750
1	1.139787	1.169993	1.201005	1.232391	1.264821
2	1.146910	1.178667	1.211525	1.244716	1.274050
3	1.154074	1.187571	1.222136	1.257163	1.293439
4	1.161272	1.196413	1.232813	1.269735	1.307990
5	1.168585	1.205386	1.243644	1.282432	1.322705
6	1.175953	1.214426	1.254539	1.295256	1.337586
7	1.183202	1.223506	1.265529	1.308209	1.352633
8	1.190597	1.232711	1.277615	1.321291	1.317851
9	1.198038	1.241956	1.287798	1.334504	1.383239
30	1.205515	1.251271	1.299079	1.347849	1.398800
1	1.213060	1.260650	1.310460	1.361327	1.414537
2	1.220642	1.270110	1.321938	1.374940	1.430450
3	1.228271	1.279626	1.333519	1.388690	1.446543
4	1.235949	1.289233	1.345200	1.402577	1.462816
5	1.243673	1.298903	1.356984	1.416603	1.479273
6	1.251445	1.308644	1.368871	1.430769	1.495915
7	1.259267	1.318456	1.380862	1.445076	1.512744
8	1.267134	1.328348	1.392991	1.459527	1.529762
9	1.275054	1.338310	1.405151	1.474122	1.546972
40	1.283026	1.348348	1.417470	1.488864	1.564376
1	1.291045	1.358460	1.427884	1.503752	1.581975
2	1.299114	1.368649	1.442413	1.518789	1.599772
3	1.307233	1.378913	1.455049	1.533977	1.617770
4	1.315404	1.389255	1.467795	1.549318	1.635969
5	1.323625	1.399675	1.480653	1.564811	1.654374
6	1.331897	1.410172	1.493623	1.580459	1.672986
7	1.340222	1.420749	1.506711	1.596263	1.691807
8	1.348598	1.431404	1.519906	1.612226	1.710840
9	1.357024	1.442140	1.533221	1.628348	1.730087
50	1.365508	1.452956	1.546652	1.644632	1.749550
1	1.374053	1.463853	1.559412	1.661078	1.769232
2	1.382630	1.474832	1.573058	1.677690	1.789137
3	1.391274	1.485893	1.586822	1.694466	1.809264
4	1.399968	1.497052	1.600707	1.711411	1.829614
5	1.408717	1.508266	1.614713	1.728525	1.850202
6	1.417521	1.519577	1.628841	1.745810	1.871016
7	1.426381	1.530974	1.643093	1.763268	1.892065
8	1.435296	1.542456	1.657471	1.780901	1.913351
9	1.444267	1.554024	1.671973	1.798711	1.934876
60	1.453293	1.565681	1.686603	1.816676	1.956644

TABLE NO. II. Continued.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	1 1-4 %	1 3-8 %	1 1-2 %	1 5-8 %	1 3-4 %
1	1.012500	1.013750	1.015000	1.016250	1.017500
2	1.025156	1.027688	1.030225	1.032764	1.035306
3	1.037971	1.041819	1.045678	1.049547	1.053424
4	1.050945	1.056144	1.061368	1.066601	1.071859
5	1.064082	1.073135	1.077284	1.083934	1.090617
6	1.077383	1.085388	1.093443	1.101548	1.109702
7	1.090951	1.100312	1.109845	1.119446	1.129122
8	1.104486	1.115442	1.126493	1.137639	1.148881
9	1.118292	1.130779	1.143390	1.156120	1.168987
10	1.132271	1.146322	1.160541	1.174913	1.189444
1	1.146424	1.162089	1.177949	1.194005	1.210260
2	1.160754	1.178068	1.195618	1.213406	1.231439
3	1.175264	1.194264	1.213552	1.233125	1.252989
4	1.189954	1.210687	1.231756	1.253164	1.274917
5	1.204829	1.227335	1.250232	1.273527	1.297228
6	1.219889	1.244210	1.268986	1.294223	1.319929
7	1.235137	1.261318	1.288020	1.315254	1.343028
8	1.250577	1.278661	1.307340	1.336630	1.366531
9	1.266209	1.296243	1.326951	1.358347	1.390445
20	1.282037	1.314066	1.346855	1.380423	1.414778
1	1.298062	1.332135	1.367058	1.402851	1.439537
2	1.317318	1.350452	1.387564	1.425648	1.464728
3	1.330716	1.369020	1.408377	1.448815	1.490361
4	1.347351	1.387844	1.429503	1.472358	1.516443
5	1.364193	1.406927	1.450945	1.496284	1.542980
6	1.381245	1.426272	1.472709	1.520599	1.569983
7	1.398511	1.449217	1.494800	1.545308	1.597457
8	1.415992	1.465765	1.517222	1.570420	1.625412
9	1.433692	1.485919	1.539980	1.595938	1.653857
30	1.451613	1.506350	1.563080	1.621872	1.682800
1	1.469758	1.527063	1.586526	1.648228	1.712249
2	1.488130	1.548060	1.610324	1.675012	1.742213
3	1.510205	1.569346	1.634479	1.702231	1.772702
4	1.525566	1.590924	1.658996	1.729892	1.803724
5	1.544636	1.612800	1.683881	1.758003	1.835289
6	1.567549	1.634973	1.709139	1.786571	1.867407
7	1.583492	1.657456	1.734777	1.815602	1.900087
8	1.603286	1.680246	1.760798	1.845106	1.933338
9	1.623328	1.703351	1.787210	1.875087	1.967172
40	1.643619	1.726771	1.814018	1.905555	2.001597
1	1.664164	1.750514	1.841229	1.936525	2.036625
2	1.684967	1.774583	1.868847	1.967993	2.072266
3	1.706029	1.798984	1.896879	1.999973	2.108530
4	1.727354	1.823720	1.925333	2.032480	2.145430
5	1.748938	1.848797	1.954213	2.065500	1.182975
6	1.770807	1.874217	1.983521	2.099064	2.221177
7	1.792942	1.899988	2.013279	2.133177	2.260048
8	1.815343	1.926113	2.043478	2.167839	2.299598
9	1.838046	1.952596	2.074130	2.203065	2.339842
50	1.861021	1.979445	2.105242	2.238865	2.380789
1	1.884295	2.006663	2.136821	2.275247	2.422452
2	1.907838	2.034254	2.168873	2.312220	2.464845
3	1.931686	2.062225	2.201406	2.349793	2.507980
4	1.955833	2.090581	2.234427	2.387977	2.551863
5	1.980280	2.119327	2.267943	2.426782	2.596527
6	2.005034	2.148467	2.301963	2.466218	2.641966
7	2.030097	2.178008	2.336492	2.506293	2.688200
8	2.055472	2.207956	2.371539	2.547022	2.735344
9	2.081166	2.238315	2.407112	2.588409	2.783110
60	2.107181	2.269093	2.443219	2.630385	2.831815

TABLE NO. II. Continued.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	1 7-8 %	2 %	2 1-4 %	2 1-2 %	2 3-4 %
1	1.018750	1.020000	1.022500	1.025000	1.027500
2	1.037851	1.040400	1.045506	1.050625	1.055756
3	1.057311	1.061208	1.069030	1.076891	1.084789
4	1.077136	1.082422	1.093833	1.103813	1.114621
5	1.097332	1.104081	1.111678	1.131308	1.145273
6	1.117967	1.126162	1.242825	1.159693	1.176768
7	1.138867	1.148686	1.168539	1.188686	1.209129
8	1.160221	1.171659	1.194831	1.218403	1.242380
9	1.181775	1.195093	1.221714	1.248863	1.276546
10	1.204137	1.218994	1.244920	1.280085	1.311651
1	1.226715	1.243374	1.277310	1.312087	1.347721
2	1.249715	1.268242	1.306050	1.344889	1.384784
3	1.273148	1.293607	1.335436	1.378511	1.422865
4	1.297019	1.319479	1.365483	1.412974	1.461994
5	1.321338	1.345868	1.396207	1.448298	1.502199
6	1.346113	1.372786	1.427621	1.484506	1.543509
7	1.371353	1.400241	1.459743	1.526618	1.585956
8	1.397283	1.428246	1.492587	1.559659	1.629569
9	1.423261	1.456811	1.526170	1.598650	1.674383
20	1.449946	1.485947	1.560509	1.638616	1.720428
1	1.477133	1.515666	1.595621	1.679582	1.767740
2	1.504829	1.545980	1.631522	1.721571	1.816353
3	1.533044	1.576899	1.668231	1.764611	1.866303
4	1.561790	1.608437	1.705766	1.808726	1.917626
5	1.591072	1.640606	1.744146	1.853944	1.970361
6	1.620905	1.673418	1.783389	1.900293	2.024546
7	1.651297	1.706886	1.823516	1.947800	2.080221
8	1.682259	1.741024	1.864545	1.996495	2.137427
9	1.713800	1.775845	1.906497	2.046407	2.196206
30	1.745931	1.811362	1.949393	2.097568	2.256602
1	1.778671	1.847589	1.993255	2.150007	2.318658
2	1.812021	1.884541	2.038103	2.203757	2.382421
3	1.845996	1.922231	2.083960	2.258851	2.447937
4	1.880687	1.960676	2.130849	2.315322	2.515256
5	1.915870	1.999889	2.178973	2.373205	2.584426
6	1.951343	2.039887	2.227816	2.432535	2.655497
7	1.988388	2.080685	2.277942	2.493349	2.728524
8	2.025671	2.122299	2.329196	2.555682	2.803558
9	2.063647	2.164745	2.381603	2.619574	2.880656
40	2.102345	2.208040	2.435189	2.685064	2959874
1	2.141764	2.252200	2.489981	2.752190	3.041270
2	2.181922	2.297244	2.546005	2.820995	3.124905
3	2.222833	2.343189	2.603290	2.891520	3.210840
4	2.264511	2.390053	2.661864	2.963808	3.299138
5	2.306960	2.437854	2.721756	3.037903	3.389865
6	2.350226	2.486611	2.782996	3.113851	3.483086
7	2.394292	2.536344	2.845613	3.191697	3.578871
8	2.439185	2.587070	2.909639	3.271490	3.677289
9	2.484920	2.638812	2.975106	3.353277	3.778415
50	2.531512	2.691588	3.042046	3.437108	3.882321
1	2.578978	2.745420	3.110492	3.523036	3.989086
2	2.627331	2.800328	3.180478	3.611112	4.098785
3	2.676595	2.856334	3.252039	3.701390	4.211502
4	2.726781	2.913461	3.325210	3.793925	4.327318
5	2.777989	2.971730	3.400027	3.888773	4.446319
6	2.829930	3.031165	3.476528	3.985992	4.568593
7	2.883057	3.091788	3.554750	4.085642	4.694231
8	2.937114	3.153624	3.634732	4.187783	4.823320
9	2.992185	3.216696	3.716513	4.292478	4.955962
60	3.048288	3.281030	3.800135	4.399790	5.092251

TABLE NO. II. Continued.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	3 %	3 1-2 %	4 %	4 1-2 %	5 %
1	1.030000	1.035000	1.040000	1.045000	1.050000
2	1.060900	1.071225	1.081600	1.092025	1.102500
3	1.092727	1.108718	1.124864	1.141166	1.157625
4	1.125509	1.147523	1.169859	1.192519	1.215506
5	1.159274	1.187686	1.216653	1.246182	1.276282
6	1.194052	1.229255	1.265319	1.302260	1.340096
7	1.229874	1.272279	1.315932	1.360862	1.407100
8	1.266770	1.316809	1.368569	1.422101	1.477455
9	1.304773	1.362897	1.423312	1.486095	1.551328
10	1.343916	1.410599	1.480244	1.552969	1.628894
1	1.384234	1.459969	1.539454	1.622853	1.710339
2	1.425761	1.511069	1.601032	1.695881	1.795856
3	1.468534	1.563956	1.665074	1.772196	1.885649
4	1.512590	1.618695	1.731676	1.851945	1.979932
5	1.557967	1.675349	1.800944	1.935282	2.078928
6	1.604706	1.733986	1.872981	2.022370	2.182875
7	1.652848	1.794676	1.947901	2.113377	2.292018
8	1.702433	1.857489	2.025817	2.208479	2.406619
9	1.753506	1.922501	2.106849	2.307850	2.526950
20	1.806111	1.989789	2.191123	2.411714	2.653298
1	1.860295	2.059431	2.278768	2.520241	2.785963
2	1.916103	2.131512	2.369918	2.633652	2.925261
3	1.973587	2.206114	2.464716	2.752166	3.071524
4	2.032794	2.283328	2.563304	2.876014	3.225100
5	2.093778	2.363245	2.665836	3.005434	3.386355
6	2.156591	2.445959	2.772470	3.140679	3.555673
7	2.221289	2.531567	2.883369	3.282010	3.733456
8	2.287928	2.620172	2.998703	3.429700	3.920129
9	2.356566	2.711878	3.118651	3.584036	4.116136
30	2.427262	2.806794	3.243397	3.745318	4.321942
1	2.500080	2.905031	3.373133	3.913857	4.538039
2	2.575083	3.006708	3.508059	4.089981	4.764942
3	2.652335	3.111942	3.648381	4.274030	5.003189
4	2.731905	3.220860	3.794316	4.466362	5.253348
5	2.813862	3.333590	3.946089	4.667348	5.516015
6	2.898278	3.450266	4.103933	4.877378	5.791816
7	2.985227	3.571025	4.268090	5.096860	6.081407
8	3.074783	3.696011	4.438813	5.326219	6.385477
9	3.167027	3.825372	4.616366	5.565899	6.704751
40	3.262038	3.959260	4.801021	5.816365	7.039989
1	3.359899	4.097833	4.993061	6.078101	7.391988
2	3.460696	4.241258	5.192784	6.351615	7.761588
3	3.564517	4.389702	5.400495	6.637438	8.149667
4	3.671452	4.543342	5.616515	6.936123	8.557150
5	3.781596	4.702359	5.841176	7.248248	8.985008
6	3.895044	4.866941	6.074823	7.574420	9.434258
7	4.011895	5.037285	6.317816	7.915268	9.905971
8	4.132252	5.213589	6.570528	8.271456	10.401270
9	4.256219	5.396065	6.833349	8.643671	10.921333
50	4.383906	5.584927	7.106683	9.032636	11.467400
1	4.515423	5.780399	7.390951	9.439105	12.040770
2	4.650886	5.982713	7.686589	9.863865	12.642808
3	4.790412	6.192108	7.994057	10.307739	13.274949
4	4.934125	6.408832	8.313814	10.771587	13.938696
5	5.082149	6.633141	8.646367	11.256308	14.635631
6	5.234613	6.865301	8.992222	11.762842	15.367412
7	5.391651	7.105587	9.351910	12.292170	16.135783
8	5.553401	7.354282	9.725987	12.845318	16.942572
9	5.720003	7.611682	10.115026	13.423357	17.789701
60	5.891603	7.878091	10.519627	14.027408	18.679186

TABLE NO. II. Concluded.

The Amount of One Dollar at Compound Interest for the Time and at the Rate Per Cent Stated. $(1+i)^n$

Periods	6 %	7 %	8 %	9 %	10 %
1	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.191016	1.225043	1.259712	1.295029	1.441000
4	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.503630	1.605782	1.713824	1.828039	1.948717
8	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.790848	1.967151	2.158925	2.367364	2.593743
1	1.898299	2.104852	2.331639	2.580426	2.853117
2	2.012196	2.252192	2.518170	2.812665	3.138428
3	2.132928	2.409845	2.719624	3.065805	3.452271
4	2.260904	2.578534	2.937194	3.341727	3.797498
5	2.396558	2.759031	3.172169	3.642483	4.177248
6	2.540352	2.952164	3.425943	3.970306	4.594973
7	2.692773	3.158815	3.700018	4.327633	5.054470
8	2.854339	3.379932	3.996019	4.717120	5.559917
9	3.025600	3.616528	4.315701	5.141661	6.115909
20	3.207135	3.869684	4.660957	5.604411	6.727500
1	3.399564	4.140562	5.033834	6.108808	7.400250
2	3.603537	4.430402	5.436540	6.658600	8.140275
3	3.819750	4.740530	5.871464	7.257875	8.954302
4	4.048935	5.072367	6.341181	7.911083	9.849733
5	4.291871	5.427433	6.848475	8.623081	10.834706
6	4.549383	5.807353	7.396353	9.399158	11.918177
7	4.822346	6.213868	7.988062	10.245082	13.109994
8	5.111687	6.648838	8.627106	11.167139	14.420994
9	5.418388	7.114257	9.317275	12.172182	15.863093
30	5.743491	7.612255	10.062657	13.267679	17.449402
1	6.088101	8.145113	10.867669	14.461770	19.194343
2	6.453387	8.715271	11.737083	15.763329	21.113777
3	6.840590	9.325340	12.676050	17.182028	23.225154
4	7.251025	9.978114	13.690134	18.728411	25.547670
5	7.686087	10.676581	14.785344	20.413968	28.102437
6	8.147252	11.423942	15.668172	22.251225	30.912681
7	8.636087	12.223618	17.245626	24.253835	34.003949
8	9.154252	13.079271	18.625276	26.436681	37.404343
9	9.703508	13.994820	20.115298	28.815982	41.144778
40	10.285718	14.974458	21.724522	31.409420	45.259256
1	10.902861	16.022670	23.462483	34.236268	49.785181
2	11.557033	17.144257	25.339482	37.317532	54.763699
3	12.250455	18.344355	27.366640	40.676110	60.240069
4	12.985482	19.628459	29.555972	44.336960	66.264076
5	13.764611	21.002452	31.920449	48.327286	72.890484
6	14.590487	22.472623	34.474085	52.676742	80.179532
7	15.465917	24.045707	37.232012	57.417649	88.197485
8	16.393872	25.728907	40.210573	62.585237	97.017234
9	17.377504	27.529930	43.427419	68.217908	106.718957
50	18.420154	29.457025	46.901612	74.357520	117.390853
1	19.525364	31.519017	50.653742	81.049697	129.129938
2	20.696885	33.725348	54.706041	88.344170	142.042932
3	21.938698	36.086122	59.082524	96.295145	156.247225
4	23.255020	38.612151	63.809126	104.961708	171.871948
5	24.650322	41.315002	68.913856	114.408262	189.059143
6	26.129341	44.207052	73.816965	124.705006	207.965057
7	27.697101	47.300155	79.722322	135.928456	228.761562
8	28.358927	50.612653	86.100107	148.162017	251.637718
9	31.120463	54.155539	92.988116	161.496599	276.801490
60	32.987691	57.946427	100.427165	176.031292	304.481639

TABLE NO. III.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.997506	.996678	.996264	.995851	.995025
2	.995019	.993366	.992619	.991718	.990075
3	.992537	.990066	.988834	.987603	.985149
4	.990062	.986777	.985140	.983505	.980248
5	.987593	.983499	.981457	.979425	.975370
6	.985130	.980231	.977793	.975361	.970518
7	.982674	.976975	.974140	.971313	.965690
8	.980223	.973729	.970434	.967283	.960885
9	.977778	.970494	.966875	.963265	.956105
10	.975340	.967270	.963263	.959272	.951348
1	.972906	.964056	.959662	.955292	.946615
2	.970781	.960853	.956080	.951328	.941905
3	.968061	.957661	.952508	.947381	.937219
4	.965647	.954479	.948949	.943449	.932556
5	.963239	.951308	.945402	.939535	.927917
6	.960837	.948148	.941872	.935636	.923300
7	.958441	.944998	.938354	.931754	.918707
8	.956050	.941858	.934848	.927888	.914136
9	.953666	.938729	.931555	.924038	.909588
20	.951288	.935611	.927876	.920204	.905063
1	.948916	.932502	.924409	.916386	.900560
2	.946549	.929404	.920956	.912583	.896080
3	.944189	.926317	.917516	.908793	.891622
4	.941834	.923239	.914088	.905025	.887186
5	.939485	.920172	.910673	.901270	.882772
6	.937142	.917115	.907271	.897530	.878380
7	.934808	.914068	.903881	.893806	.874010
8	.932474	.911031	.900505	.890098	.869662
9	.930147	.908005	.897141	.886404	.865335
10	.927829	.904988	.893789	.882726	.861030
1	.925515	.901987	.890450	.879063	.856746
2	.923217	.898985	.887123	.875416	.852484
3	.920903	.895998	.887809	.871783	.848242
4	.918608	.893021	.880507	.868166	.844022
5	.916318	.890054	.877218	.864564	.839823
6	.914032	.887097	.873948	.860976	.835645
7	.911753	.884150	.870676	.857404	.831487
8	.909479	.881213	.867423	.853846	.827351
9	.907211	.878285	.864183	.850303	.823235
40	.904939	.875367	.860954	.846775	.819139
1	.902692	.872459	.857738	.843261	.815064
2	.900441	.869561	.854533	.839762	.811009
3	.898195	.866672	.851341	.836278	.806974
4	.895955	.863792	.848161	.832808	.802959
5	.893721	.860923	.844992	.829352	.798964
6	.891492	.858062	.841835	.825911	.794989
7	.889269	.855212	.838690	.822484	.791034
8	.887051	.852371	.835556	.819071	.787098
9	.884839	.849539	.832435	.815672	.783183
50	.882634	.846716	.829326	.812287	.779286
1	.880432	.843903	.826227	.808917	.775409
2	.878236	.841099	.823131	.805561	.771551
3	.876046	.838305	.820066	.802218	.767713
4	.873861	.835520	.817002	.798890	.763893
5	.871682	.832744	.813950	.795574	.760093
6	.869508	.829978	.810909	.792273	.756312
7	.867340	.827220	.807878	.788986	.752548
8	.865177	.824472	.804861	.785712	.748804
9	.863019	.821733	.801855	.782452	.745079
60	.860868	.819003	.798852	.779205	.741372

TABLE NO. III. Continued.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	1-4 %	1-5 %	3-8 %	5-12 %	1-2 %
61	.858722	.816284	.795865	.775972	.737684
2	.856581	.813570	.792895	.772752	.734014
3	.854444	.810867	.789932	.769546	.730362
4	.852314	.808173	.786981	.766353	.726728
5	.850188	.805488	.784041	.763173	.723113
6	.848068	.802812	.781116	.760006	.719515
7	.845953	.800145	.778194	.756853	.715935
8	.843844	.797487	.775286	.753712	.712374
9	.841739	.794838	.772390	.750585	.708829
70	.839640	.792194	.769504	.747470	.705303
1	.837546	.789565	.766629	.744369	.701794
2	.835458	.786942	.763765	.741280	.698302
3	.833375	.784327	.760912	.738204	.694828
4	.831296	.781722	.758069	.735141	.691371
5	.829223	.779125	.755237	.732091	.687932
6	.827155	.776536	.752415	.729053	.684509
7	.825093	.773956	.749604	.726028	.681104
8	.823035	.771388	.746804	.723015	.677715
9	.820982	.768822	.744014	.720015	.674343
80	.818935	.766268	.741234	.717024	.670988
1	.816893	.763722	.738465	.714053	.667650
2	.814846	.761185	.735706	.711090	.664329
3	.812824	.758656	.732957	.708139	.661023
4	.810794	.756136	.730319	.705201	.657735
5	.808775	.753624	.727491	.702275	.654462
6	.806015	.751120	.724773	.699361	.651206
7	.804746	.748625	.722065	.696459	.647967
8	.802737	.746137	.719368	.693569	.644743
9	.800737	.743659	.716680	.690791	.641535
90	.798741	.741188	.714003	.687825	.638343
1	.796749	.738725	.711335	.684971	.635168
2	.794762	.736271	.708677	.682129	.632008
3	.792779	.733825	.706030	.679298	.628863
4	.790803	.731387	.703392	.676480	.625735
5	.788841	.728957	.700764	.673673	.622621
6	.786863	.726536	.698146	.670877	.619524
7	.784901	.724122	.695538	.668094	.616442
8	.782944	.721716	.692939	.665321	.613375
9	.780991	.719320	.690351	.662561	.610323
100	.779044	.716929	.687772	.659476	.607287
1	.777101	.714547	.685202	.657074	.604265
2	.775163	.712173	.682642	.654347	.601259
3	.773230	.709807	.680092	.651632	.598268
4	.771302	.707450	.677551	.648928	.595291
5	.769378	.705098	.675019	.646236	.592330
6	.767460	.702756	.672498	.643554	.589383
7	.765546	.700421	.669986	.640884	.586451
8	.763637	.698094	.667482	.638225	.583533
9	.761732	.695775	.664989	.635576	.580630
110	.759833	.693463	.662504	.632939	.577741
1	.757937	.691159	.660029	.630313	.574867
2	.756048	.688863	.657563	.627698	.572007
3	.754162	.686575	.655107	.625093	.569161
4	.752281	.684294	.652659	.622499	.566329
5	.750406	.682020	.650221	.619916	.563512
6	.748534	.679754	.647792	.617344	.560708
7	.746668	.677496	.645371	.614782	.557919
8	.744806	.675245	.642960	.612231	.555143
9	.742948	.673002	.640602	.609691	.552381
120	.741096	.670768	.638165	.607161	.549633

TABLE NO. III. Continued.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	5-8 %	3-4 %	7-8 %	1 %	1 1-8 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.993789	.992556	.991326	.990099	.988875
2	.987616	.985167	.982727	.980296	.977874
3	.981482	.977834	.974203	.970590	.966995
4	.975387	.970551	.965752	.960980	.956238
5	.969327	.963330	.957375	.951466	.945600
6	.963307	.956159	.949071	.942045	.935080
7	.957324	.949041	.940839	.932718	.924678
8	.951377	.941976	.932678	.923483	.914391
9	.945468	.934964	.924588	.914340	.904219
10	.939596	.928004	.916568	.905287	.894159
1	.733759	.921096	.908617	.896324	.884214
2	.927960	.914239	.900736	.887449	.874376
3	.922196	.907434	.892923	.876663	.864648
4	.916468	.900679	.885177	.869963	.855029
5	.910860	.893974	.877499	.816349	.845517
6	.905119	.887319	.869888	.852821	.836111
7	.899497	.880714	.862342	.844377	.826809
8	.893910	.874158	.854862	.836017	.817611
9	.888359	.867651	.847447	.827740	.808516
20	.882840	.861190	.840096	.819544	.799466
1	.877356	.854781	.832809	.811430	.790627
2	.871907	.848418	.825585	.803396	.781831
3	.866491	.842102	.818424	.795442	.773134
4	.861110	.835834	.811325	.787566	.764523
5	.855770	.829612	.804287	.779768	.756027
6	.850446	.823436	.797311	.772048	.747617
7	.845164	.817307	.790395	.764404	.739299
8	.839914	.811223	.783539	.756836	.731076
9	.834697	.805184	.776743	.749342	.722942
30	.829512	.799190	.770005	.741923	.714899
1	.824360	.793241	.763326	.734577	.706946
2	.819240	.787336	.756705	.727304	.699083
3	.814152	.781475	.750141	.720103	.691305
4	.809095	.775657	.743634	.712973	.683624
5	.804069	.769884	.737184	.705914	.676009
6	.799075	.764152	.730789	.698925	.668488
7	.794112	.758462	.724451	.692005	.661052
8	.789179	.752818	.718167	.685153	.653699
9	.784278	.747212	.711937	.678369	.646425
40	.779406	.741652	.705762	.671653	.639234
1	.774564	.736131	.699640	.665003	.632123
2	.769755	.730651	.693571	.658419	.625091
3	.764973	.725212	.687555	.651900	.618137
4	.760222	.719814	.681591	.645445	.611260
5	.755500	.714455	.675679	.639055	.604460
6	.750807	.709137	.669818	.632728	.597735
7	.746144	.703858	.664008	.626463	.591086
8	.741510	.698618	.658248	.620260	.584541
9	.736904	.693418	.652539	.614119	.578007
50	.732327	.688256	.646878	.608039	.571577
1	.727778	.683133	.642170	.602019	.565219
2	.723258	.678047	.635707	.596058	.558931
3	.718766	.672999	.630193	.590157	.552713
4	.714301	.667990	.624725	.584314	.546564
5	.709874	.663018	.619306	.578528	.540484
6	.705455	.658082	.613923	.572800	.534471
7	.701074	.653183	.608607	.567129	.528525
8	.696719	.648321	.603328	.561514	.522645
9	.692392	.643495	.598094	.555955	.516831
60	.688191	.638705	.592904	.550450	.511081

TABLE NO. III. Continued.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	1 1-4 %	1 3-8 %	1 1-2 %	1 5-8 %	1 3-4 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.987654	.986437	.985222	.984010	.982801
2	.975463	.973057	.970662	.968274	.965898
3	.963419	.959859	.956317	.952792	.949285
4	.951525	.946840	.942184	.937557	.932959
5	.939777	.933998	.928260	.922565	.916913
6	.928775	.921330	.914542	.907813	.901143
7	.916716	.908833	.901027	.893297	.885644
8	.905499	.896501	.887711	.879013	.870412
9	.894221	.884347	.874592	.764957	.855441
10	.883182	.872352	.861667	.851126	.840729
1	.872278	.860520	.848933	.837517	.816269
2	.861509	.848849	.836387	.824124	.812058
3	.850873	.837336	.824027	.810946	.798091
4	.840369	.825979	.811849	.797979	.784365
5	.829994	.814776	.799852	.785219	.770875
6	.819747	.803726	.788031	.772664	.757616
7	.809627	.792823	.776385	.760308	.744586
8	.799632	.782070	.764912	.748151	.731779
9	.789760	.771463	.753607	.731289	.719194
20	.780100	.760999	.742470	.724416	.706825
1	.770380	.750677	.731498	.712932	.694668
2	.760869	.740495	.720688	.701434	.682720
3	.751476	.730452	.710037	.690216	.670978
4	.742198	.720545	.699544	.679181	.659438
5	.733034	.710771	.689206	.668321	.648096
6	.723986	.701131	.679021	.657634	.636950
7	.715048	.691621	.668986	.647118	.625995
8	.706220	.681241	.659099	.636771	.615228
9	.697501	.672987	.649359	.626589	.604647
30	.688890	.663759	.639762	.616569	.694248
1	.680386	.654855	.630308	.606710	.584027
2	.671986	.645973	.620993	.597009	.573982
3	.663689	.637212	.611816	.587463	.564111
4	.655496	.628569	.602774	.578069	.554408
5	.647403	.620043	.593866	.568825	.544873
6	.639411	.611633	.585090	.559730	.535502
7	.631511	.603338	.576443	.550780	.526292
8	.623720	.595154	.567924	.541972	.517240
9	.616020	.587082	.559531	.533306	.508344
40	.608415	.579114	.551262	.524778	.499601
1	.600904	.571265	.543116	.516387	.491008
2	.593472	.563516	.535089	.508130	.482563
3	.586158	.555873	.527182	.501158	.474264
4	.578922	.548333	.519391	.492009	.466107
5	.571775	.540896	.511715	.484142	.458090
6	.564716	.533560	.504153	.476401	.450212
7	.557744	.526323	.496702	.468883	.442469
8	.550859	.519184	.489362	.461287	.434858
9	.544058	.512143	.482130	.453911	.427379
50	.537341	.505196	.475005	.446654	.420029
1	.530707	.498344	.467982	.439511	.412805
2	.524155	.491585	.461066	.432483	.405705
3	.517684	.484917	.454252	.425567	.398724
4	.511293	.478340	.447538	.418763	.391869
5	.504980	.471852	.440925	.412066	.385129
6	.498748	.465452	.434419	.405476	.379505
7	.492589	.459139	.427989	.398993	.371995
8	.486508	.453016	.421664	.392614	.365597
9	.480502	.446769	.415430	.386356	.359309
60	.474569	.440708	.409293	.380159	.353130

TABLE NO. III. Continued.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	1 7-8 %	2 %	2 1-4 %	2 1-2 %	2 3-4 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.981595	.980392	.977995	.975609	.973236
2	.963529	.961169	.956474	.951814	.947188
3	.945795	.942322	.935427	.928599	.921838
4	.928387	.923845	.914843	.905951	.897166
5	.911300	.905731	.894712	.883854	.873154
6	.894528	.887971	.875024	.862297	.849785
7	.878064	.870560	.855769	.841265	.827041
8	.861903	.853490	.836938	.820747	.804906
9	.846040	.836755	.818522	.800728	.783364
10	.830468	.820348	.800510	.781198	.726398
1	.815183	.804263	.782895	.762145	.741993
2	.800180	.788493	.765667	.743556	.722134
3	.785453	.773033	.748819	.725420	.702807
4	.770998	.757875	.732341	.707727	.683997
5	.756806	.743015	.716226	.690466	.665691
6	.742877	.728446	.700466	.673625	.647874
7	.729204	.714163	.685052	.657195	.630535
8	.715783	.700159	.669978	.641166	.613658
9	.702609	.686431	.655235	.625528	.597235
20	.689678	.672971	.640816	.610271	.681251
1	.676984	.659776	.626715	.595386	.565694
2	.664514	.646839	.612952	.580865	.550554
3	.652293	.634156	.599437	.566697	.535819
4	.640288	.621721	.586247	.552875	.521478
5	.628503	.609531	.573346	.539391	.507521
6	.616936	.597597	.560730	.526235	.493937
7	.605581	.585862	.548391	.513400	.480718
8	.594425	.574374	.536324	.500878	.467852
9	.583494	.563112	.524522	.488661	.455331
30	.572756	.552071	.512980	.476743	.443144
1	.562215	.541246	.501692	.465115	.431284
2	.551866	.530633	.490652	.453771	.419741
3	.541708	.520229	.479856	.442703	.408507
4	.531739	.510028	.469296	.431905	.397573
5	.521952	.500028	.458969	.421371	.386933
6	.512345	.490223	.448870	.411094	.376577
7	.502916	.480611	.438993	.401067	.366499
8	.493656	.471187	.429333	.391285	.356690
9	.484574	.461948	.419885	.381741	.347143
40	.475655	.452890	.410646	.372431	.337852
1	.466900	.444010	.401610	.363347	.328810
2	.458307	.435304	.392772	.354485	.320010
3	.449872	.426769	.384129	.345839	.311445
4	.441592	.418401	.375676	.337404	.303109
5	.433465	.410197	.367410	.329174	.294997
6	.425487	.402154	.359325	.321146	.287102
7	.417655	.394268	.351418	.313313	.279418
8	.409969	.386538	.343685	.305671	.271939
9	.402423	.378958	.336122	.298216	.264661
50	.395016	.371528	.328726	.290942	.257578
1	.387746	.364246	.321492	.283845	.250795
2	.380610	.357104	.314418	.276922	.243986
3	.373604	.350102	.307499	.270168	.237456
4	.366728	.343237	.300733	.263578	.231101
5	.359978	.336507	.294115	.257150	.224905
6	.353353	.329909	.287643	.250877	.218896
7	.346849	.323440	.281313	.244759	.213027
8	.340466	.317098	.275124	.238789	.207326
9	.334199	.310881	.269070	.232964	.201777
60	.328048	.304782	.263149	.227284	.196377

TABLE NO. III. Continued.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	3 %	3 1-2 %	4 %	4 1-2 %	5 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.970874	.966184	.961538	.956938	.952381
2	.942596	.933511	.924556	.915730	.907029
3	.915142	.901943	.888996	.876297	.863838
4	.888487	.871442	.854804	.838561	.822702
5	.862609	.841973	.821927	.802451	.783526
6	.837484	.813501	.790315	.767896	.746215
7	.813092	.785991	.759918	.734828	.710681
8	.789409	.759412	.730690	.703185	.676839
9	.766417	.733731	.702587	.672904	.644609
10	.744094	.708919	.675564	.643928	.613913
1	.722421	.684946	.649581	.616199	.584679
2	.701380	.661783	.624597	.589664	.556837
3	.680951	.639404	.600574	.564272	.530321
4	.661118	.617782	.577475	.539973	.505068
5	.641862	.596891	.555265	.516720	.481017
6	.623167	.576706	.533908	.494469	.458112
7	.605016	.557204	.513373	.473176	.436297
8	.587395	.538361	.493628	.452800	.415521
9	.570286	.520156	.474642	.433302	.395734
20	.553676	.502566	.456387	.414643	.376889
1	.537549	.485571	.438834	.396787	.358942
2	.521893	.469151	.421955	.379701	.341850
3	.506692	.452286	.405726	.363350	.325571
4	.491934	.437957	.390121	.347703	.310068
5	.477606	.423147	.375117	.332731	.295303
6	.463695	.408838	.360689	.318402	.281241
7	.450189	.395012	.346817	.304691	.267848
8	.437077	.381654	.334477	.291571	.255094
9	.424346	.368748	.320651	.279015	.242946
30	.411987	.356278	.308319	.267000	.231377
1	.399987	.344230	.296460	.255502	.220359
2	.388337	.332590	.285058	.244500	.209866
3	.377026	.321343	.274094	.233971	.199873
4	.366045	.310476	.263552	.223896	.190355
5	.355383	.299977	.253415	.214254	.181290
6	.345032	.289833	.243669	.205028	.172657
7	.334983	.280032	.234297	.196199	.164436
8	.325226	.270562	.225285	.187750	.156605
9	.315754	.261413	.216621	.179665	.149148
40	.306557	.252572	.208289	.171929	.142046
1	.297628	.244031	.200278	.164525	.135282
2	.288959	.235779	.192575	.157440	.128840
3	.280543	.227806	.185168	.150661	.122704
4	.272372	.220102	.178046	.144173	.116861
5	.264439	.212659	.171198	.137964	.111297
6	.256737	.205468	.164614	.132023	.105997
7	.249259	.198520	.158283	.126338	.100949
8	.241999	.191806	.152195	.120898	.096142
9	.234959	.185320	.146341	.115692	.091564
50	.228107	.179053	.140713	.110710	.087204
1	.221463	.172998	.135301	.105942	.083051
2	.215013	.167148	.130097	.101380	.079096
3	.208750	.161496	.125093	.097014	.075330
4	.202670	.156035	.120282	.092837	.071743
5	.196767	.150758	.115656	.088839	.068326
6	.191036	.145660	.111207	.085013	.065073
7	.185472	.140734	.106930	.081353	.061974
8	.180070	.135975	.102817	.077849	.059023
9	.174825	.131377	.098863	.074497	.056212
60	.169733	.126934	.095060	.071289	.053536

TABLE NO. III. Concluded.

The Present Value of One Dollar Payable at Any One of the Given Periods, Discounted at the Rate Per Cent Stated. v^n

Periods	6 %	7 %	8 %	9 %	10 %
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	.943396	.934579	.925926	.917431	.909091
2	.889996	.873439	.857339	.841680	.826446
3	.839619	.816298	.794832	.772183	.751315
4	.792094	.762895	.735030	.708425	.683031
5	.747258	.712986	.680583	.649931	.620922
6	.704961	.666342	.630170	.596267	.564584
7	.665057	.622750	.583490	.547034	.513258
8	.627412	.582009	.540269	.501866	.466507
9	.591898	.543934	.500249	.460428	.424098
10	.558395	.508349	.473193	.422411	.385538
1	.526788	.475093	.428883	.387533	.351494
2	.496969	.444012	.397114	.355535	.318631
3	.468839	.414964	.367688	.326179	.289664
4	.442301	.387817	.340461	.299246	.263331
5	.417265	.362446	.315242	.274538	.239393
6	.393646	.338734	.291890	.251870	.217629
7	.371364	.316574	.270269	.231073	.197844
8	.350344	.295864	.250249	.211994	.179859
9	.330513	.276508	.231712	.194489	.163508
20	.311805	.258419	.215658	.178431	.148644
1	.294155	.241513	.208656	. 163698	.135130
2	.277505	.225713	.183941	.150182	.122846
3	.261797	.210947	.170315	.137781	.111678
4	.246979	.197147	.157699	.126405	.101526
5	.232999	.184294	.146018	.115968	.092296
6	.219810	.172195	.135202	.106393	.083905
7	.207368	.160930	.125187	.097608	.076278
8	.195630	.150402	.115913	.087584	.069334
9	.184557	.140563	.107328	.082155	.063139
30	.174110	.131367	.099377	.075371	.057308
1	.164255	.122773	.092016	.069146	.052099
2	.154957	.114741	.085200	.063437	.047363
3	.146186	.107234	.078889	.058199	.043056
4	.137912	.100219	.073046	.053394	.039143
5	.130105	.093663	.067634	.048985	.035584
6	.122741	.087535	.062625	.044941	.032349
7	.115793	.081809	.057986	.041229	.029409
8	.109239	.076457	.053790	.037826	.026734
9	.103056	.071455	.049723	.034702	.024305
40	.097222	.066780	.046031	.031837	.022095
1	.091719	.062412	.042622	.029208	.020086
2	.086527	.058329	.039464	.026797	.018260
3	.081630	.054513	.036551	.024584	.016601
4	.077009	.059946	.033834	.022554	.015091
5	.072650	.047613	.031328	.020692	.013719
6	.068538	.044499	.029007	.018983	.012472
7	.064658	.041587	.026858	.017416	.011338
8	.060998	.038867	.024869	.015978	.010308
9	.057546	.036324	.023027	.014659	.009370
50	.054288	.033948	.021322	.013448	.008618
1	.051215	.031727	.019742	.012338	.007745
2	.048316	.029651	.018279	.011319	.007040
3	.045582	.027711	.016926	.010385	.006400
4	.043001	.025899	.015771	.009527	.005818
5	.040567	.024204	.014511	.008741	.005290
6	.038271	.022621	.013436	.008019	.004809
7	.036105	.021141	.012441	.007357	.004362
8	.034061	.019758	.011519	.006749	.003966
9	.032133	.018465	.011519	.006192	.003605
60	.030314	.017257	.009875	.005681	.003277

TABLE NO. IV.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.002500	2.003333	2.003750	2.004167	2.005000
3	3.007506	3.010011	3.011264	3.012517	3.015025
4	4.015025	4.020044	4.022556	4.025069	4.030100
5	5.025063	5.033445	5.037638	5.041840	5.050251
6	6.037626	6.050323	6.056529	6.062848	6.075502
7	7.052720	7.070490	7.079241	7.088110	7.105879
8	8.070352	8.094058	8.105788	8.117646	8.141409
9	9.090528	9.121038	9.136184	9.151470	9.1'2116
10	10.113254	10.151441	10.170444	10.189601	10.228026
1	11.138537	11.185279	11.208580	11.232079	11.279166
2	12.166383	12.222563	12.250612	12.278879	12.335562
3	13.196799	13.263305	13.296551	13.330041	13.397240
4	14.229791	14.307516	14.346413	14.385584	14.464226
5	15.265365	15.355207	15.400212	15.445524	15.536548
6	16.303528	16.406391	16.457960	16.509880	16.614230
7	17.344285	17.461079	17.519677	17.578669	17.697301
8	18.387646	18.519284	18.585375	18.651913	18.785787
9	19.433615	19.581015	19.655070	19.729629	19.879717
20	20.482199	20.646283	20.728776	20.811896	20.979115
1	21.533405	21.715104	21.806509	21.898612	22.084011
2	22.587239	22.787487	22.888283	22.989856	23.194431
3	23.643707	23.863445	23.974114	24.085446	24.310403
4	24.702816	24.942989	25.064018	25.185802	25.431955
5	25.764573	26.026132	26.158008	26.290743	26.559115
6	26.828984	27.112886	27.256100	27.400288	27.691911
7	27.896056	28.203265	28.358340	28.514457	28.830370
8	28.965797	29.297276	29.464683	29.633263	29.974522
9	30.038211	30.394933	30.575175	30.756737	31.124395
30	31.113306	31.496249	31.689831	31.884890	32.280066
1	32.191090	32.601236	32.808667	33.017631	33.441417
2	33.271568	33.709906	33.931699	34.155205	34.608624
3	34.354746	34.822272	35.058942	35.297519	35.781667
4	35.440633	35.938346	36.190437	36.444593	36.960575
5	36.529235	37.058140	37.326152	37.596451	38.145378
6	37.620558	38.181667	38.466144	38.753104	39.336105
7	38.714609	39.308939	39.610392	39.914576	40.532785
8	39.811396	40.439968	40.758930	41.080888	41.735449
9	40.910925	41.574768	41.911776	42.252059	42.944127
40	42.013202	42.713350	43.068948	43.428110	44.158847
1	43.118235	43.855727	44.230452	44.609061	45.379642
2	44.226031	45.001912	45.396316	45.794933	46.606540
3	45.336601	46.151918	46.566552	46.985746	47.839572
4	46.449943	47.305757	47.741176	48.181521	49.078770
5	47.567568	48.463443	48.920205	49.382278	50.324164
6	48.686486	49.624987	50.103655	50.583038	51.575785
7	49.808199	50.790403	51.291543	51.798822	52.833664
8	50.932716	51.959704	52.483886	53.014652	54.097832
9	52.060044	53.132902	53.680700	54.235547	55.368321
50	53.190190	54.310011	54.882002	55.461529	56.645163
1	54.323162	55.491047	56.087809	56.692619	57.928389
2	55.459066	56.676016	57.298138	57.928839	59.218028
3	56.597707	57.864936	58.513006	59.170210	60.514118
4	57.739197	59.057819	59.732429	60.416754	61.816689
5	58.883541	60.254678	60.956425	61.668491	63.125773
6	60.030746	61.455527	62.185011	62.925444	64.441402
7	61.180819	62.660378	63.418205	64.187634	65.763609
8	62.333767	63.869246	64.656023	65.455084	67.092427
9	63.489597	65.082140	65.898483	66.727814	68.427889
60	64.648317	66.299080	67.145502	68.005847	69.770028

TABLE NO. IV. Continued.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
61	65.809934	67.520076	68.407298	69.294553	71.118878
2	66.974455	68.745143	69.663759	70.588628	72.474472
3	68.141887	69.974293	70.924961	71.888095	73.836844
4	69.312238	71.207540	72.190863	73.192815	75.206028
5	70.485514	72.444898	73.461541	74.503134	76.582058
6	71.661724	73.686381	74.736984	75.818906	77.964968
7	72.840874	74.932002	76.017211	77.140136	79.354793
8	74.022972	76.181975	77.302238	78.466902	80.751565
9	75.208030	77.435917	78.592054	79.799196	82.155323
70	76.396046	78.694036	79.886737	81.137042	83.566099
1	77.587032	79.956348	81.186275	82.480462	84.983929
2	78.781023	81.222868	82.490687	83.829480	86.408849
3	79.977971	82.493610	83.799990	85.184120	87.840893
4	81.177912	83.768588	85.114203	86.544403	89.280097
5	82.380853	85.047816	86.433344	87.910254	90.726498
6	83.586801	86.331307	87.757432	89.281992	92.180130
7	84.795764	87.619077	89.086482	90.659349	93.641031
8	86.007749	88.911140	90.420520	92.042445	95.109237
9	87.222764	90.207509	91.759560	93.431304	96.584783
80	88.440817	91.508200	93.103590	94.825949	98.067707
1	89.661915	92.813226	94.452692	96.226406	99.558046
2	90.886066	94.122602	95.806853	97.632698	101.055836
3	92.113277	95.436343	97.166092	99.044849	102.561115
4	93.343556	96.754463	98.530428	100.462884	104.073921
5	94.576911	98.076977	99.899880	101.886828	105.594290
6	95.813349	99.403899	101.274465	103.316705	107.122261
7	97.052878	100.735244	102.654207	104.752540	108.657872
8	98.295506	102.071027	104.039123	106.194357	110.201161
9	99.541244	103.411263	105.429233	107.642182	111.752167
90	100.790094	104.755966	106.824556	109.096040	113.310928
1	102.042065	106.105151	108.225111	110.555955	114.877483
2	103.297166	107.458834	109.630919	112.021953	116.451870
3	104.555405	108.817032	111.041998	113.494060	118.034129
4	105.816790	110.179754	112.458769	114.972300	119.624300
5	107.081328	111.547019	113.880651	116.456700	121.222418
6	108.349027	112.918841	115.307661	117.947285	122.828530
7	109.619895	114.295236	116.740026	119.444077	123.442672
8	110.893941	115.676219	118.177762	120.947109	125.064885
9	112.171172	117.061806	119.620890	122.456404	126.695210
100	113.451596	118.452111	121.069429	123.971992	128.333686
1	114.735220	119.846950	122.523400	125.493891	129.980354
2	116.022054	121.246438	123.982824	127.022128	131.635256
3	117.312105	122.650591	125.447718	128.556936	133.298432
4	118.605381	124.059425	126.918108	130.097738	134.979924
5	119.901890	125.472955	128.394012	131.645161	136.659774
6	121.201641	126.891197	129.875451	133.199032	138.338023
7	122.504641	128.314167	131.362445	134.759377	140.034713
8	123.810901	129.741879	132.855015	136.326224	141.749887
9	125.120424	131.174350	134.353182	137.899599	143.453586
110	126.433221	132.611597	135.856969	139.479530	145.175854
1	127.749300	134.053634	137.366387	141.066044	146.906733
2	129.068669	135.500478	138.881472	142.659166	148.646267
3	130.391336	136.952143	140.402239	144.259028	150.394498
4	131.719310	138.408645	131.928708	145.865456	152.151471
5	133.046599	139.870006	143.460901	147.478577	153.917229
6	134.379211	141.336241	144.998840	149.098420	155.691815
7	135.715155	142.807360	146.542547	150.725012	157.475274
8	137.054439	144.283383	148.092042	152.358382	159.267651
9	138.397071	145.764326	149.647348	153.998557	161.068989
120	139.743059	147.250206	150.208487	155.645187	162.879334

TABLE NO. IV. Continued.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	5-8 %	3-4 %	7-8 %	1 %	1 1-8 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.006250	2.007500	2.008750	2.010000	2.011250
3	3.018789	3.022500	3.026327	3.030100	3.033913
4	4.037654	4.045190	4.052807	4.060401	4.068044
5	5.062889	5.075329	5.088269	5.101005	5.113809
6	6.094532	6.113596	6.132791	6.152015	6.171340
7	7.132623	7.159448	7.186453	7.213535	7.240767
8	8.177199	8.213146	8.249335	8.285671	8.322225
9	9.228305	9.274745	9.321516	9.368527	9.415849
10	10.285984	10.344281	10.403080	10.462213	10.521776
11	11.350271	11.421863	11.494107	11.566835	11.640145
12	12.421210	12.507529	12.594680	12.628503	12.771096
13	13.498843	13.601336	13.704884	13.809328	13.914770
14	14.583211	14.703346	14.824801	14.947421	15.071310
15	15.674356	15.813621	15.954518	16.096896	16.240834
16	16.772321	16.932224	17.094120	17.257864	17.423543
17	17.877147	18.059216	18.243694	18.430443	18.619557
18	18.988879	19.194660	19.403326	19.614748	19.829024
19	20.107462	20.338620	20.573105	20.810895	21.052100
20	21.233134	21.491160	21.753120	22.019004	22.288935
21	22.365842	22.652342	22.943460	23.239194	23.539685
22	23.505629	23.822335	24.144215	24.471586	24.804506
23	24.652539	25.001002	25.355477	25.716302	26.078556
24	25.806618	26.188573	26.577337	26.973465	27.371995
25	26.967910	27.384986	27.809889	28.243199	28.679985
26	28.136460	28.590372	29.053226	29.525632	30.002690
27	29.312413	29.804798	30.307441	30.820888	31.340376
28	30.495615	31.028304	31.572631	32.129097	32.692909
29	31.686212	32.261015	32.848892	33.450388	34.060760
30	32.884250	33.502971	34.136320	34.784892	35.443999
31	34.099765	34.754242	35.435012	36.132740	36.842799
32	35.312825	36.014892	36.745069	37.494068	38.257336
33	36.533467	37.285002	38.066558	38.869009	39.687786
34	37.761738	38.564628	39.399671	40.257699	41.134329
35	38.997687	39.853861	40.744418	41.660276	42.597145
36	40.241360	41.152764	42.100932	43.076878	44.076418
37	41.492805	42.461408	43.469315	44.507647	45.572333
38	42.752072	43.779864	44.849671	45.952724	47.085077
39	44.019206	45.108212	46.242106	47.412251	48.614839
40	45.294260	46.446522	47.646724	48.886373	50.161811
41	46.577286	47.794870	49.063633	50.375237	51.726187
42	47.868331	49.153330	50.492940	51.878989	53.308162
43	49.167445	50.521979	51.934753	53.397779	54.907934
44	50.474678	51.900892	53.389182	54.931757	56.525704
45	51.790082	53.290147	54.856338	56.481075	58.161673
46	53.113717	54.689822	56.336331	58.045885	59.816047
47	54.445614	56.099994	57.829273	59.626344	61.489033
48	55.785836	57.520743	59.335280	61.222608	63.180840
49	57.134434	58.952147	60.854463	62.834834	64.891767
50	58.491458	60.394287	62.386940	64.463182	66.621767
51	59.856966	61.847243	63.932826	66.107814	68.371317
52	61.231019	63.311096	65.492238	67.768892	70.140549
53	62.613649	64.785944	67.065296	69.446582	72.649686
54	64.004923	66.271805	68.652118	71.141048	73.738950
55	65.404891	67.768857	70.252825	72.852459	75.568564
56	66.813608	69.277123	71.867538	74.580984	77.418766
57	68.231129	70.796700	73.496379	76.326794	79.289782
58	69.657510	72.327674	75.139472	78.090062	81.181847
59	71.092806	73.870130	76.796943	79.870963	83.095198
60	72.537073	75.424154	78.468916	81.669674	85.030074

TABLE NO. IV. Continued.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	1 1-4 %	1 3-8 %	1 1-2 %	1 5-8 %	1 3-4 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.012500	2.013750	2.015000	2.016250	2.017500
3	3.037656	3.041438	3.045225	3.049014	3.052806
4	4.075626	4.083257	4.090903	4.098561	4.106230
5	5.126572	5.139401	5.152267	5.165162	5.178089
6	6.190654	6.212536	6.229551	6.249096	6.268706
7	7.268037	7.297924	7.322994	7.350644	7.378408
8	8.358988	8.393236	8.432839	8.470092	8.507530
9	9.463474	9.513678	9.559332	9.607731	9.656412
10	10.581766	10.644457	10.702722	10.763856	10.825399
11	11.714037	11.790779	11.863262	11.938770	12.014844
12	12.860461	12.952868	13.041211	13.132775	13.225104
13	14.021215	14.130936	14.236830	14.346182	14.456543
14	15.196479	15.325200	15.450382	15.579308	15.709532
15	16.386433	16.535887	16.682138	16.832472	16.984449
16	17.591262	17.763222	17.932370	18.106000	18.281677
17	18.811151	19.007432	19.201355	19.400223	19.601607
18	20.046288	20.268750	20.489375	20.715477	20.944635
19	21.296865	21.547411	21.796716	22.052103	22.311166
20	22.563074	22.843654	23.123667	23.410450	23.701611
21	23.845111	24.157720	24.470522	24.790870	25.116389
22	25.143173	25.489855	25.837580	26.193689	26.555926
23	26.460491	26.840307	27.225144	27.619337	28.020655
24	27.791207	28.209327	28.633521	29.068152	29.511016
25	29.138558	29.597171	30.063024	30.540510	31.027459
26	30.502751	31.004098	31.513969	32.036794	32.570440
27	31.883996	32.430370	32.986678	33.557393	34.140422
28	33.282507	33.879587	34.481479	35.102705	35.737880
29	34.698499	35.345352	35.998701	36.673124	37.363293
30	36.132191	36.831271	37.538681	38.269063	39.017150
31	37.583804	38.337621	39.101762	39.890936	40.699950
32	39.153562	39.864684	40.688288	41.539164	42.412199
33	40.541692	41.412744	42.298612	43.214176	44.154413
34	42.051897	42.982090	43.933091	44.916407	45.927115
35	43.577463	44.573014	45.592088	46.646299	47.730840
36	45.122099	46.185814	47.275969	48.404302	49.566129
37	46.689648	47.820787	48.985109	50.190873	51.433537
38	48.273140	49.478243	50.719885	52.006475	53.333624
39	49.876426	51.158489	52.480684	53.851581	55.266692
40	51.499754	52.861840	54.267894	55.726670	57.234134
41	53.143373	54.588611	56.081912	57.632229	59.235731
42	54.807537	56.339125	57.923141	59.568753	61.272357
43	56.492504	58.113708	59.791988	61.536746	63.344623
44	58.198533	59.912692	61.688868	63.536721	65.453154
45	59.925887	61.736412	63.614201	65.569193	67.598584
46	61.696694	63.585209	65.568414	67.634693	69.781559
47	63.467501	65.459426	67.551940	69.733757	72.002736
48	65.260443	67.359414	69.565219	71.866931	74.262784
49	67.075786	69.285527	71.608697	74.034769	76.562383
50	68.913832	71.238123	73.682828	76.237835	78.902225
51	70.774853	73.217568	75.788070	78.476700	81.283014
52	72.659148	75.224231	77.924891	80.751947	83.705466
53	74.566986	77.258485	80.093764	83.064167	86.170311
54	76.498672	79.320710	82.295170	85.413960	88.678291
55	78.454505	81.411291	84.529597	87.801937	91.230154
56	80.434785	83.530618	86.797541	90.228719	93.826681
57	82.439819	85.679085	89.099503	92.694936	96.468647
58	84.469916	87.857093	91.436996	95.201228	99.156847
59	86.525388	90.065049	93.807536	97.748249	101.892191
60	88.606554	92.303364	96.214649	100.336658	104.675301

TABLE NO. IV. Continued.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	1 7-8 %	2 %	2 1-4 %	2 1-2 %	2 3-4 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.018750	2.020000	2.022500	2.025000	2.027500
3	3.056601	3.060400	3.068006	3.075625	3.083256
4	4.113912	4.121608	4.137036	4.152516	4.168046
5	5.191048	5.204040	5.230120	5.256329	5.282667
6	6.288380	6.308121	6.347797	6.387737	6.427940
7	7.406347	7.434283	7.490622	7.547430	7.604709
8	8.545214	8.582969	8.659162	8.736116	8.813838
9	9.705435	9.754628	9.853993	9.954519	10.056219
10	10.887210	10.949721	11.075708	11.203382	11.332765
11	12.091347	12.168715	12.324911	12.483466	12.644416
12	13.318062	13.412090	13.602222	13.795553	13.992137
13	14.567777	14.680332	14.908272	15.140442	15.376921
14	15.840925	15.973938	16.243708	16.518953	16.799786
15	17.137944	17.293417	17.609191	17.931927	18.261781
16	18.459282	18.639285	19.005398	19.380225	19.763979
17	19.805395	20.012071	20.433020	20.864730	21.307489
18	21.176784	21.412312	21.892762	22.386349	22.893445
19	22.574031	22.840559	23.385350	23.946007	24.523015
20	23.997292	24.297370	24.911520	25.544658	26.197398
21	25.447238	25.783317	26.472029	27.183274	27.917826
22	26.924371	27.298984	28.067650	28.862856	29.685566
23	28.429200	28.844963	29.699172	30.584427	31.501919
24	29.962244	30.421862	31.367403	32.349038	33.368222
25	31.524034	32.030300	33.073170	34.157764	35.285848
26	33.115106	33.670906	34.817316	36.011708	37.256209
27	34.736011	35.344324	36.600706	37.912001	39.280755
28	36.387308	37.051210	38.424222	39.859801	41.360975
29	38.069567	38.792234	40.288767	41.856296	43.498402
30	39.783367	40.568079	42.195264	43.902703	45.694608
31	41.529298	42.379441	44.144657	46.000271	47.951210
32	43.307969	44.227030	46.137912	48.150278	50.269868
33	45.119990	46.111570	48.176015	50.354034	52.652290
34	46.965986	48.033802	50.259976	52.612885	55.100228
35	48.846673	49.994478	52.390825	54.928207	57.615484
36	50.762543	51.994367	54.569619	57.301413	60.199910
37	52.713886	54.034254	56.797435	59.733948	62.855407
38	54.702274	56.114940	59.075377	62.227297	65.583930
39	56.727945	58.237238	61.404573	64.782979	68.387489
40	58.791592	60.401983	63.786176	67.402553	71.268145
41	60.893937	62.610023	66.221365	70.087617	74.228019
42	63.035701	64.862223	68.711346	72.839808	77.269289
43	65.217623	67.159468	71.257351	75.660803	80.394195
44	67.440456	69.502657	73.860642	78.552323	83.605035
45	69.704967	71.892710	76.522506	81.516131	86.904174
46	72.011927	74.330564	79.244262	84.554034	90.294039
47	74.362153	76.817176	82.027258	87.667885	93.777125
48	76.756445	79.353519	84.872872	90.859582	97.355996
49	79.195630	81.940590	87.782511	94.131072	101.033285
50	81.680550	84.579401	90.757618	97.484349	104.811701
51	84.212062	87.270989	93.799664	100.921457	108.694022
52	86.791040	90.016409	96.910166	104.444493	112.683108
53	89.418371	92.816737	100.090644	108.055605	116.781893
54	92.094966	95.673071	103.342683	111.756995	120.993395
55	94.821747	98.586532	106.667893	115.550920	125.320713
56	97.599736	101.558262	110.067920	119.439693	129.767032
57	100.429666	104.589427	113.544448	123.425685	134.335625
58	103.312923	107.681215	117.099198	127.511327	139.029856
59	106.249837	110.834839	120.733930	131.699110	143853176
60	109.242022	114.051535	124.450433	135.991588	148.809138

TABLE NO. IV. Continued.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}|$

Periods	3 %	3 1-2 %	4 %	4 1-2 %	5 %
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.03000	2.03500	2.04000	2.04500	2.05000
3	3.09090	3.10623	3.12160	3.13703	3.15250
4	4.18363	4.21494	4.24646	4.27819	4.31013
5	5.30914	5.56247	5.41632	5.47071	5.52563
6	6.46841	6.55015	6.63298	6.71689	6.80191
7	7.66246	7.77941	7.89829	8.01915	8.14201
8	8.89234	9.05169	9.21423	9.38001	9.54911
9	10.15911	10.36850	10.58280	10.80211	11.02656
10	11.46388	11.73139	12.00611	12.28821	12.57789
11	12.80780	13.14199	13.48635	13.84118	14.20679
12	14.19203	14.60196	15.02581	15.46403	15.91713
13	15.61779	16.11303	16.62684	17.15991	17.71298
14	17.08632	17.67699	18.29191	17.93211	19.59863
15	18.59891	19.29568	20.02359	20.78405	21.57856
16	20.15688	20.97103	21.82453	22.71934	23.65749
17	21.76159	22.70502	23.69751	24.74171	25.84037
18	23.41444	24.49969	25.64541	26.85508	28.13238
19	25.11687	26.35718	27.67123	29.06356	30.53900
20	26.87037	28.27968	29.77808	31.37142	33.06595
21	28.67649	30.26947	31.96920	33.78314	35.71925
22	30.53678	32.32890	34.24797	36.30338	38.50521
23	32.45288	34.46041	36.61789	38.93703	41.43048
24	34.42647	36.66653	39.08260	41.68920	44.50200
25	36.45926	39.94986	41.64591	44.56521	47.72710
26	38.55304	41.31310	44.31174	47.57064	51.11345
27	40.70963	43.75906	47.08421	50.71132	54.66913
28	42.93092	46.29063	49.96758	53.99333	58.40258
29	45.21885	48.91080	52.96629	57.42303	62.32271
30	47.57542	51.62268	56.08494	61.00707	66.43885
31	50.00268	54.42947	59.32834	64.75239	70.76079
32	52.50276	57.33450	62.70147	68.66625	75.29883
33	55.07784	60.34121	66.20953	72.75623	80.06377
34	57.73018	63.45315	69.85791	77.03026	85.06696
35	60.46208	66.67401	73.65222	81.49662	90.32031
36	63.27594	70.00760	77.59831	86.16397	95.83632
37	66.17422	73.45787	81.70225	91.04134	101.62814
38	69.15945	77.02889	85.97034	96.13820	107.70955
39	72.23423	80.72491	90.40915	101.46442	114.09502
40	75.40126	84.55028	95.02552	107.03032	120.79977
41	78.66330	88.50954	99.82654	112.84669	127.83976
42	82.02320	92.60737	104.81960	118.92479	135.23175
43	85.48389	96.84863	110.01238	125.27640	142.99334
44	89.04841	101.23833	115.41288	131.91384	151.14301
45	92.71986	105.78167	121.02939	138.84997	159.70016
46	96.50146	110.48403	126.87057	146.09821	168.68516
47	100.39650	115.35097	132.94539	153.67263	178.11942
48	104.40840	120.38826	139.26321	161.58790	188.02539
49	108.54065	125.60185	145.83373	169.85936	198.42660
50	112.79687	130.99791	152.66708	178.50303	209.34800
51	117.18077	136.58284	159.77377	187.53566	220.81540
52	121.69620	142.36324	167.16472	196.97477	232.85617
53	126.34708	148.34595	174.85131	206.83863	245.49879
54	131.13749	154.53806	182.84536	217.14637	258.77392
55	136.07162	160.94689	181.15917	227.91796	272.71262
56	141.15377	167.58003	199.80544	239.17427	287.34825
57	146.38838	174.44533	208.79776	250.93711	302.71566
58	151.78003	181.55092	218.14967	263.22928	318.85144
59	157.33343	188.90520	227.87566	276.07460	335.79402
60	163.05344	196.51688	237.99069	289.49795	353.58372

TABLE NO. IV. Concluded.

The Amount of One Dollar Payable at the End of Each Period, Improved at Compound Interest at the Rate and for the Terms Stated. $\overline{Sn}$

Periods	6 %	7 %	8 %	9 %	10 %
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.06000	2.07000	2.08000	2.09000	2.10000
3	3.18360	3.21490	3.24640	3.27810	3.31000
4	4.37462	4.43994	4.50611	4.57312	4.64100
5	5.63709	5.75074	5.86660	5.98470	6.10510
6	6.97532	7.15329	7.33593	7.52332	7.71561
7	8.39384	8.65402	8.92280	9.20042	9.48717
8	9.89747	10.25980	10.63662	11.02846	11.43589
9	11.49132	11.97799	12.48755	13.02102	13.57948
10	13.18079	13.81645	14.48655	15.19291	15.93743
11	14.97164	15.78360	16.64548	17.56027	18.53117
12	16.86994	17.88845	18.97712	20.14070	21.38429
13	18.88214	20.14067	21.49529	22.95336	24.52272
14	21.01507	22.55052	24.21491	26.01916	27.97499
15	23.27597	25.12905	27.15210	29.36087	31.77249
16	25.67253	27.88808	30.32427	33.00337	35.94974
17	28.21288	30.84024	33.75021	36.97367	40.54471
18	30.90565	33.99906	37.45023	41.30130	45.59918
19	33.75999	37.37899	41.44625	46.01842	51.15910
20	36.78559	40.99552	45.76195	51.16008	57.27501
21	39.99273	44.86520	50.42291	56.76449	64.00251
22	43.39229	49.00576	55.45674	62.87330	71.40276
23	46.99583	53.43616	60.89328	69.53190	79.54303
24	50.81538	38.17669	66.76474	76.78977	88.49733
25	54.86451	63.24906	73.10592	84.70085	98.34706
26	59.15638	68.67649	79.95440	93.32393	109.18177
27	63.70577	74.48384	87.35075	102.72309	121.09995
28	68.52811	80.69771	95.33881	112.96817	134.20994
29	73.63980	87.34655	103.96592	124.13531	148.63093
30	79.05819	94.46081	113.28319	136.30749	164.49402
31	84.80168	102.07307	123.34585	149.57517	181.94342
32	90.88978	110.21818	134.21352	164.03694	201.13776
33	97.34316	118.93345	145.95060	179.80027	222.25154
34	104.18375	128.25879	158.62665	196.98230	245.47669
35	111.43478	138.23690	172.31678	215.71071	271.02436
36	119.12087	148.91348	187.10212	236.12468	299.12680
37	127.26812	160.33742	203.07029	258.37591	330.03948
38	135.90421	172.56104	220.31592	282.62975	364.04344
39	145.05846	185.64031	238.94120	309.06643	401.44773
40	154.76197	199.63513	259.05650	337.88241	442.59256
41	165.04768	214.60959	280.78102	369.29183	487.85182
42	175.95054	230.63226	304.24350	403.52810	537.63700
43	187.50758	247.77652	329.58298	440.84563	592.40070
44	199.75803	266.12087	356.94962	481.52174	652.64077
45	212.74351	285.74933	386.50559	525.85870	718.90485
46	226.50812	306.75178	418.42604	574.18599	791.79533
47	241.09861	329.22440	452.90013	626.86273	871.97486
48	256.56453	353.27011	490.13214	684.28038	960.17235
49	272.95840	378.99902	530.34271	746.86562	1057.18958
50	290.33590	406.52895	573.77013	815.08353	1163.90854
51	308.75606	435.98598	620.67174	889.44105	1281.29939
52	328.28142	467.50500	671.32548	970.49074	1410.42933
53	348.97831	501.23035	726.03152	1058.83491	1552.47226
54	370.91701	537.31647	785.11404	1155.13006	1708.71949
55	394.17203	575.92862	848.92317	1260.09177	1880.59144
56	418.82235	617.24362	917.83703	1374.50003	2069.65058
57	444.95169	661.45067	992.26400	1499.20503	2277.61563
58	472.64879	708.75221	1072.64513	1635.13348	2506.37719
59	502.00772	759.36485	1159.45685	1783.29549	2758.01491
60	553.12818	813.52038	1253.21351	1944.49208	3034.81640

TABLE NO. V.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a_{\overline{n}|}$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
1	.997506	.996678	.996264	.995851	.995025
2	1.992525	1.990044	1.988883	1.987569	1.985100
3	2.985062	2.980050	2.977717	2.975172	2.970249
4	3.975124	3.966827	3.962857	3.958677	3.950497
5	4.962717	4.950326	4.944314	4.938102	4.925867
6	5.947847	5.930557	5.922107	5.913463	5.896385
7	6.930521	6.907532	6.896247	6.884776	6.862075
8	7.910744	7.881261	7.866681	7.852059	7.822960
9	8.888522	8.851755	8.833556	8.815324	8.779065
10	9.863862	9.819025	9.796819	9.774596	9.730413
11	10.836768	10.783081	10.756481	10.729888	10.677028
12	11.807549	11.743936	11.712561	11.681216	11.618933
13	12.775610	12.701596	12.665069	12.628597	12.556152
14	13.741257	13.656075	13.614018	13.572046	13.488708
15	14.704496	14.607383	14.559420	14.511581	14.416626
16	15.665333	15.555531	15.501292	15.447217	15.339925
17	16.623774	16.500529	16.439646	16.378971	16.258632
18	17.579824	17.442387	17.374494	17.306859	17.172768
19	18.533490	18.381116	18.406049	18.230897	18.082356
20	19.484778	19.316727	19.233925	19.151101	18.987419
21	20.433694	20.249229	20.158334	20.067487	19.887979
22	21.380248	21.178633	21.079290	20.980070	20.784059
23	22.324432	22.104950	21.996806	21.888863	21.675681
24	23.266266	23.028189	22.910894	22.793888	22.562867
25	24.205751	23.948361	23.821567	23.695158	23.445639
26	25.142893	24.865476	24.728838	24.592688	24.324019
27	26.077701	25.779544	25.632719	25.486494	25.198029
28	27.010175	26.690575	26.533224	26.376592	26.067691
29	27.940322	27.598580	27.430365	27.262996	26.933026
30	28.868149	28.503586	28.324154	28.145722	27.794056
31	29.793366	29.405549	29.214604	29.024785	28.650802
32	30.716883	30.304534	30.101727	29.900201	29.503286
33	31.637786	31.200532	30.985536	30.771984	30.351528
34	32.556394	32.093553	31.866043	31.640150	31.195550
35	33.472712	32.983607	32.743261	32.504714	32.035373
36	34.386744	33.870704	33.617209	33.365690	32.871018
37	35.298497	34.754854	34.487885	34.223094	33.702505
38	36.207976	35.636067	35.355308	35.076940	34.529856
39	37.115187	36.514352	36.219491	35.927243	35.353091
40	38.020126	37.389719	37.080445	36.774018	36.172230
41	38.922818	38.262178	37.938183	37.617279	36.987294
42	39.823259	39.131739	38.792716	38.457041	37.798303
43	40.721454	39.998411	39.644057	39.293319	38.605277
44	41.617409	40.862203	40.492218	40.126127	39.408236
45	42.511130	41.723126	41.337210	40.955479	40.207200
46	43.402622	42.581188	42.179045	41.781390	41.002187
47	44.291891	43.436400	43.017735	42.603874	41.793223
48	45.178942	44.288771	43.853291	43.422945	42.579321
49	46.063781	45.138310	44.685726	44.238617	43.363504
50	46.946413	45.985026	45.515052	45.050904	44.142790
51	47.826847	46.828929	46.341279	45.859821	44.918199
52	48.705083	47.670028	47.164410	46.665382	45.689750
53	49.581129	48.508333	47.984476	47.467600	46.457463
54	50.454990	49.343853	48.801478	48.266490	47.221356
55	51.326672	50.176597	49.615428	49.062064	47.981448
56	52.196180	51.006575	50.426337	49.854337	48.737759
57	53.063520	51.833795	51.234216	50.643323	49.490308
58	53.928697	52.658267	52.039077	51.429035	50.239112
59	54.791716	53.480000	52.840932	52.211488	50.984192
60	55.652583	54.299003	53.639791	52.990695	51.725564

TABLE NO. V. Continued.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
61	56.511305	55.115287	54.435656	53.766667	52.463248
62	57.367886	55.928857	55.228551	54.539419	53.197262
63	58.222330	56.739724	56.018483	55.308965	53.927624
64	59.074644	57.547897	56.805464	56.075318	54.654352
65	59.924832	58.353385	57.589505	56.838491	55.377465
66	60.727900	59.156197	58.370621	57.598497	56.096980
67	61.618853	59.956342	59.148815	58.355350	56.812915
68	62.461697	60.753829	59.924101	59.109062	57.525289
69	63.303436	61.548667	60.696491	59.859647	58.234118
70	64.143076	62.340861	61.465995	60.607117	58.939421
71	64.980622	63.130426	62.232624	61.351486	59.641215
72	65.816080	63.917368	62.996389	62.092766	60.339517
73	66.649455	64.701695	63.757301	62.830970	61.034345
74	67.470751	65.483417	64.515370	63.566111	61.725716
75	68.299974	66.262542	65.270670	64.298202	62.413648
76	69.127129	67.039078	66.023022	65.027255	63.098157
77	69.952222	67.813034	66.772626	65.753283	63.779261
78	70.775257	68.584422	67.519430	66.476298	64.456976
79	71.596239	69.353244	68.263444	67.196313	65.131319
80	72.415174	70.119512	69.004678	67.913337	65.802307
81	73.232067	70.883234	69.743143	68.627390	66.469957
82	74.046913	71.644419	70.478849	69.338480	67.134286
83	74.859737	72.403075	71.211806	70.046619	67.795309
84	75.670531	73.159211	71.942125	70.751820	68.453044
85	76.479306	73.912835	72.669616	71.454095	69.107506
86	77.285321	74.663955	73.394389	72.153456	69.758712
87	78.089067	75.412580	74.116454	72.849915	70.406679
88	78.891806	76.158717	74.835822	73.543484	71.051422
89	79.692548	76.892376	75.552502	74.234275	71.692957
90	80.491284	77.633564	76.266505	74.922100	72.331300
91	81.288033	78.372289	76.977840	75.607071	72.966468
92	82.082795	79.108460	77.686517	76.289200	73.598476
93	82.875574	99.842285	78.392547	76.968498	74.227339
94	83.666377	80.573672	79.095939	77.644978	74.853074
95	84.455218	81.302629	79.796703	78.318651	75.475695
96	85.242081	82.029165	80.494849	78.989528	76.095219
97	86.026982	82.753287	81.190387	79.657622	76.711661
98	86.809926	83.475003	81.883326	80.322942	77.325036
99	87.590917	84.194323	82.573677	80.985503	77.935359
100	88.369961	84.911252	83.261449	81.644979	78.542646
101	89.147062	85.625799	83.946651	82.301053	79.146911
102	89.922225	86.337972	84.529293	82.955400	79.748170
103	90.695455	87.047779	85.209385	83.607032	80.346438
104	91.466757	87.755229	85.886936	84.255960	80.941729
105	92.236135	88.460327	86.561955	84.902196	81.534059
106	93.003595	89.163083	87.234453	85.545750	82.123442
107	93.769141	89.863504	87.904439	86.186634	82.709893
108	94.532778	90.561598	88.571921	86.824859	83.293426
109	95.294510	91.257373	89.236910	87.460435	83.874056
110	96.054343	91.950836	89.899414	88.093374	84.451797
111	96.812280	92.641995	90.559443	88.823687	85.026664
112	97.568328	93.330858	91.217006	89.341385	85.598671
113	98.322490	94.017433	91.872113	89.966478	86.167832
114	99.074771	94.701727	92.524772	90.588977	86.734161
115	99.825177	95.383747	93.174993	91.208893	87.297673
116	100.573711	96.063501	93.822783	91.826237	87.858381
117	101.320379	96.740997	94.468156	92.441019	88.416300
118	102.065185	97.416242	95.111116	93.053250	88.971443
119	102.808133	98.089244	95.751718	93.662941	89.523824
120	103.549229	98.760012	96.389883	94.270102	90.083457

TABLE NO. V. Continued.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|\,j$

Periods	5-8 %	3-4 %	7-8 %	1 %	1 1-8 %
1	.993789	.992556	.991326	.990099	.988875
2	1.981405	1.977723	1.974053	1.970395	1.966749
3	2.962887	2.955557	2.948256	2.940985	2.933744
4	3.938274	3.926108	3.914008	3.901966	3.889982
5	4.907601	4.889438	4.871381	4.853431	4.835582
6	5.870908	5.845597	5.820452	5.795476	5.770662
7	6.828232	6.794638	6.761291	6.728195	6.695340
8	7.779609	7.736614	7.693969	7.651678	7.609731
9	8.725077	8.671578	8.618557	8.566018	8.513950
10	9.664673	9.599582	9.535125	9.471305	9.408109
11	10.598432	10.520678	10.443742	10.367628	10.292323
12	11.526392	11.434917	11.344478	11.255074	11.166699
13	12.448588	12.342351	12.237401	12.133740	12.031347
14	13.365056	13.243030	13.122578	13.003703	12.886376
15	14.275916	14.137004	14.000077	13.865053	13.731893
16	15.181035	15.024323	14.869965	14.717874	14.568004
17	16.080532	15.905037	15.732307	15.562251	15.394813
18	16.974442	16.779195	16.587169	16.398269	16.212424
19	17.862801	17.646846	17.434616	17.226009	17.020940
20	18.755641	18.508036	18.274712	18.045553	17.820406
21	19.622997	19.362817	19.107521	18.856983	18.611033
22	20.494904	20.211235	19.933106	19.660379	19.392864
23	21.361395	21.053337	20.751530	20.455821	20.165998
24	22.222505	21.889171	21.562855	21.243387	20.930521
25	23.078275	22.718783	22.367142	22.023156	21.686548
26	23.928421	23.542219	23.164453	22.795204	22.434165
27	24.773585	24.359526	23.954848	23.559608	23.173464
28	25.613499	25.170749	24.738387	24.316443	23.904540
29	26.448196	25.975933	25.515130	25.065785	24.627482
30	27.277708	26.775123	26.285135	25.807708	25.342371
31	28.102068	27.568364	27.048461	26.542285	26.049317
32	28.921308	28.355700	27.805166	27.269589	26.748400
33	29.735460	29.137175	28.555307	27.989693	27.439705
34	30.544555	29.912832	29.298941	28.702666	28.123329
35	31.348624	30.682716	30.036125	29.408580	28.799338
36	32.147699	31.446868	30.766914	30.107505	29.467826
37	32.941811	32.205330	31.491365	30.799510	30.128878
38	33.730990	32.958148	32.209532	31.484663	30.782576
39	34.515268	33.705360	32.921469	32.163033	31.429001
40	35.294674	34.447012	33.627231	32.834686	32.068235
41	36.069238	35.183143	34.326871	33.499689	32.700358
42	36.838993	35.913794	35.020442	34.158108	33.325449
43	37.603966	36.639006	35.707997	34.810008	33.943586
44	38.364188	37.358820	36.389588	35.455454	34.554846
45	39.119688	38.073275	37.065267	36.094508	35.159306
46	39.870495	38.782412	37.735085	36.727236	35.757041
47	40.616639	39.486270	38.399093	37.353699	36.348127
48	41.358149	40.184888	39.057341	37.937959	36.932637
49	42.095053	40.878306	39.709880	38.588079	37.510644
50	42.827380	41.566562	40.356758	39.196118	38.082221
51	43.555158	42.249695	40.998928	39.798136	38.644440
52	44.278416	42.927742	41.634635	40.394193	39.206371
53	44.997182	43.600741	42.264828	40.984349	39.759084
54	45.711483	44.268731	42.889553	41.568662	40.305648
55	46.421357	44.931749	43.508859	42.147193	40.846132
56	47.126812	45.589831	44.122782	42.719990	41.381603
57	47.827886	46.243014	44.731389	43.287118	41.910128
58	48.524605	46.891325	45.334717	43.848634	42.432773
59	49.216997	47.534830	45.932811	44.404588	42.949604
60	49.905188	48.173535	46.525715	44.955038	43.460685

TABLE NO. V. Continued.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|$

Periods	1 1-4 %	1 3-8 %	1 1-2 %	1 5-8 %	1 3-4 %
1	.987654	.986437	.985222	.984010	.982801
2	1.963117	1.959494	1.955883	1.952284	1.948699
3	2.926536	2.919353	2.912200	2.905076	2.897984
4	3.878061	3.866193	3.854385	3.842633	3.830943
5	4.817838	4.800191	4.782645	4.765198	4.747855
6	5.746613	5.721521	5.697187	5.673011	5.648998
7	6.663329	6.630354	6.598214	6.566308	6.534641
8	7.568828	7.526861	7.485925	7.445321	7.405053
9	8.463049	8.411208	8.360517	8.310278	8.260494
10	9.346231	9.283560	9.222185	9.161404	9.101223
11	10.218509	10.144080	10.071118	9.998921	9.927492
12	11.080018	10.992929	10.907505	10.823045	10.739550
13	11.930891	11.830265	11.731532	11.633991	11.537641
14	12.771260	12.656244	12.543381	12.431970	12.322006
15	13.601254	13.471020	13.343233	13.217189	13.092880
16	14.421001	14.274746	14.131264	13.989853	13.850497
17	15.230628	15.067569	14.907649	14.750161	14.595083
18	16.030260	15.849639	15.672561	15.498312	15.326863
19	16.820020	16.621102	16.426168	16.234601	16.046057
20	17.600120	17.382101	17.168639	16.959017	16.752881
21	18.370500	18.132778	17.900137	17.671849	17.447549
22	19.131369	18.873273	18.620824	18.373283	18.130269
23	19.882845	19.603725	19.330861	19.063499	18.801248
24	20.625043	20.324270	20.030405	19.742680	19.460686
25	21.358077	21.035041	20.719611	20.411001	20.108782
26	22.082063	21.736172	21.398632	21.068635	20.745732
27	22.797111	22.427793	22.067617	21.715753	21.371726
28	23.503331	23.110034	22.726717	22.352524	21.986954
29	24.200832	23.783021	23.376076	22.979113	22.591602
30	24.889722	24.446780	24.015838	23.595682	23.185849
31	25.570108	25.101635	24.646146	24.202392	23.769877
32	26.242094	25.747608	25.267139	24.799401	24.343858
33	26.905783	26.384820	25.878954	25.386864	24.907970
34	27.561279	27.012389	26.481728	25.964933	25.462377
35	28.208682	27.632432	27.075595	26.533758	26.007251
36	28.848093	28.244065	27.660684	27.093488	26.542753
37	29.479664	28.847403	28.237127	27.644268	27.069045
38	30.103324	29.442557	28.805052	28.186240	27.586285
39	30.719344	30.029639	29.364583	28.719546	28.094629
40	31.327759	30.608753	29.915845	29.244324	28.594229
41	31.928663	31.180018	30.458961	29.760711	29.085238
42	32.522135	31.743534	30.994050	30.268841	29.567801
43	33.108293	32.299407	31.521232	30.769999	30.042065
44	33.687215	32.847740	32.040622	31.262008	30.508172
45	34.258990	33.388636	32.552337	31.746150	30.966262
46	34.823706	33.922196	33.056490	32.222551	31.416474
47	35.381450	34.448519	33.553192	32.691434	31.858943
48	35.932209	34.967703	34.042554	33.152721	32.293801
49	36.476267	35.479846	34.524683	33.606632	32.721181
50	37.013608	35.985042	34.999688	34.053286	33.141209
51	37.544315	36.483386	35.467650	34.492797	33.554015
52	38.068470	36.974971	35.928716	34.925280	33.959711
53	38.586154	37.459888	36.382968	35.350847	34.358446
54	39.097447	37.938228	36.830506	35.769610	34.750316
55	39.602427	38.410080	37.271431	36.181676	35.135446
56	40.101175	38.875532	37.705850	36.587152	35.513952
57	40.593764	39.334671	38.133839	36.986145	35.885949
58	41.080272	39.787687	38.555503	37.378759	36.251547
59	41.560774	40.234456	38.970933	37.765115	36.610856
60	42.035343	40.675164	39.380226	38.145274	36.963986

TABLE NO. V. Continued.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|$

Periods	1 7-8 %	2 %	2 1-4 %	2 1-2 %	2 3-4 %
1	.981595	.980392	.977995	.975610	.973236
2	1.945124	1.941561	1.934469	1.927424	1.920424
3	2.890979	2.883883	2.869897	2.856024	2.842262
4	3.819306	3.807720	3.784740	3.761974	3.739428
5	4.730606	4.713460	4.679453	4.645829	4.612582
6	5.625134	5.601431	5.554477	5.508125	5.462367
7	6.503198	6.471991	6.410246	6.349891	6.289408
8	7.365101	7.325481	7.247185	7.170137	7.094314
9	8.211141	8.162237	8.065706	7.970866	7.877678
10	9.041609	8.982585	8.866216	8.752064	8.640076
11	9.856792	9.786848	9.649111	9.514209	9.382069
12	10.656972	10.575341	10.414778	10.257765	10.104204
13	11.442425	11.348374	11.163598	10.983185	10.807011
14	12.213423	12.106249	11.895939	11.690912	11.491008
15	12.970229	12.849264	12.612166	12.381378	12.156699
16	13.713106	13.577709	13.312631	13.055003	12.804573
17	14.442310	14.291872	13.997683	13.712198	13.435108
18	15.158093	14.992031	14.667661	14.353364	14.048767
19	15.860702	15.678462	15.322896	14.978891	14.646002
20	16.550380	16.351433	15.963712	15.589162	15.227252
21	17.227364	17.011209	16.590428	16.184549	15.792946
22	17.891978	17.658048	17.203352	16.765413	16.343500
23	18.544171	18.292204	17.802789	17.332110	16.879319
24	19.184459	18.913926	18.389036	17.884986	17.400797
25	19.812962	19.523456	18.962383	18.424376	17.908318
26	20.429898	20.121036	19.523113	18.950611	18.402255
27	21.035479	20.706898	20.071504	19.464011	18.882974
28	21.629904	21.281272	20.607828	19.964889	19.350826
29	22.213398	21.844385	21.132350	20.453550	19.806157
30	22.786154	22.396456	21.645330	20.930293	20.249301
31	23.348369	22.937701	22.147022	21.395407	20.680585
32	23.900235	23.468335	22.637674	21.849178	21.100326
33	24.441943	23.988564	23.117530	22.291881	21.508833
34	24.973682	24.498592	23.586826	22.723786	21.906407
35	25.495634	24.998619	24.045796	23.145157	22.293340
36	26.007979	25.488842	24.494666	23.556251	22.669918
37	26.510895	25.969453	24.933658	23.957318	23.036416
38	27.004551	26.440641	25.362991	24.348603	23.393106
39	27.489125	26.902589	25.782876	24.730344	23.740249
40	27.964780	27.355479	26.193522	25.102775	24.078101
41	28.431680	27.799489	26.595131	25.466122	24.406911
42	28.889987	28.234794	26.987904	25.820607	24.726921
43	29.339859	28.661562	27.372033	26.166446	25.038366
44	29.781451	29.079963	27.747710	26.503849	25.341475
45	30.214916	29.490160	28.115120	26.833024	25.636472
46	30.640403	29.892314	28.474445	27.154170	25.923574
47	31.058058	30.286582	28.825863	27.467483	26.202992
48	31.468027	30.673119	29.169548	27.773154	26.474931
49	31.870450	31.052078	29.505670	28.071369	26.739592
50	32.265466	31.423606	29.834396	28.362312	26.997169
51	32.653212	31.787849	30.155888	28.646158	27.247853
52	33.033822	32.144950	30.470306	28.923081	27.491827
53	33.407426	32.495049	30.777805	29.193250	27.729272
54	33.774154	32.838283	31.078538	29.456829	27.960362
55	34.134132	33.174787	31.372653	29.713980	28.185267
56	34.487485	33.504693	31.660296	29.964858	28.404153
57	34.834334	33.828130	31.941609	30.209618	28.617180
58	35.174800	34.145225	32.216733	30.448407	28.824506
59	35.508999	34.456103	32.485803	30.681373	29.026283
60	35.837047	34.760885	32.748952	30.908657	29.222660

TABLE NO. V. Continued.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|$

Periods	3 %	3 1-2 %	4 %	4 1-2 %	5 %
1	.970874	.966184	.961538	.956938	.95238
2	1.913470	1.899694	1.886095	1.872668	1.85941
3	2.828611	2.801637	2.775091	2.748964	2.72325
4	3.717098	3.673079	3.629895	3.587526	3.54595
5	4.579707	4.515052	4.451822	4.389977	4.32948
6	5.417191	5.328553	5.242137	5.157872	5.07569
7	6.230283	6.114544	6.002055	5.892701	5.78637
8	7.019692	6.873956	6.732745	6.595886	6.46321
9	7.786109	7.607687	7.435332	7.268791	7.10782
10	8.530203	8.316605	8.110896	7.912718	7.72174
11	9.252624	9.001551	8.760477	8.528917	8.30641
12	9.954004	9.663334	9.385074	9.118581	8.86325
13	10.634955	10.302738	9.985648	9.682852	9.39357
14	11.296073	10.920520	10.563123	10.222825	9.89864
15	11.937935	11.517411	11.118387	10.739546	10.37966
16	12.561102	12.094117	11.652296	11.234015	10.83777
17	13.166118	12.651321	12.165669	11.707191	11.27407
18	13.753513	13.189682	12.659297	12.159992	11.68959
19	14.323799	13.709837	13.133930	12.593294	12.08532
20	14.877475	14.212403	13.590326	13.007936	12.46221
21	15.415024	14.697974	14.029160	13.404724	12.82115
22	15.936917	15.167125	14.451115	13.784425	13.16300
23	16.443608	15.620410	14.856842	14.147775	13.48857
24	16.935542	16.058368	15.246963	14.495478	13.79864
25	17.413148	16.481515	15.622080	14.828209	14.09395
26	17.876842	16.890352	15.982769	15.146611	14.37519
27	18.327031	17.285365	16.329586	15.451303	14.64303
28	18.764108	17.667019	16.663063	15.742873	14.89813
29	19.188455	18.035767	16.983715	16.021889	15.14107
30	19.600441	18.392045	17.292033	16.288888	15.37245
31	20.000428	18.736276	17.588494	16.544391	15.59281
32	20.388766	19.068865	17.873552	16.788891	15.80268
33	20.765792	19.390208	18.147646	17.022862	16.00255
34	21.131837	19.700684	18.411198	17.246758	16.19290
35	21.487220	20.000661	18.664613	17.461012	16.37419
36	21.832253	20.290494	18.908282	17.666041	16.54685
37	22.167235	20.570525	19.142579	17.862239	16.71129
38	22.492461	20.841087	19.367864	18.049990	16.86789
39	22.808215	21.102500	19.584485	18.229656	17.01704
40	23.114772	21.355072	19.792774	18.401584	17.15909
41	23.412400	21.599104	19.993052	18.566109	17.29437
42	23.701359	21.834883	20.185627	18.723550	17.42321
43	23.981902	22.062689	20.370795	18.874210	17.54591
44	24.254274	22.282791	20.548841	19.018383	17.66277
45	24.518713	22.495450	20.720040	19.156347	17.77407
46	24.775449	22.700918	20.884654	19.288370	17.88007
47	25.024708	22.899438	21.042936	19.414709	17.98102
48	25.266707	23.091244	21.195131	19.535607	18.07716
49	25.501657	23.276565	21.341472	19.651298	18.16872
50	25.729764	23.455618	21.482185	19.962008	18.25593
51	25.951227	23.628616	21.617485	19.867949	18.33898
52	26.166240	23.795764	21.747582	19.969329	18.41807
53	26.374990	23.957260	21.872675	20.066344	18.49340
54	26.577660	24.113395	21.992957	20.159180	18.56515
55	26.774427	24.264053	22.108612	20.248019	18.63347
56	26.965463	24.409713	22.219819	20.333032	18.69854
57	27.150935	24.550447	22.326749	20.414385	18.76052
58	27.331005	24.686422	22.429566	20.492234	18.81954
59	27.505830	24.817899	22.528429	20.566731	18.87575
60	27.675563	24.944733	22.623489	20.638020	18.92929

TABLE NO. V. Concluded.

The Present Value of One Dollar Payable at the End of Each Period at the Rates and for the Terms Stated. $a\overline{n}|$

Periods	6 %	7 %	8 %	9 %	10 %
1	.94339	.93458	.92593	.91743	.90909
2	1.83340	1.80802	1.78327	1.75911	1.73554
3	2.67301	2.62432	2.57710	2.53129	2.48685
4	3.46511	3.38721	3.31213	3.23972	3.16987
5	4.21236	4.10020	3.99271	3.88965	3.79079
6	4.91732	4.76654	4.62288	4.48592	4.35526
7	5.58238	5.38929	5.20637	5.03295	4.86842
8	6.20979	5.97130	5.74664	5.53482	5.33493
9	6.80169	6.51523	6.24689	5.99525	5.75902
10	7.36009	7.02358	6.71008	6.41766	6.14457
11	7.88687	7.49867	7.13896	6.80519	6.49506
12	8.38384	7.92469	7.53608	7.16072	6.81369
13	8.85268	8.35765	7.90378	7.48690	7.10336
14	9.29498	8.74547	8.24424	7.78615	7.36669
15	9.71225	9.10791	8.55948	8.06069	7.60608
16	10.10589	9.44665	8.85137	8.31256	7.82371
17	10.47726	9.76322	9.12164	8.54362	8.02155
18	10.82760	10.05909	9.37189	8.75562	8.20141
19	11.15812	10.33559	9.60360	8.95011	8.36492
20	11.46992	10.59401	9.81815	9.12854	8.51356
21	11.76408	10.83553	10.01680	9.29224	8.64869
22	12.04158	11.06124	10.20074	9.44242	8.77154
23	12.30338	11.27219	10.37106	9.58021	8.88322
24	12.55036	11.46933	10.52876	9.70661	8.98474
25	12.78336	11.65358	10.67478	9.82258	9.07704
26	13.00317	11.82578	10.80998	9.92897	9.16095
27	13.21053	11.98671	10.93552	10.02658	9.23722
28	13.40616	12.13711	11.05108	10.11616	9.30657
29	13.59072	12.27767	11.15841	10.19832	9.36961
30	13.76483	12.40904	11.25728	10.27369	9.42691
31	13.92909	12.53181	11.34980	10.34284	9.47901
32	14.08404	12.64656	11.43499	10.40627	9.52638
33	14.23023	12.75379	11.51389	10.46447	9.56943
34	14.36814	12.85401	11.58694	10.51787	9.60858
35	14.49825	12.94767	11.65457	10.56685	9.64416
36	14.62099	13.03521	11.71719	10.61179	9.67651
37	14.73678	13.11702	11.77518	10.65302	9.70592
38	14.84602	13.19347	11.82887	10.69085	9.73265
39	14.94908	13.26493	11.87858	10.72555	9.75696
40	15.04630	13.33171	11.92461	10.75739	9.77905
41	15.13802	13.39412	11.96724	10.78659	9.79914
42	15.22454	13.45245	12.00670	10.81339	9.81740
43	15.30617	13.50696	12.04324	10.83797	9.83399
44	15.38318	13.55791	12.07707	10.86053	9.84909
45	15.45583	13.60552	12.10840	10.88122	9.86281
46	15.52437	13.65002	12.13741	10.90020	9.87528
47	15.58903	13.69161	12.16427	10.91762	9.88662
48	15.65003	13.73047	12.18914	10.93360	9.89693
49	15.70757	13.76680	12.21216	10.94826	9.90630
50	15.76186	13.80075	12.23348	10.96170	9.91481
51	15.81308	13.83247	12.25323	10.97404	9.92256
52	15.86139	13.86212	12.27151	10.98536	9.92960
53	15.90697	13.88984	12.28843	10.99575	9.93599
54	15.94998	13.91574	12.30410	11.00527	9.94182
55	15.99054	13.93994	12.31861	11.01401	9.94711
56	16.02881	13.96256	12.33205	11.02203	9.95192
57	16.06492	13.98370	12.34449	11.02939	9.95629
58	16.09898	14.00346	12.35601	11.03614	9.96026
59	16.13111	14.02193	12.36667	11.04233	9.96387
60	16.16143	14.03918	12.37655	11.04801	9.96715

TABLE NO. VI.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.499401	.499102	.499068	.498982	.498753
3	.332491	.332206	.332094	.331979	.331672
4	.249053	.248713	.248597	.248465	.248133
5	.198997	.198653	.198507	.198353	.198010
6	.165629	.165270	.165111	.164951	.164595
7	.141788	.141421	.141259	.141106	.140729
8	.123910	.123526	.123370	.122913	.122829
9	.110006	.109772	.109706	.109281	.108907
10	.098888	.098499	.098326	.098146	.097771
1	.089780	.089404	.089218	.089031	.088659
2	.082194	.081816	.081630	.081253	.081066
3	.075776	.075396	.075208	.075017	.074642
4	.070276	.069894	.069704	.069514	.069136
5	.065508	.065125	.064937	.064744	.064364
6	.061336	.060952	.060761	.060572	.060189
7	.057655	.057269	.057079	.056887	.056506
8	.054384	.053998	.053806	.053614	.053232
9	.051457	.051071	.050878	.050648	.050303
20	.048822	.048435	.048242	.048049	.047666
1	.046439	.046051	.045858	.045665	.045282
2	.044273	.043884	.043691	.043589	.043114
3	.042295	.041906	.041711	.041519	.041135
4	.040481	.040092	.039898	.039705	.039321
5	.038813	.038423	.038229	.038036	037652
6	.037273	.036882	.036678	.036528	.036112
7	.035847	.035457	.035264	.035071	.034686
8	.034524	.034133	.034017	.033746	.033362
9	.033291	.032900	.032706	.032511	.032129
30	.032140	.031970	.031483	.031389	.030978
1	.031064	.030674	.030480	.030286	.029903
2	.030056	.029665	.029571	.029278	.028895
3	.029108	.028717	.028518	.028331	.027947
4	.028216	.027826	.027695	.027438	.027056
5	.027375	.026985	.026787	.026589	.026216
6	.026581	.026191	.025997	.025804	.025422
7	.025830	.025381	.025246	.025053	.024671
8	.025118	.024928	.024535	.024342	.023960
9	.024443	.024053	.023860	.023667	.023286
40	.023802	.023412	.023272	.023026	.022646
1	.023197	.022750	.022609	.022417	.022036
2	.022611	.022221	.022028	.021836	.021456
3	.022057	.021668	.021475	.021283	.020903
4	.021529	.021139	.020946	.020851	.020375
5	.021023	.020634	.020488	.020250	.019871
6	.020540	.020151	.019959	.019767	.019389
7	.020077	.019689	.019495	.019305	.018927
8	.019634	.019246	.019053	.018863	.018485
9	.019209	.018821	.018629	.018438	.018061
50	.018801	.018413	.018221	.018030	.017654
1	.018395	.018021	.017788	.017639	.017261
2	.018032	.017644	.017453	.017262	.016886
3	.017669	.017282	.017090	.016901	.016525
4	.017320	.016933	.016741	.016552	.016214
5	.016983	.016596	.016405	.016216	.015841
6	.016659	.016272	.016081	.015892	.015516
7	.016345	.015959	.015768	.015579	.015206
8	.016043	.015657	.015466	.015278	.014905
9	.015751	.015365	.015175	.014986	.014614
60	.015469	.015083	.014893	.014705	.014333

TABLE NO. VI. Continued.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $Sn\overline{|}^{-1}$

Periods	1-4 %	1-3 %	3-8 %	5-12 %	1-2 %
61	.015196	.014810	.014622	.014432	.014061
2	.014932	.014546	.014357	.014169	.013798
3	.014676	.014291	.013974	.013913	.013543
4	.014428	.014043	.013854	.013666	.013297
5	.014187	.013804	.013614	.013427	.013058
6	.013955	.013571	.013382	.013195	.012826
7	.013729	.013335	.013157	.012971	.012602
8	.013509	.013126	.012937	.012751	.012384
9	.013297	.012914	.012726	.012539	.012172
70	.013090	.012708	.012519	.012333	.011967
1	.012887	.012507	.012319	.012133	.011767
2	.012694	.012312	.012124	.011938	.011576
3	.012504	.012122	.011935	.011749	.011384
4	.012319	.011938	.011750	.011565	.011201
5	.012139	.011758	.011598	.011386	.011022
6	.011965	.011583	.011396	.011212	.010848
7	.011793	.011413	.011226	.011042	.010679
8	.011627	.011245	.011086	.010876	.010514
9	.011465	.011086	.010900	.010715	.010354
80	.011307	.010928	.010742	.010558	.010197
1	.011153	.010774	.010579	.010404	.010044
2	.011008	.010624	.010439	.010255	.009896
3	.010856	.010478	.010293	.010110	.009750
4	.010713	.010335	.010150	.009967	.009609
5	.010574	.010196	.010104	.009828	.009470
6	.010437	.010060	.009875	.009693	.009335
7	.010314	.009927	.009742	.009560	.009203
8	.010173	.009797	.009613	.009431	.009074
9	.010046	.009670	.009486	.009304	.008948
90	.009922	.009546	.009362	.009181	.008825
1	.009800	.009425	.009241	.009060	.008705
2	.009681	.009306	.009122	.008941	.008587
3	.009564	.009190	.008998	.008826	.008472
4	.009494	.009076	.008889	.008713	.008060
5	.009339	.008965	.008782	.008602	.008249
6	.009230	.008856	.008673	.008493	.008441
7	.009123	.008749	.008567	.008387	.008036
8	.009017	.008645	.008463	.008287	.007902
9	.008915	.008540	.008361	.008180	.007831
100	.008815	.008442	.008260	.008081	.007732
1	.008716	.008344	.008162	.007984	.007635
2	.008619	.008248	.008066	.007887	.007539
3	.008524	.008153	.007972	.007794	.007446
4	.008431	.008061	.007880	.007702	.007355
5	.008340	.007970	.007789	.007612	.007265
6	.008251	.007881	.007700	.007524	.007177
7	.008163	.007793	.007613	.007436	.007090
8	.008077	.007709	.007528	.007351	.007006
9	.007992	.007624	.007444	.007267	.006922
110	.007909	.007541	.007361	.007185	.006841
1	.007828	.007461	.007280	.007104	.006761
2	.007748	.007382	.007201	.007025	.006682
3	.007669	.007302	.007123	.006946	.006605
4	.007592	.007225	.007046	.006871	.006529
5	.007516	.007150	.006971	.006796	.006455
6	.007442	.007075	.006897	.006722	.006382
7	.007369	.007002	.006824	.006647	.006310
8	.007297	.006931	.006753	.006579	.006240
9	.007226	.006861	.006682	.006509	.006170
120	.007156	.006791	.006614	.006444	.006102

TABLE NO. VI. Continued.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	5-8 %	3-4 %	7-8 %	1 %	1 1-8 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.498524	.498140	.497822	.497512	.497202
3	.331266	.330848	.330434	.330022	.329612
4	.247672	.247207	.246743	.246281	.245821
5	.197516	.197021	.196530	.196040	.195555
6	.164081	.163570	.163138	.162548	.162041
7	.140201	.139672	.139151	.138628	.138108
8	.122290	.121755	.121222	.120690	.120162
9	.108362	.107819	.107279	.106740	.106205
10	.097220	.096671	.096125	.095582	.095056
1	.088104	.087550	.087001	.086454	.085910
2	.080507	.079910	.079399	.078849	.078302
3	.074080	.073523	.072967	.072415	.071867
4	.068572	.068012	.067455	.066901	.066362
5	.063798	.063210	.062678	.062124	.061573
6	.059623	.059059	.058500	.057945	.057394
7	.055937	.059373	.054813	.054258	.053708
8	.052706	.052098	.051538	.050982	.050431
9	.049733	.049164	.048607	.048052	.047501
20	.046988	.046531	.045970	.045415	.047501
1	.044711	.044120	.043585	.043031	.042480
2	.042543	.041978	.041418	.040864	.040316
3	.040564	.039985	.039439	.038886	.038338
4	.038774	.038185	.037626	.037073	.036527
5	.037081	.036517	.035958	.035407	.034862
6	.035521	.034977	.034420	.033869	.033325
7	.034115	.033556	.032995	.032446	.031903
8	.032867	.032229	.031673	.031124	.030583
9	.031560	.030998	.030442	.029895	.029355
30	.030411	.029848	.029294	.028748	.028210
1	.029334	.028774	.028221	.027676	.027139
2	.028326	.027765	.027215	.026671	.026135
3	.027380	.026822	.026270	.025727	.025194
4	.026489	.025931	.025381	.024840	.024308
5	.025649	.025092	.024543	.024004	.023473
6	.024856	.024300	.023752	.023214	.022685
7	.024106	.023551	.023005	.022468	.021941
8	.023396	.022842	.022297	.021761	.021236
9	.022723	.022169	.021625	.021092	.020568
40	.022083	.021531	.020988	.020455	.019934
1	.021414	.020924	.020382	.019851	.019332
2	.020875	.020345	.019805	.019276	.018757
3	.020343	.019794	.019255	.018727	.018211
4	.019816	.019267	.018730	.018204	.017690
5	.019312	.018765	.018229	.017795	.017192
6	.018831	.018285	.017751	.017298	.016719
7	.018371	.017825	.017292	.016771	.016262
8	.017929	.017383	.016853	.016333	.015826
9	.017506	.016963	.016433	.015915	.015409
50	.017099	.016558	.016029	.015513	.015009
1	.016706	.016169	.015641	.015127	.014629
2	.016334	.015795	.015269	.014756	.014256
3	.015973	.015436	.014911	.014400	.013902
4	.015626	.015089	.014566	.014057	.013560
5	.015292	.014756	.014234	.013726	.013232
6	.014969	.014435	.013994	.013408	.012917
7	.014658	.014125	.013606	.013102	.012611
8	.014358	.013826	.013309	.012806	.012317
9	.014067	.013537	.013021	.012520	.012164
60	.013786	.013258	.012744	.012244	.011760

TABLE NO. VI. Continued.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	1 1-4 %	1 3-8 %	1 1-2 %	1 5-8 %	1 3-4 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.496900	.496587	.496278	.495971	.495663
3	.329200	.328450	.328383	.327971	.327567
4	.245356	.244897	.244445	.243990	.243532
5	.195062	.194569	.194089	.193604	.193121
6	.161534	.161184	.160525	.160022	.159523
7	.137437	.137071	.136556	.136045	.135531
8	.119909	.119107	.118584	.118062	.117543
9	.105760	.105139	.104610	.104087	.103558
10	.094285	.093984	.093434	.092904	.092375
1	.085369	.084830	.084294	.083761	.083230
2	.077759	.077218	.076680	.076146	.075614
3	.071321	.070779	.070240	.069705	.069173
4	.065805	.065262	.064723	.064188	.063656
5	.061026	.060483	.059944	.059409	.058877
6	.056844	.056307	.055765	.055230	.054699
7	.053160	.052619	.052080	.051546	.051016
8	.049885	.049343	.048806	.048273	.047745
9	.046956	.046415	.045878	.045347	.044821
20	.044320	.043780	.043246	.042716	.042191
1	.041938	.041399	.040865	.040337	.039815
2	.039393	.039235	.038703	.038177	.037656
3	.037797	.037251	.036731	.036206	.035688
4	.035936	.035452	.034924	.034403	.033886
5	.034243	.033790	.033263	.032744	.032230
6	.032787	.032256	.031732	.031214	.030703
7	.031295	.030838	.030315	.029800	.029291
8	.030049	.029521	.029001	.028488	.027982
9	.028822	.028297	.027779	.027268	.026764
30	.027679	.027155	.026639	.026131	.025630
1	.026616	.026088	.025574	.025068	.024570
2	.025608	.025088	.024577	.024070	.023578
3	.024668	.024151	.023641	.023141	.022648
4	.023784	.023267	.022762	.022264	.021774
5	.022951	.022438	.021934	.021438	.020951
6	.022165	.021654	.021152	.020659	.020175
7	.021373	.020914	.020414	.019878	.019443
8	.202720	.020213	.019716	.019228	.018750
9	.020054	.019549	.019055	.018570	.018094
40	.019420	.018919	.018427	.017945	.017472
1	.018821	.018321	.017813	.017351	.016882
2	.018249	.017751	.017264	.016787	.016321
3	.017705	.017209	.016725	.016250	.015787
4	.017146	.016694	.016210	.015739	.015278
5	.016990	.016199	.015720	.015251	.014793
6	.016217	.015728	.015251	.014785	.014330
7	.015764	.015278	.014803	.014340	.013888
8	.015331	.014830	.014375	.013915	.013466
9	.014916	.014434	.013965	.013507	.013061
50	.014518	.014039	.013571	.013117	.012674
1	.014136	.013659	.013195	.012743	.012303
2	.013769	.013295	.012833	.012384	.011947
3	.013417	.012945	.012485	.012039	.011605
4	.013078	.012608	.012151	.011708	.011277
5	.012747	.012284	.011830	.011392	.010961
6	.012436	.011964	.011521	.011082	.010658
7	.012135	.011672	.011224	.010788	.010366
8	.011845	.011363	.010937	.010504	.010085
9	.011563	.011104	.010660	.010230	.009814
60	.011289	.010834	.010394	.009967	.009553

TABLE NO. VI. Continued.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	1 7-8 %	2 %	2 1-4 %	2 1-2 %	2 3-4 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.495363	.495050	.494438	.493827	.493218
3	.327162	.326755	.325945	.325137	.324332
4	.243077	.242624	.241719	.240818	.239921
5	.192639	.192158	.191200	.190247	.189298
6	.158932	.158526	.157535	.156550	.155571
7	.135021	.134512	.133500	.132495	.131497
8	.117025	.116510	.115485	.114467	.113458
9	.103149	.102515	.101482	.100457	.099441
10	.091850	.091327	.090288	.089259	.088240
1	.082703	.082178	.081136	.080106	.079086
2	.075086	.074560	.073517	.072487	.071469
3	.068644	.068118	.067077	.066048	.065033
4	.063021	.062602	.061562	.060537	.059525
5	.058350	.057825	.056789	.055766	.054757
6	.054173	.053650	.052617	.051599	.050597
7	.050491	.049970	.048940	.047928	.046932
8	.047191	.046702	.045677	.044670	.043681
9	.044299	.043782	.042762	.041761	.040778
20	.041672	.041157	.040142	.039147	.038171
1	.039297	.038785	.037776	.036787	.035819
2	.037141	.036631	.035628	.034647	.033686
3	.035175	.034668	.033671	.032696	.031744
4	.033375	.032871	.031880	.030913	.029969
5	.031722	.031220	.030236	.029276	.028340
6	.030198	.029699	.028721	.027769	.026841
7	.028789	.028293	.027322	.026377	.025458
8	.027482	.026990	.026025	.025088	.024177
9	.026268	.025778	.024821	.023891	.022989
30	.025136	.024650	.023699	.022778	.021884
1	.024024	.023596	.022653	.021739	.020854
2	.023091	.022611	.021674	.020768	.019892
3	.022163	.021687	.020757	.019859	.018993
4	.021290	.020819	.019897	.019007	.018149
5	.020472	.020002	.019087	.018206	.017356
6	.019709	.019233	.018325	.017452	.016611
7	.018970	.018507	.017606	.016741	.015910
8	.018461	.017821	.016927	.016070	.015248
9	.017628	.017171	.016285	.015436	.014623
40	.017009	.016556	.015677	.014836	.014032
1	.016322	.015972	.015101	.014268	.013472
2	.015864	.015417	.014554	.013729	.012942
3	.015690	.014890	.014034	.013217	.012439
4	.014828	.014388	.013539	.012730	.011961
5	.014346	.013910	.013068	.012268	.011507
6	.013887	.013453	.012619	.011827	.011075
7	.013448	.013018	.012191	.011407	.010664
8	.013027	.012602	.011782	.011101	.010272
9	.012627	.012204	.011392	.010623	.009898
50	.012243	.011823	.011018	.010258	.009541
1	.011875	.011459	.010661	.009909	.009200
2	.011522	.011109	.010319	.009575	.008874
3	.011183	.010774	.009991	.009255	.008563
4	.010858	.010452	.009676	.008952	.008265
5	.010544	.010143	.009375	.008654	.007980
6	.010246	.009884	.009082	.008372	.007706
7	.009957	.009561	.008807	.008102	.007443
8	.009679	.009287	.008540	.007843	.007193
9	.009412	.009022	.008283	.007593	.006952
60	.009154	.008768	.008035	.007353	.006720

TABLE NO. VI. Continued.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	3 %	3 1-2 %	4 %	4 1-2 %	5 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.492611	.491400	.490196	.488998	.487805
3	.323530	.321934	.320348	.318733	.317209
4	.239027	.237251	.235490	.233744	.232012
5	.188355	.186481	.184627	.182792	.180975
6	.154598	.152668	.150762	.148078	.147017
7	.130506	.128544	.126610	.124701	.122820
8	.112456	.110477	.108528	.106610	.104722
9	.098434	.096446	.094493	.092574	.090690
10	.087230	.085241	.083290	.081379	.079505
1	.078077	.076092	.074149	.072248	.070389
2	.070462	.068484	.066552	.064666	.062825
3	.064029	.062062	.060144	.058275	.056456
4	.058526	.056571	.054669	.052820	.051024
5	.053767	.051825	.049941	.048114	.046342
6	.049611	.047685	.045820	.044015	.042270
7	.045952	.044043	.042198	.040418	.038699
8	.042709	.040817	.038933	.037237	.035546
9	.039814	.037940	.036139	.034407	.032745
20	.037216	.035361	.033582	.031876	.030243
1	.034872	.033037	.031280	.029601	.027996
2	.032747	.030932	.029199	.027546	.025970
3	.030814	.029019	.027309	.025682	.024137
4	.029047	.027273	.025587	.023987	.022471
5	.027428	.025674	.024012	.022439	.020952
6	.025938	.024205	.022567	.021021	.019564
7	.024564	.022852	.021238	.019719	.018292
8	.023293	.021603	.020013	.018521	.017123
9	.022115	.020445	.018880	.017415	.016046
30	.021019	.019371	.017830	.016392	.015051
1	.019999	.018372	.016855	.015443	.014132
2	.019046	.017442	.015949	.014563	.013280
3	.018156	.016572	.015104	.013744	.012490
4	.017322	.015759	.014315	.012982	.011755
5	.016539	.014998	.013577	.012270	.011072
6	.015804	.014284	.012887	.011606	.010434
7	.015112	.013613	.012239	.010984	.009840
8	.014459	.012982	.011632	.010402	.009284
9	.013844	.012388	.011061	.009856	.008764
40	.013262	.011827	.010523	.009343	.008278
1	.012712	.011298	.010017	.008862	.007822
2	.012192	.010798	.009540	.008409	.007395
3	.011698	.010325	.009090	.007982	.006993
4	.011230	.009878	.008664	.007581	.006616
5	.010785	.009453	.008262	.007202	.006262
6	.010363	.009051	.007882	.006845	.005928
7	.009961	.008669	.007522	.006507	.005614
8	.009578	.008306	.007181	.006189	.005318
9	.009213	.007962	.006857	.005887	.005040
50	.008866	.007634	.006550	.005602	.004777
1	.008534	.007322	.006259	.005332	.004529
2	.008217	.007024	.005982	.005077	.004294
3	.007915	.006741	.005719	.004835	.004073
4	.007626	.006471	.005469	.004605	.003864
5	.007349	.006213	.005231	.004388	.003667
6	.007084	.005967	.005005	.004181	.003480
7	.006831	.005732	.004789	.003985	.003303
8	.006588	.005508	.004584	.003799	.003136
9	.006356	.005294	.004388	.003622	.002978
60	.006133	.005089	.004202	.003454	.002828

TABLE NO. VI. Concluded.

The Annuity Which Will Amount to One Dollar in a Stated Term at the Rates of Interest Stated. $\overline{Sn}|^{-1}$

Periods	6 %	7 %	8 %	9 %	10 %
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	.485437	.483091	.480769	.478469	.476190
3	.314110	.311056	.308039	.305055	.302115
4	.228591	.225226	.221920	.218668	.215471
5	.177396	.173891	.170456	.167092	.163797
6	.143363	.139791	.136316	.132917	.129607
7	.119135	.115554	.112073	.108691	.105406
8	.101036	.097467	.094015	.090674	.087444
9	.087022	.083866	.080080	.076799	.073641
10	.075868	.073278	.069065	.065820	.062745
1	.066793	.063503	.060076	.056934	.053963
2	.059277	.055901	.052695	.049652	.046763
3	.052960	.049765	.046522	.043567	.040779
4	.047585	.044345	.041297	.038433	.035747
5	.042963	.039795	.036830	.034059	.031474
6	.038952	.035858	.032977	.030300	.027817
7	.035445	.032425	.029617	.027046	024664
8	.032357	.029413	.026702	.024212	.021930
9	.029621	.026753	.024128	.021730	.019547
20	.027185	.024393	.021852	.019546	.017460
1	.025005	.022289	.019832	.017617	.015624
2	.023046	.020405	.018032	.015905	.014005
3	.021278	.018714	.016421	.014382	.012572
4	.019679	.017189	.014978	.013023	.011300
5	.018227	.015681	.013679	.011806	.010168
6	.016904	.014561	.012507	.010715	.009159
7	.015697	.013426	.011448	.009735	.008277
8	.014593	.012392	.010489	.008852	.007451
9	.013580	.011449	.009619	.008056	.006728
30	.012649	.010586	.008827	.007336	.006079
1	.011792	.009797	.008107	.006686	.005496
2	.011002	.009073	.007451	.006096	.004972
3	.010273	.008408	.006852	.005562	.004499
4	.009598	.007797	.006304	.005077	.004074
5	.008974	.007234	.005803	.004636	.003690
6	.008395	.006715	.005345	.004235	.003343
7	.007857	.006237	.004924	.003870	.003030
8	.007358	.005795	.004539	.003538	.002747
9	.006894	.005387	.004185	.003236	.002491
40	.006462	.005009	.003860	.002960	.002259
1	.006059	.004660	.003562	.002708	.002050
2	.005683	.004336	.003287	.002478	.001860
3	.005333	.004036	.003034	.002268	.001688
4	.005006	.003958	.002802	.002077	.001532
5	.004701	.003500	.002587	.001902	.001391
6	.004415	.003260	.002389	.001742	.001263
7	.004148	.003045	.002208	.001595	.001147
8	.003898	.002831	.002040	.001461	.001042
9	.003664	.002638	.001886	.001339	.0009459
50	.003444	.002460	.001743	.001227	.0008592
1	.003239	.002294	.001611	.001124	.0007805
2	.003046	.002139	.001490	.001030	.0007092
3	.002866	.001995	.001377	.0009444	.0006441
4	.002696	.001861	.001273	.0008657	.0005852
5	.002537	.001736	.001178	.0007936	.0005317
6	.002388	.001620	.001090	.0007275	.0004832
7	.002247	.001512	.001008	.0006670	.0004392
8	.002116	.001411	.0009323	.0006116	.0003990
9	.001992	.001317	.0008625	.0005608	.0003626
60	.001876	.001229	.0007980	0005142	.0003295

TABLE NO. VII

Carlisle Mortality Table

x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px	x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px
0	10000	1539	.15390	.84610	53	4211	68	.01615	.98385
1	8461	682	.08061	.91939	54	4143	70	.01690	.98310
2	7779	505	.06492	.93508	55	4073	73	.01792	.98208
3	7274	276	.03794	.96206	56	4000	76	.01900	.98100
4	6998	201	.02872	.97128	57	3924	82	.02090	.97910
5	6797	121	.01780	.98220	58	3842	93	.02421	.97579
6	6676	82	.01228	.98772	59	3749	106	.02827	.97173
7	6594	58	.00880	.99120	60	3643	122	.03349	.96651
8	6536	43	.00658	.99342	61	3521	126	.03579	.96421
9	6493	33	.00508	.99492	62	3395	127	.03741	.96259
10	6460	29	.00449	.99551	63	3268	125	.03825	.96175
11	6431	31	.00482	.99518	64	3143	125	.03977	.96023
12	6400	32	.00500	.99500	65	3018	124	.04109	.95891
13	6368	33	.00518	.99482	66	2894	123	.04250	.95749
14	6335	35	.00553	.99447	67	2771	123	.04439	.95561
15	6300	39	.00619	.99381	68	2648	123	.04645	.95355
16	6261	42	.00671	.99329	69	2525	124	.04911	.95089
17	6219	43	.00691	.99309	70	2401	124	.05165	.94835
18	6176	43	.00696	.99304	71	2277	134	.05885	.94115
19	6133	43	.00701	.99299	72	2143	146	.06813	.93187
20	6090	43	.00706	.99294	73	1997	156	.07812	.92188
21	6047	42	.00695	.99305	74	1841	166	.09017	.90983
22	6005	42	.00699	.99301	75	1675	160	.09552	.90448
23	5963	42	.00704	.99296	76	1515	156	.10297	.89703
24	5921	42	.00709	.99291	77	1359	146	.10743	.89257
25	5879	43	.00731	.99269	78	1213	132	.10882	.89118
26	5836	43	.00737	.99263	79	1081	128	.11841	.89159
27	5793	45	.00777	.99223	80	953	116	.12172	.87829
28	5748	50	.00870	.99130	81	837	112	.13381	.86619
29	5698	56	.00983	.99017	82	725	102	.14069	.85931
30	5642	57	.01010	.98990	83	623	94	.15088	.84912
31	5585	57	.01021	.98979	84	529	84	.15879	.84121
32	5528	56	.01013	.98987	85	445	78	.17528	.82472
33	5472	55	.01005	.98995	86	367	71	.19346	.80654
34	5417	55	.01015	.98985	87	296	64	.21622	.78378
35	5362	55	.01026	.98974	88	232	51	.21983	.78017
36	5307	56	.01055	.98945	89	181	39	.21547	.78453
37	5251	57	.01086	.98914	90	142	37	.26056	.73944
38	5194	58	.01117	.98883	91	105	30	.28571	.71429
39	5136	61	.01188	.98812	92	75	21	.28000	.72000
40	5075	66	.01301	.98699	93	54	14	.25926	.74074
41	5009	69	.01378	.98622	94	40	10	.25000	.75000
42	4940	71	.01437	.98563	95	30	7	.23333	.76667
43	4869	71	.01458	.98542	96	23	5	.21739	.78261
44	4798	71	.01480	.98520	97	18	4	.22222	.77778
45	4727	70	.01481	.98519	98	14	3	.21429	.78571
46	4657	69	.01482	.98518	99	11	2	.18182	.81818
47	4588	67	.01460	.98540	100	9	2	.22222	.77778
48	4521	63	.01394	.98606	101	7	2	.28571	.71429
49	4458	61	.01368	.98632	102	5	2	.40000	.60000
50	4397	59	.01342	.98658	103	3	2	.66667	.33333
51	4338	62	.01429	.98571	104	1	1	1.00000	.00000
52	4276	65	.01520	.98480					

TABLE NO. VIII.

Northampton Mortality Table

x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px	x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px
0	11650	3000	.25751	.74249	49	2936	79	.02691	.97309
1	8650	1367	.15804	.84196	50	2857	81	.02835	.97165
2	7283	502	.06893	.93107	51	2776	82	.02954	.97046
3	6781	335	.04940	.95060	52	2694	82	.03044	.96956
4	6446	197	.03056	.96944	53	2612	82	.03139	.96861
5	6249	184	.02945	.97055	54	2530	82	.03241	.96759
6	6065	140	.02308	.97692	55	2448	82	.03350	.96650
7	5925	110	.01857	.98143	56	2366	82	.03466	.96534
8	5815	80	.01376	.98624	57	2284	82	.03590	.96410
9	5735	60	.01046	.98954	58	2202	82	.03724	.96276
10	5675	52	.00916	.99084	59	2120	82	.03868	.96132
11	5623	50	.00889	.99111	60	2038	82	.04024	.95976
12	5573	50	.00897	.99103	61	1956	82	.04192	.95808
13	5523	50	.00905	.99095	62	1874	81	.04322	.95678
14	5473	50	.00914	.99086	63	1793	81	.04518	.95482
15	5423	50	.00922	.99078	64	1712	80	.04673	.95327
16	5373	53	.00986	.99014	65	1632	80	.04902	.95098
17	5320	58	.01090	.98910	66	1552	80	.05155	94845
18	5262	63	.01197	.98803	67	1472	80	.05435	.94565
19	5199	67	.01289	.98711	68	1392	80	.05747	.94253
20	5132	72	.01403	.98597	69	1312	80	.06098	.93902
21	5060	75	.01482	.98518	70	1232	80	.06493	.93507
22	4985	75	.01505	.98495	71	1152	80	.06944	.93056
23	4910	75	.01528	.98472	72	1072	80	.07463	.92537
24	4835	75	.01551	.98449	73	992	80	.08065	.91935
25	4760	75	.01576	.98424	74	912	80	.08772	.91228
26	4685	75	.01601	.98399	75	832	80	.09615	.90385
27	4610	75	.01627	.98373	76	752	77	.10239	.89761
28	4535	75	.01654	.98346	77	675	73	.10815	.89185
29	4460	75	.01682	.98318	78	602	68	.11296	.88704
30	4385	75	.01710	.98290	79	534	65	.12172	.87828
31	4310	75	.01740	.98260	80	469	63	.13433	.86567
32	4235	75	.01771	.98229	81	406	60	.14778	.85222
33	4160	75	.01803	.98197	82	346	57	.16474	.83526
34	4085	75	.01836	.98164	83	289	55	.19031	.80969
35	4010	75	.01870	.98130	84	234	48	.20513	.79487
36	3935	75	.01906	.98094	85	186	41	.22043	.77957
37	3860	75	.01943	.98057	86	145	34	.23448	.76552
38	3785	75	.01981	.98019	87	111	28	.25225	.74775
39	3710	75	.02022	.97978	88	83	21	.25301	.74699
40	3635	76	.02091	.97909	89	62	16	.25807	.74193
41	3559	77	.02164	.97836	90	46	12	.26087	.73913
42	3482	78	.02240	.97750	91	34	10	.29412	.70588
43	3404	78	.02291	.97709	92	24	8	.33333	.66667
44	3326	78	.02345	.97655	93	16	7	.43750	.56250
45	3248	78	.02402	.97598	94	9	5	.55556	.44444
46	3170	78	.02461	.97539	95	4	3	.75000	.25000
47	3092	78	.02523	.97477	96	1	1	1.00000	.00000
48	3014	78	.02588	.97412					

TABLE NO. IX.

Actuaries Mortality Table

x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px	x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px
10	100000	676	.00676	.99324	55	63469	1375	.02166	.97834
11	99324	674	.00679	.99321	56	62094	1436	.02313	.97687
12	98650	672	.00681	.99319	57	60658	1497	.02468	.97532
13	97978	671	.00685	.99315	58	59161	1561	.02639	.97361
14	97307	671	.00690	.99310	59	57600	1627	.02825	.97175
15	96636	671	.00694	.99306	60	55973	1698	.03034	.96966
16	95965	672	.00700	.99300	61	54275	1770	.03261	.96739
17	95293	673	.00706	.99294	62	52505	1844	.03512	.96488
18	94620	675	.00713	.99287	63	50661	1917	.03784	.96216
19	93945	677	.00721	.99279	64	48744	1990	.04083	.95917
20	93268	680	.00729	.99271	65	46754	2061	.04408	.95592
21	92588	683	.00738	.99262	66	44693	2128	.04761	.95239
22	91905	685	.00746	.99254	67	42565	2191	.05147	.94853
23	91219	690	.00756	.99244	68	40374	2246	.05563	.94437
24	90529	694	.00767	.99233	69	38128	2291	.06009	.93991
25	89835	698	.00777	.99223	70	35837	2327	.06493	.93507
26	89137	703	.00789	.99211	71	33510	2357	.07016	.92984
27	88434	708	.00801	.99199	72	31159	2362	.07580	.92420
28	87726	714	.00814	.99186	73	28797	2358	.08188	.91812
29	87012	720	.00828	.99172	74	26439	2339	.08847	.91153
30	86292	727	.00842	.99158	75	24100	2303	.09556	.90444
31	85565	734	.00858	.99142	76	21797	2249	.10318	.89682
32	84831	742	.00875	.99125	77	19548	2179	.11147	.88853
33	84089	750	.00892	.99108	78	17369	2092	.12044	.87956
34	83339	758	.00910	.99090	79	15277	1987	.13006	.86994
35	82581	767	.00929	.99071	80	13290	1866	.14041	.85957
36	81814	776	.00948	.99052	81	11424	1730	.15144	.84856
37	81038	785	.00969	.99031	82	9694	1582	.16319	.83681
38	80253	795	.00991	.99009	83	8112	1427	.17591	.82409
39	79458	805	.01013	.98987	84	6685	1268	.18968	.81032
40	78653	815	.01036	.98964	85	5417	1111	.20510	.79490
41	77838	826	.01061	.98939	86	4306	958	.22248	.77752
42	77012	839	.01089	.98911	87	3348	811	.24223	.75777
43	76173	857	.01125	.98875	88	2537	673	.26527	.73473
44	75316	881	.01170	.98830	89	1864	545	.29238	.70762
45	74435	909	.01221	.98779	90	1319	427	.32373	.67627
46	73526	944	.01284	.98716	91	892	322	.36099	.63901
67	72582	981	.01352	.98648	92	570	231	.40526	.59474
48	71601	1021	.01426	.98574	93	339	155	.45723	.54277
49	70580	1063	.01506	.98494	94	184	95	.51630	.48370
50	69517	1108	.01594	.98406	95	89	52	.58427	.41573
51	68409	1156	.01690	.98310	96	37	24	.64865	.35135
52	67253	1207	.01795	.98205	97	13	9	.69231	.30769
53	66046	1261	.01909	.98091	98	4	3	.75000	.25000
54	64785	1316	.02031	.97969	99	1	1	1.00000	.00000

TABLE NO. IX.

Actuaries Mortality Table—Makehamized

x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px	x Age	No. living lx	No. dying dx	Yearly Probability of living qx	Yearly Probability of dying px
10	100030	691	.99310	.00690	57	60504	1487	.97543	.02457
11	99339	688	.99307	.00693	58	59017	1549	.97375	.02625
12	98651	687	.99304	.00696	59	57468	1614	.97191	.02809
13	97964	684	.99301	.00699	60	55854	1682	.96990	.03010
14	97280	683	.99298	.00702	61	54172	1750	.96770	.03230
15	96597	681	.99295	.00705	62	52422	1818	.96530	.03470
16	95916	680	.99291	.00709	63	50604	1889	.96263	.03832
17	95236	679	.99287	.00712	64	48715	1957	.95982	.04018
18	94557	679	.99282	.00718	65	46758	2024	.95670	.04230
19	93878	679	.99277	.00723	66	44734	2090	.95330	.04670
20	93199	678	.99272	.00728	67	42644	2150	.94958	.05042
21	92521	680	.99266	.00734	68	40494	2206	.94553	.05447
22	91841	681	.99259	.00741	69	38288	2254	.94111	.05889
23	91161	682	.99252	.00748	70	36034	2296	.93630	.06370
24	90479	684	.99244	.00756	71	33738	2326	.93106	.06894
25	89795	686	.99236	.00768	72	31412	2344	.92536	.07464
26	89109	690	.99226	.00774	73	29068	2351	.91915	.08085
27	88419	693	.99216	.00784	74	26717	2340	.91240	.08760
28	87726	698	.99205	.00795	75	24377	2314	.90507	.09493
29	87028	703	.99192	.00808	76	22063	2270	.89712	.10288
30	86325	709	.99179	.00821	77	19792	2207	.88848	.11152
31	85616	716	.99164	.00836	78	17596	2126	.87913	.12087
32	84900	724	.99148	.00852	79	15460	2025	.86900	.12100
33	84176	732	.99130	.00870	80	13425	1907	.85804	.14196
34	83444	743	.99110	.00890	81	11528	1773	.84620	.15380
35	82701	753	.99089	.00911	82	9755	1625	.83342	.16658
36	81948	765	.99066	.00934	83	8130	1464	.81965	.18035
37	81183	780	.99040	.00960	84	6666	1303	.80484	.19516
38	80403	794	.99012	.00988	85	5363	1132	.78893	.21107
39	79609	810	.98982	.01018	86	4231	965	.77187	.22813
40	78799	829	.98948	.01052	87	3266	805	.75361	.24639
41	77970	848	.98912	.01088	88	2461	654	.73411	.26589
42	77122	870	.98872	.01128	89	1807	518	.71333	.28667
43	76252	894	.98828	.01172	90	1289	398	.69128	.30872
44	75358	919	.98780	.01270	91	891	296	.66790	.33210
45	74439	948	.98727	.01273	92	595	212	.64321	.35679
46	73491	977	.98670	.01330	93	383	147	.61723	.38277
47	72514	1010	.98607	.01393	94	236	97	.58998	.41001
48	71504	1045	.98538	.01462	95	139	61	.56152	.43848
49	70459	1083	.98463	.01537	96	78	36	.53193	.46807
50	69376	1123	.98381	.01618	97	42	21	.50132	.49868
51	68253	1167	.98291	.01709	98	21	11	.46983	.53017
52	67086	1212	.98192	.01808	99	10	6	.43760	.56240
53	65874	1262	.98085	.01915	100	4	2	.37180	.62820
54	64612	1314	.97967	.02023	101	2	1	.33869	.66131
55	63298	1386	.97838	.02162	102	1	1	.00000	1.00000
56	61930	1426	.97697	.02303					

TABLE NO. X.

American Experience Mortality Table.

x Age	No. living lx	No Dying dx	Yearly Probability of dying qx	Yearly Probability of living px	x Age	No. living lx	No. dying dx	Yearly Probability of dying qx	Yearly Probability of living px
10	100000	749	.00749	.99251	53	66797	1091	.01633	.98367
11	99251	746	.00752	.99248	54	65706	1143	.01740	.98260
12	98505	743	.00754	.99246	55	64563	1199	.01857	.98143
13	97762	740	.00757	.99343	56	63364	1260	.01988	.98012
14	97022	737	.00760	.99240	57	62104	1325	.02134	.97866
15	96285	735	.00763	.99237	58	60779	1394	.02294	.97706
16	95550	732	.00766	.99234	59	59385	1468	.02472	.97528
17	94818	729	.00769	.99231	60	57917	1546	.02669	.97331
18	94089	727	.00773	.99227	61	56371	1628	.02888	.97112
19	93362	725	.00777	.99223	62	54743	1713	.03129	.96871
20	92637	723	.00781	.99219	63	53030	1800	.03394	.96606
21	91914	722	.00786	.99214	64	51230	1889	.03687	.96313
22	91192	721	.00791	.99209	65	49341	1980	.04013	.95987
23	90471	720	.00796	.99204	66	47361	2070	.04371	.95629
24	89751	719	.00801	.99199	67	45291	2158	.04765	.95235
25	89032	718	.00807	.99193	68	43133	2243	.05200	.94800
26	88314	718	.00813	.99187	69	40890	2321	.05676	.94324
27	87596	718	.00820	.99180	70	38569	2391	.06199	.93801
28	86878	718	.00826	.99174	71	36178	2448	.06767	.93233
29	86160	719	.00835	.99165	72	33730	2487	.07373	.92627
30	85441	720	.00843	.99157	73	31243	2505	.08018	.91982
31	84721	721	.00851	.99149	74	28738	2501	.08703	.91297
32	84000	723	.00861	.99139	75	26237	2476	.09437	.90563
33	83277	726	.00872	.99128	76	23761	2431	.10231	.89769
34	82551	729	.00883	.99117	77	21330	2369	.11106	.88894
35	81822	732	.00895	.99105	78	18961	2291	.12083	.87917
36	81090	737	.00909	.99091	79	16670	2196	.13173	.86827
37	80353	742	.00923	.99077	80	14474	2091	.14447	.85553
38	79611	749	.00941	.99059	81	12383	1964	.15861	.84139
39	78862	756	.00959	.99041	82	10419	1816	.17430	.82570
40	78106	765	.00979	.99021	83	8603	1648	.19156	.80844
41	77341	774	.01001	.98999	84	6955	1470	.21136	.78864
42	76567	785	.01025	.98975	85	5485	1292	.23555	.76445
43	75782	797	.01052	.98948	86	4193	1114	.26568	.73432
44	77985	812	.01083	.98917	87	3079	933	.30302	.69698
45	74173	828	.01116	.98884	88	2146	744	.34669	.65331
46	73345	848	.01156	.98844	89	1402	555	.39586	.60414
47	72497	870	.01200	.98800	90	847	385	.45455	.54545
48	71627	896	.01251	.98749	91	462	246	.53247	.46753
49	70731	927	.01311	.98689	92	216	137	.63426	.36574
50	69804	762	.01378	.98622	93	79	58	.73418	.26582
51	68842	1001	.01454	.98546	94	21	18	.85714	.14286
52	67842	1041	.01539	.98461	95	3	3	1.00000	.00000

TABLE NO. XI.

National Fraternal Congress Table of Mortality.

x Age	lx Number Living	dx Number Dying	qx Yearly Probability of Dying	1000 qx Yearly Death Rate Per 1000	1000 qxv Discounted yearly Rate Per 1000	ex Expectation of Life	Average Duration of Life	Annuity Due $(1+a_x)$
20	100000	500	.0050000	5.000	4.808	45.6	49.7	20.4916
21	99500	501	.0050352	5.035	4.841	44.9	48.8	20.3731
22	98999	502	.0050708	5.071	4.876	44.1	47.9	20.2500
23	98497	503	.0051068	5.107	4.911	43.3	47.0	20.1220
24	97994	505	.0051535	5.154	4.956	42.5	46.1	19.9890
25	97489	507	.0052006	5.201	5.001	41.8	45.2	19.8508
26	96982	510	.0052587	5.259	5.057	41.0	44.3	19.7074
27	96472	513	.0053176	5.318	5.113	40.2	43.4	19.5585
28	95959	517	.0053877	5.388	5.181	39.4	42.5	19.4040
29	95442	522	.0054693	5.469	5.259	38.6	41.6	19.2439
30	94920	527	.0055520	5.552	5.338	37.8	40.7	19.0780
31	94393	533	.0056466	5.647	5.430	37.0	39.8	18.9060
32	93860	540	.0057532	5.753	5.532	36.2	38.9	18.7280
33	93320	548	.0058723	5.872	5.646	35.4	38.0	18.5439
34	92772	557	.0060040	6.004	5.773	34.6	37.1	18.3534
35	92215	567	.0061487	6.149	5.912	33.9	36.2	18.1565
36	91648	578	.0063067	6.307	6.064	33.1	35.3	17.9532
37	91070	591	.0064895	6.490	6.240	32.3	34.4	17.7432
38	90479	606	.0066977	6.698	6.440	31.5	33.5	17.5267
39	89873	622	.0069209	6.921	6.655	30.7	32.6	17.3036
40	89251	640	.0071708	7.171	6.895	29.9	31.7	17.0740
41	88611	660	.0074483	7.448	7.162	29.1	30.9	16.8376
42	87951	683	.0077657	7.766	7.467	28.3	30.0	16.5948
43	87268	708	.0081129	8.113	7.801	27.5	29.1	16.3455
44	86560	734	.0084797	8.480	8.154	26.8	28.2	16.0898
45	85826	761	.0088668	8.867	8.526	26.0	27.4	15.8276
46	85065	790	.0092870	9.287	8.930	25.2	26.5	15.5587
47	84275	822	.0097538	9.754	9.379	24.4	25.6	15.2830
48	83453	857	.0102693	10.269	9.874	23.7	24.8	15.0006
49	82596	894	.0108238	10.824	10.408	22.9	23.9	14.7117
50	81702	935	.0114440	11.444	11.004	22.2	23.1	14.4162
51	80767	981	.0121460	12.146	11.679	21.4	22.2	14.1144
52	79786	1029	.0128970	12.897	12.401	20.7	21.4	13.8067
53	78757	1083	.0137512	13.751	13.222	19.9	20.6	13.4930
54	77674	1140	.0146767	14.677	14.112	19.2	19.8	13.1738
55	76534	1202	.0157054	15.705	15.101	18.5	19.0	12.8494
56	75332	1270	.0168587	16.859	16.211	17.8	18.2	12.5200
57	74062	1342	.0181200	18.120	17.423	17.1	17.4	12.1862
58	72720	1418	.0194994	19.499	18.749	16.4	16.6	11.8484
59	71302	1501	.0210513	21.051	20.241	15.7	15.8	11.5067

TABLE NO. XI. (Concluded)

National Fraternal Congress Table of Mortality.

x Age	lx Number Living	dx Number Dying	qx Yearly Probability of Dying	1000 qx Yearly Death Rate Per 1000	1000 qx v Discounted Yearly Rate Per 1000	ex Expectation of Life	Average Duration of Life	Annuity Due $(1+a_x)$
60	69801	1588	.0227504	22.750	21.875	15.0	15.1	11.1619
61	68213	1681	.0246434	24.643	23.695	14.4	14.4	10.8144
62	66532	1778	.0267240	26.724	25.696	13.7	13.6	10.4649
63	64754	1880	.0290330	29.033	27.916	13.1	12.9	10.1138
64	62874	1985	.0315711	31.571	30.357	12.4	12.2	9.7617
65	60889	2094	.0343904	34.390	33.067	11.8	11.6	9.4092
66	58795	2206	.0375202	37.520	36.077	11.2	10.9	9.0571
67	56589	2318	.0409620	40.962	39.387	10.7	10.3	8.7060
68	54271	2430	.0447753	44.775	43.053	10.1	9.7	8.3566
69	51841	2539	.0489767	48.977	47.093	9.5	9.1	8.0095
70	49302	2645	.0536489	53.649	51.586	9.0	8.5	7.6653
71	46657	2744	.0588122	58.812	56.550	8.5	7.9	7.3248
72	43913	2832	.0644912	64.491	62.011	8.0	7.4	6.9889
73	41081	2909	.0708113	70.811	68.087	7.5	6.9	6.6578
74	38172	2969	.0777795	77.780	74.788	7.0	6.4	6.3325
75	45203	3009	.0854757	85.476	82.188	6.6	6.0	6.0135
76	32194	3026	.0939927	93.993	90.377	6.2	5.5	5.7014
77	29168	3016	.1034010	103.401	99.424	5.7	5.1	5.3967
78	26152	2977	.1138345	113.835	109.457	5.3	4.7	5.0999
79	23175	2905	.1253506	125.351	120.530	5.0	4.3	4.8117
80	20270	2799	.1380858	138.086	132.775	4.6	4.0	4.5323
81	17471	2659	.1521951	152.195	146.341	4.3	3.6	4.2621
82	14812	2485	.1677694	167.769	161.316	3.9	3.3	4.0016
83	12327	2280	.1849599	184.960	177.846	3.6	3.0	3.7510
84	10047	2050	.2040410	204.041	196.193	3.3	2.8	3.5102
85	7997	1800	.2250844	225.084	216.427	3.0	2.5	3.2799
86	6197	1539	.2483460	248.346	238.794	2.8	2.3	3.0598
87	4658	1277	.2741520	274.152	263.608	2.5	2.0	2.8500
88	3381	1023	.3025732	302.573	290.035	2.3	1.8	2.6507
89	2358	788	.3341815	334.182	321.329	2.1	1.7	2.4614
90	1570	579	.3687898	368.790	354.606	1.9	1.5	2.2828
91	991	404	.4076690	407.669	391.989	1.7	1.4	2.1135
92	587	264	.4497445	449.745	432.447	1.5	1.2	1.9551
93	323	161	.4984520	498.452	479.281	1.4	1.0	1.8051
94	162	89	.5493827	549.383	528.253	1.2	.9	1.6695
95	73	44	.6027397	602.740	579.557	1.1	.8	1.5452
06	29	19	.6551724	655.172	629.973	1.0	.8	1.4272
97	10	7	.7000000	700.000	673.077	.8	.7	1.2885
98	3	3	1.0000000	1000.000	961.538	.5	.5	1.0000

TABLE NO. XII.

Commutation Columns for Annuities
Actuaries or Combined Experience.
One Life.

x Age	Five Per Cent. Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx
10	61391.32	1016386.90	55839.49	801152.40
11	58072.68	958314.22	52322.65	748829.75
12	54932.01	903382.21	49026.02	699803.73
13	51959.83	851422.38	45935.92	653867.81
14	49146.65	802275.73	43038.98	610828.83
15	46483.57	755792.16	40322.83	570506.00
16	43962.68	711829.48	37776.27	532729.73
17	41576.03	670253.45	35388.43	497341.30
18	39316.56	630936.89	33149.53	464191.77
19	37177.23	593759.66	31050.04	433141.73
20	35151.73	558607.93	29081.41	404060.32
21	33233.76	525374.17	27235.26	376825.06
22	31417.71	493956.46	25504.11	351320.95
23	29698.29	464258.17	23880.83	327440.12
24	28070.14	436188.03	22358.72	305081.40
25	26528.52	409659.51	20931.43	284149.97
26	25068.95	384590.56	19593.21	264556.76
27	23686.90	360963.66	18338.38	246218.38
28	22378.34	338525.32	17161.85	229056.53
29	21139.25	317386.07	16058.65	212997.88
30	19966.02	297420.05	15024.31	197073.57
31	18855.06	278564.99	14054.47	183919.10
32	17803.16	260761.83	13145.19	170773.91
33	16807.08	243954.75	12292.65	158481.26
34	15863.97	228090.78	11493.40	146987.86
35	14971.13	213119.65	10744.22	136243.64
36	14125.79	198993.86	10041.91	126201.733
37	13325.53	185668.33	9383.648	116818.085
38	12568.05	173100.28	8766.746	108051.339
39	11851.00	161249.28	8188.585	99862.754
40	11172.32	150076.96	7646.818	92215.936
41	10530.05	139546.906	7139.228	85076.708
42	9922.196	129624.710	6663.618	78413.060
43	9346.763	120277.947	6217.974	72195.086
44	8801.527	111476.420	5800.015	66395.071
45	5284.357	103192.063	5407.775	60987.296
46	7793.512	95398.551	5033.512	55953.784
47	7327.695	88071.456	4693.028	51260.756
48	6883.872	81187.584	4367.546	46893.210
49	6462.582	74725.002	4061.573	42831.637
50	6062.142	68662.860	3773.963	39057.674
51	5681.468	62981.392	3503.597	35554.071
52	5319.447	57661.945	3249.426	32304.651
53	4975.236	52686.709	3010.479	29294.172
54	4647.552	48038.857	2785.851	26508.321

TABLE NO. XII. (Concluded)

Commutation Columns for Annuities.
Actuaries or Combined Experience.
One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx
55	4336.607	43702.248	2574.774	23933.547
56	4040.625	39661.623	2376.409	21557.138
57	3759.223	35902.400	2190.048	19367.090
58	3491.854	32410.546	2015.094	17351.996
59	3237.828	29172,718	1850.872	15501.124
60	2996.544	26176.174	1696.784	13804.340
61	2767.277	23408.897	1552.180	12252.160
62	2549.554	20859.343	1416.567	10835.593
63	2342.869	18516.474	1289.449	9546.144
64	2146.871	16369.603	1170.431	8375.713
65	1961.166	14408.437	1059.101	7316.612
66	1785.442	12622.995	955.1076	6361.504
67	1619.458	11003.537	858.1431	5503.361
68	1462.950	9540.587	767.8968	4735.464
69	1315.777	8224.810	684.1311	4051.333
70	1177.825	7046.985	606.6261	3444.707
71	1048.900	5998.085	535.1283	2909.579
72	928.8678	5069.2172	469.4197	2440.159
73	817.5765	4251.6407	409.2785	2030.880
74	714.8862	3536.7545	354.4957	1676.384
75	620.6111	2916.1434	304.8436	1371.540
76	534.5765	2381.5669	260.1064	1111.434
77	456.5900	1924.9769	220.0649	891.369
78	386.3754	1538.6015	184.4664	706.903
79	323.6558	1214.9457	153.0645	553.838
80	268.1521	946.7936	125.6191	428.219
81	219.5255	727.2681	101.8691	326.3498
82	177.4109	549.8572	81.5496	244.8002
83	141.3891	408.4681	64.3785	180.4217
84	110.9686	297.4995	50.0505	130.3712
85	85.6384	211.8611	38.2614	92.1098
86	64.8327	147.0284	28.6926	63.4172
87	48.0083	99.0201	21.0463	42.3709
88	34.6467	64.3733	15.0454	27.3255
89	24.2437	40.1297	10.4286	16.8970
90	16.3383	23.7913	6.9617	9.9352
91	10.5230	13.2684	4.4415	5.4937
92	6.4041	6.8642	2.6775	2.8162
93	3.6273	3.2369	1.5023	1.3139
94	1.8751	1.3618	.7692	.5446
95	.8638	.4980	.3510	.1936
96	.3420	.1560	.1377	.0560
97	.1144	.0415	.0456	.0103
98	.0335	.0080	.0103	.0000
99	.0080	.0000		

TABLE NO. XIII.

Carlisle Table.

Annuity Commutation Columns. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
1	8058.094	112772.078	7982.076	96415.031	1
2	7055.767	105716.311	6923.282	89491.749	2
3	6283.555	99432.756	6107.390	83384.359	3
4	5757.271	93675.485	5543.070	77841.289	4
5	5325.628	88349.857	5079.112	72762.177	5
6	4981.733	83368.124	4706.316	68055.861	6
7	4686.232	78681.892	4385.385	63670.476	7
8	4423.821	74258.071	4100.767	59569.709	8
9	4185.445	70072.626	3843.196	55726.513	9
10	3965.879	66106.747	3607.229	52119.284	10
11	3760.078	62346.669	3387.770	48731.514	11
12	3563.760	58782.909	3180.604	45550.910	12
13	3377.086	55405.823	2985.567	42565.343	13
14	3199.605	52206.218	2801.976	39763.367	14
15	3030.407	49175.811	2628.770	37134.597	15
16	2868.236	46307.575	2464.619	34669.978	16
17	2713.330	43594.245	2309.515	32360.463	17
18	2566.235	41028.010	2163.723	30196.740	18
19	2427.036	38600.974	2027.036	28169.704	19
20	2295.256	36305.718	1898.890	26270.814	20
21	2170.525	34135.193	1778.757	24492.057	21
22	2052.804	32082.389	1666.418	22825.639	22
23	1941.381	30141.008	1561.097	21264.542	23
24	1835.912	28305.096	1462.360	19802.182	24
25	1736.085	26569.011	1369.799	18432.383	25
26	1641.320	24927.691	1282.810	17149.573	26
27	1551.645	23376.046	1201.282	15948.291	27
28	1466.278	21909.768	1124.482	14823.809	28
29	1384.308	20525.460	1051.604	13772.2048	29
30	1305.431	19220.029	982.3293	12789.8755	30
31	1230.707	17989.322	917.3634	11872.5121	31
32	1160.140	16829.182	856.6044	11015.9077	32
33	1093.702	15735.480	799.9311	10215.9766	33
34	1031.151	14704.3291	747.0667	9468.9099	34
35	972.0784	13732.2507	697.6242	8771.2857	35
36	916.2929	12815.9578	651.3853	8119.9004	36
37	863.4514	11952.5064	608.0300	7511.8704	37
38	813.4081	11139.0983	567.3864	6944.4840	38
39	766.0238	10373.0745	529.2931	6415.1909	39
40	720.8816	9652.1929	493.4026	5921.7883	40
41	677.6254	8974.5675	459.4206	5462.3677	41
42	636.4676	8338.0999	427.4452	5034.9225	42
43	597.4476	7740.6523	397.4546	4637.4679	43
44	560.7005	7179.9518	369.4895	4267.9784	44
45	526.0986	6653.8532	343.4168	3924.5616	45
46	493.6265	6160.2267	319.1805	3605.3811	46
47	463.1548	5697.0719	296.6522	3308.7289	47
48	434.6585	5262.4134	275.7737	3032.9552	48
49	408.1919	4854.2215	256.5385	2776.4167	49
50	383.4349	4470.7866	238.7059	2537.7108	50
51	360.2757	4110.5109	222.1725	2315.5383	51
52	338.2159	3772.2950	206.6011	2108.9372	52

TABLE NO. XIII. (Concuded)

Carlisle Table.

Annuity Commutation Columns. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
53	317.2140	3455.0810	191.9439	1916.9933	53
54	297.2300	3157.8510	178.1550	1738.8383	54
55	278.2934	2879.5576	165.2311	1573.6072	55
56	260.2910	2619.2666	153.0845	1420.5227	56
57	243.1861	2376.0805	141.6754	1278.8473	57
58	226.7660	2149.3145	130.8631	1147.9842	58
59	210.7398	1938.5747	120.4674	1027.5168	59
60	195.0299	1743.5448	110.4351	917.0817	60
61	179.5224	1564.0224	100.6951	816.38658	61
62	164.8554	1399.1670	91.59592	724.79066	62
63	151.1319	1248.0351	83.17877	641.61189	63
64	138.4293	1109.6058	75.46905	566.14284	64
65	126.5944	983.0114	68.36566	497.77718	65
66	115.6124	867.3990	61.84596	435.93122	66
67	105.4297	761.96934	55.86549	380.06573	67
68	95.95015	666.01919	50.36388	329.70185	68
69	87.13644	578.88275	45.30610	284.39575	69
70	78.91167	499.97108	40.64261	243.75314	70
71	71.27262	428.69846	36.36190	207.39124	71
72	63.88407	364.81439	32.28496	175.10628	72
73	56.69689	308.11750	28.38244	146.72384	73
74	49.77894	258.33856	24.68424	122.03960	74
75	43.13377	215.20479	21.18726	100.85234	75
76	37.15572	178.04907	18.07870	82.77364	76
77	31.74267	146.30640	15.29918	67.47446	77
78	26.98333	119.32307	12.88260	54.59186	78
79	22.90187	96.42120	10.83084	43.76102	79
80	19.22866	77.19254	9.00790	34.75312	80
81	16.08393	61.10861	7.46364	27.28948	81
82	13.26830	47.84031	6.09898	21.19050	82
83	10.85866	36.98165	4.94426	16.24623	83
84	8.78121	28.20044	3.96062	12.28561	84
85	7.03509	21.16535	3.14312	9.14249	85
86	5.52569	15.63966	2.44547	6.69702	86
87	4.24446	11.39520	1.86072	4.83630	87
88	3.16832	8.22688	1.37585	3.46045	88
89	2.35413	5.87275	1.01264	2.44781	89
90	1.75894	4.11381	.74948	1.69833	90
91	1.23869	2.87512	.52282	1.17551	91
92	.84265	2.03247	.35231	.82320	92
93	.57782	1.45465	.23930	.58390	93
94	.40763	1.04702	.16723	.41667	94
95	.29116	.75586	.11832	.29835	95
96	.21260	.54326	.08558	.21277	96
97	.15846	.38480	.06318	.14959	97
98	.11738	.26742	.04636	.10323	98
99	.08783	.17959	.03437	.06886	99
100	.06844	.11115	.02653	.04233	100
101	.05070	.06045	.01946	.02287	101
102	.03445	.02596	.01312	.00975	102
103	.01970	.00626	.00742	.00233	103
104	.00626	.00000	.00233	.00000	104

TABLE NO. XIV.

Commutation Columns for Annuities.

Northampton Experience. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
1	8238.094	95264.271	8160.380	82473.4470	1
2	6605.891	88658.380	6481.845	75991.602	2
3	5857.682	82800.698	5693.453	70298.149	3
4	5303.130	77497.568	5105.833	65192.316	4
5	4896.289	72601.279	4669.626	60522.690	5
6	4526.005	68075.274	4275.782	56246.908	6
7	4210.787	63864.487	3940.464	52306.444	7
8	3935.921	59928.566	3648.403	48658.041	8
9	3696.816	56231.750	3394.538	45263.503	9
10	3483.958	52747.792	3168.891	42094.612	10
11	3287.423	49460.369	2962.133	39132.479	11
12	3103.254	46357.115	2769.609	36362.870	12
13	2928.964	43428.151	2589.398	33773.472	13
14	2764.236	40663.915	2417.705	31355.767	14
15	2608.555	38055.360	2262.828	29092.939	15
16	2461.433	35593.927	2115.061	26977.878	16
17	2321.099	33272.828	1975.659	25002.219	17
18	2186.469	31086.359	1843.589	23158.630	18
19	2057.425	29028.934	1718.337	21440.293	19
20	1934.200	27094.734	1600.182	19840.111	20
21	1816.249	25278.485	1488.426	18351.685	21
22	1704.122	23574.363	1383.363	16968.322	22
23	1598.555	21975.807	1285.424	15682.898	23
24	1499.178	20476.629	1194.141	14488.757	24
25	1405.642	19070.981	1009.074	13379.683	25
26	1318.613	17752.374	1029.809	12349.874	26
27	1234.781	16517.593	955.9648	11393.909	27
28	1156.849	15360.744	887.1829	10506.726	28
29	1083.541	14277.203	823.1231	9683.603	29
30	1014.590	13262.6132	763.4728	8920.130	30
31	949.7496	12312.8636	707.9384	8212.192	31
32	888.7835	11424.0797	656.2445	7555.947	32
33	831.4696	10592.6196	608.1354	6947.812	33
34	777.5995	9815.0106	563.3687	6384.443	34
35	726.9740	9088.0366	521.7219	5862.721	35
36	679.4069	8408.6297	482.9850	5379.736	36
37	634.7215	7773.9082	446.9616	4932.774	37
38	592.7512	7181.1570	413.4690	4519.305	38
39	553.3388	6627.8182	382.3360	4136.969	39
40	516.3360	6111.4822	353.4028	3783.566	40
41	481.4672	5630.0150	326.4280	3457.138	41
42	448.6196	5181.3954	301.2884	3155.850	42
43	417.6858	4763.7096	277.8672	2877.983	43
44	388.6907	4375.0189	256.1322	2621.851	44
45	361.4911	4013.5278	235.9674	2385.884	45
46	336.0095	3677.5183	217.2648	2168.619	46
47	312.1350	3365.3833	199.9235	1968.695	47
48	289.7723	3075.6110	183.8491	1784.8455	48

TABLE NO. XIV. (Concluded.

Commutation Columns for Annuities.

Northampton Experience. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
49	268.8317	2806.7793	168.9541	1615.8914	49
50	249.1239	2557.6554	155.1018	1460.7896	50
51	230.5501	2327.1053	142.1741	1318.6155	51
52	213.0855	2114.0198	130.1643	1188.4512	52
53	196.7616	1917.2582	119.0590	1069.3922	53
54	181.5096	1735.7492	109.0445	960.3477	54
55	167.2630	1568.4862	99.3090	861.0387	55
56	153.9621	1314.5224	90.5495	770.4892	56
57	141.5486	1272.9755	82.4637	688.0257	57
58	129.9684	1143.0071	75.0027	613.0230	58
59	119.1701	1923.8370	68.1225	544.9005	59
60	109.1053	914.7317	61.7806	483.1199	60
61	99.7290	815.0027	55.9385	427.1814	61
62	90.0983	724.9044	50.5599	376.6215	62
63	82.9191	641.9853	45.6363	230.9852	63
64	75.4030	566.5823	41.1081	289.8770	64
65	68.4567	498.1256	36.9690	252.9079	65
66	62.0009	436.1247	33.1669	219.7410	66
67	56.0047	380.1200	29.6766	190.0644	67
68	50.4390	329.6810	26.4752	163.5891	68
69	45.2764	284.4046	23.5412	140.0479	69
70	40.4911	243.9135	20.8545	119.1934	70
71	36.0589	207.8546	18.3965	100.7969	71
72	31.9569	175.8977	16.1500	84.6469	72
73	28.1639	147.7338	14.0988	70.5481	73
74	24.6596	123.0742	12.2281	58.3199	74
75	21.4253	101.6489	10.5241	47.7958	75
76	18.4430	83.1059	8.9737	38.8221	76
77	15.7762	67.4297	7.5989	31.2232	77
78	13.3916	54.0381	6.3935	24.8297	78
79	11.3132	42.7249	5.3503	19.4794	79
80	9.4630	33.2619	4.4330	15.0463	80
81	7.8017	25.4602	3.6204	11.4259	81
82	6.3322	19.1280	2.9107	8.5152	82
83	5.0371	14.0909	2.2936	6.2216	83
84	3.8843	10.2065	1.7520	4.4697	84
85	2.9405	7.2660	1.3138	3.1559	85
86	2.1832	5.0829	.9661	2.1897	86
87	1.5917	3.4912	.6977	1.4919	87
88	1.1335	2.3577	.4922	.9997	88
89	.8062	1.5515	.3469	.6528	89
90	.5698	.9817	.2428	.4100	90
91	.4011	.5806	.1693	.2407	91
92	.2696	.3109	.1127	.1280	92
93	.1712	.1398	.0709	.0571	93
94	.0917	.0481	.0376	.0195	94
95	.0388	.0092	.0158	.0037	95
96	.0092	.0000	.0037	.0000	96

TABLE NO. XV.

Commutation Columns for Annuities.

American Experience. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
10	61391.48	1013248.23	55839.45	798121.48	10
11	58030.00	955218.23	52284.19	745837.29	11
12	54851.27	900366.96	48953.98	696883.31	12
13	51845.27	848521.69	45834.73	651048.58	13
14	49002.70	799518.99	42912.92	608135.66	14
15	46314.72	753204.27	40176.37	567959.29	15
16	43772.56	709431.71	37612.91	530346.38	16
17	41368.79	668062.92	35212.03	495134.35	17
18	39095.93	628966.99	32963.50	462170.85	18
19	36946.51	592020.48	30857.35	431313.50	19
20	34913.91	557106.57	28884.66	402428.84	20
21	32991.93	524114.74	27037.00	375391.84	21
22	31174.00	492940.74	25306.24	350085.60	22
23	29454.76	463485.98	23685.01	326400.59	23
24	27828.90	435657.08	22166.57	304234.02	24
25	26291.45	409365.63	20744.34	283489.68	25
26	24837.50	384528.13	19412.30	264077.38	26
27	23462.45	361065.68	18164.60	245912.78	27
28	22162.02	338903.66	16995.96	228916.82	28
29	20932.25	317971.41	15901.41	213015.41	29
30	19769.12	298202.29	14876.14	198139.27	30
31	18669.08	279533.21	13915.84	184223.43	31
32	17628.76	261904.45	13016.43	171207.00	32
33	16644.74	245259.71	12173.95	159033.05	33
34	15714.00	229545.71	11384.73	147648.32	34
35	14833.53	214712.18	10645.48	137002.84	35
36	14000.78	200711.40	9953.05	127049.79	36
37	13212.90	187498.50	9304.329	117745.461	37
38	12467.51	175030.99	8696.614	109048.847	38
39	11762.08	163268.91	8127.166	100921.681	39
40	11094.62	152174.29	7593.637	93328.044	40
41	10462.81	141711.483	7093.643	86234.401	41
42	9864.861	131846.622	6225.142	79609.259	42
43	9298.785	122547.837	6186.057	73423.202	43
44	8762.846	113784.991	5774.526	67648.676	44
45	8255.196	105529.795	5388.673	62260.003	45
46	7774.327	97755.468	5021.127	57238.887	46
47	7318.514	90436.954	4687.533	52551.349	47
48	6886.373	83550.581	4369.132	48182.217	48
49	6476.407	77074.174	4070.262	44111.955	49
50	6087.169	70987.005	3789.544	40322.411	50
51	5717.409	65269.596	3525.773	36796.638	52
52	5365.974	59903.622	3277.836	33518.802	52

TABLE NO. XV. (Concluded.)

Commutation Columns for Annuities.

American Experience. One Life.

x Age	Five Per Cent Dx	Five Per Cent Nx	Six Per Cent Dx	Six Per Cent Nx	x Age
53	5031.809	54871.813	3444.712	30474.090	53
54	4713.926	50157.887	2825.455	27648.635	54
55	4411.358	45746.529	2619.154	25029.481	55
56	4123.270	41623.259	2425.014	22604.467	56
57	3848.837	37774.422	2242.256	20362.211	57
58	3587.354	34187.068	2070.205	18292.006	58
59	3338.167	30848.901	1908.230	16383.776	59
60	3100.617	27748.284	1755.717	14628.059	60
61	2874.143	24874.141	1612.122	13015.937	61
62	2658.227	22215.914	1471.947	11538.990	62
63	2452.424	19763.490	1349.746	10189.244	63
64	2256.365	17507.125	1230.124	8959.120	64
65	2069.681	15437.444	1117.702	7841.418	65
66	1892.026	13545.418	1012.124	6829.2939	66
67	1723.173	11822.245	913.1015	5916.1924	67
68	1562.922	10259.323	820.3721	5095.8203	68
69	1411.092	8848.231	733.6896	4362.1307	69
70	1267.615	7580.616	652.8718	3709.2589	70
71	1132.411	6448.205	577.7342	3131.5247	71
72	1005.511	5442.6945	508.1526	2623.3721	72
73	887.021	4555.6735	444.0423	2179.3298	73
74	777.0489	3778.6246	385.3208	1794.0090	74
75	675.6422	3102.9824	331.8748	1462.1342	75
76	582.7441	2520.2373	283.5431	1178.5911	76
77	498.2129	2022.0244	240.1262	938.4649	77
78	421.7896	1600.2348	201.3742	737.0907	78
79	353.1676	1247.0672	167.0214	570.0693	79
80	292.0417	955.0255	136.8104	433.2589	80
81	237.9538	717.0717	110.4208	322.83808	81
82	190.6792	526.3925	87.64858	235.18950	82
83	149.9471	376.4454	68.27522	166.91428	83
84	115.4506	260.9948	52.07200	114.84228	84
85	86.7134	174.28137	38.74165	76.10063	85
86	63.13136	111.15001	27.93640	48.16099	86
87	44.15102	66.99899	19.35527	28.80572	87
88	29.30700	37.69199	12.72664	16.07908	88
89	18.23477	19.45722	7.84380	8.23528	89
90	10.49171	8.96551	4.47049	3.76479	90
91	5.45024	3.51527	2.30043	1.46436	91
92	2.42682	1.08844	1.01464	.44972	92
93	.84532	.24312	.35009	.09963	93
94	.21401	.02912	.08779	.01183	94
95	.02912	.00000	.01183	.00000	95

TABLE NO. XVI.

Carlisle Table, Makeham's Formula.

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cent Nxxx
10	2561958	37854571	10	1655025	22124056
11	2415546	35439025	11	1552616	20571440
12	2277030	33161995	12	1456090	19115350
13	2145939	31016056	13	1365074	17750276
14	2021770	28994286	14	1279154	16471122
15	1904240	27090046	15	1198135	15272987
16	1792878	25297168	16	1121622	14151365
17	1687426	23609748	17	1049407	131019579
18	1587487	22022261	18	9812252	121207327
19	1492728	20529533	19	9167883	112039444
20	1406003	19126530	20	8560157	103479287
21	1318035	17808495	21	7986898	95492389
22	1237681	16570814	22	7444571	88047818
23	1160987	15409827	23	6932952	81114866
24	1088476	14321351	24	6449110	74665756
25	1018388	13302963	25	5980486	68685270
26	9526080	123503550	26	5544306	63140964
27	8909353	114594197	27	5138682	58002282
28	8332344	106261853	28	4761798	53240484
29	7789530	98472323	29	4410713	48829771
30	7281153	91191170	30	4084509	44745262
31	6804023	84387147	31	3780792	40964470
32	6356768	78030379	32	3498510	37465960
33	5937148	72093231	33	3235864	34230096
34	5543553	66549678	34	2991579	31238517
35	5174056	61375622	35	2764135	28474382
36	4827451	56548171	36	2552616	25921766
37	4502264	52045907	37	2355854	23565912
38	4197070	47848837	38	2172781	21393131
39	3910741	43938096	39	2002564	19390567
40	3641913	40296183	40	1844083	17546484
41	3389493	36906690	41	1696607	15849877
42	3152430	33754260	42	1559350	14290527
43	2930041	30824219	43	1431795	12858733
44	2721168	28103051	44	1313097	11545636
45	2524998	25578053	45	1202682	10342954
46	2340869	23237184	46	1100068	92428860
47	2167877	21069307	47	1004613	82382734
48	2005546	19063761	48	9158931	73223803
49	1853020	17210741	49	8336002	64887801
50	1709820	15500921	50	7571081	57316720
51	1575368	13925553	51	6861200	50455520
52	1449257	12476296	52	6203544	44251976
53	1330845	11145451	53	5593808	38658168
54	1219741	99257100	54	5029357	33628811
55	1115641	88100691	55	4508072	29120799
56	1018024	77920452	56	4026590	25094209
57	9266926	68653526	57	3583428	21510781

TABLE NO. XVI. (Concluded)

Carlisle Table, Makeham's Formula,
Annuity Commutation Columns.

x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cent Nxxx
58	8411580	60241946	58	3175456	18335325
59	7612907	52629039	59	2801626	15533699
60	6867078	45161961	60	2459444	13074255
61	6172150	39589811	61	2147476	10926779
62	5525524	34064287	62	1863921	9062858
63	4925399	29138888	63	1607404	7455454
64	4369970	24768918	64	1376497	6078957
65	3857167	20911751	65	1169647	4909310
66	3385249	17526502	66	9854458	39238642
67	2952683	14573819	67	8225583	31013059
68	2558061	12015758	68	6796769	24216290
69	2199847	9815911	69	5554173	18662117
70	1876408	7939503	70	4483486	14178631
71	1586449	6353054	71	3571576	1060755
72	1328203	5024851	72	2803572	7803483
73	1100262	39245894	73	2165976	5637507
74	9009650	30236244	74	1644621	3992886
75	7283686	22952558	75	1224970	27679162
76	5806591	17145967	76	8934602	18744560
77	4559099	12586868	77	6369517	12375043
78	3519872	9066996	78	4427647	7947396
79	2668951	6398045	79	2995632	4951764
80	1983143	4414902	80	1966088	2985676
81	1441794	2973108	81	1248882	17367942
82	1023407	1949700.9	82	7653038	9714904
83	707462.5	1242238.4	83	4507244	5207660
84	475301.2	766937.2	84	2543337	2664323
85	309413.9	457523.3	85	1368847	12954856
86	194589.0	262934.3	86	6995471	5959385
87	117865.4	145068.79	87	3379202	2580133
88	68523.19	96545.60	88	1534919	10452637
89	38076.09	38469.51	89	6514819	39378180
90	20136.47	18333.049	90	2567401	13704174
91	10093.86	8239.189	91	9336817	4367357
92	4761.532	3477.657	92	3099757	1267599.6
93	2109.411	1368.2463	93	936578.5	331021.1
94	868.9034	499.3429	94	253714.0	77307.14
95	331.7701	167.5728	95	61451.39	15855.75
96	116.0245	51.5483	96	12986.18	2869.567
97	37.1483	14.4000	97	2417.569	451.9977
98	10.7199	3.6801	98	391.1616	60.8361
99	2.8976	.7825	99	54.7671	6.0689
100	.6325	.1500	100	5.5437	.5253
101	.1159	.0342	101	.4635	.0618
102	.0276	.0066	102	.0552	.0066
103	.0066	.0000	103	.0066	.0000

TABLE NO. XVII.

Carlisle Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx
10	2330270	30324753	10	1550355	17926548
11	2176372	28148391	11	1398885	16527663
12	2032217	26116164	12	1299542	15228121
13	1897152	24219012	13	1206816	14021305
14	1770516	22448496	14	1120188	12901117
15	1651869	20796627	15	1039340	118617765
16	1540586	19256041	16	9637749	108980016
17	1436288	17819753	17	8932276	100047740
18	1338479	16481274	18	8273181	91774559
19	1246711	15234563	19	7656923	84117636
20	1160720	14073843	20	7081900	77035736
21	1080137	12993706	21	6545308	70490428
22	1004487	119892185	22	6043297	64447131
23	9335691	110556494	23	5574901	58872230
24	8670044	101886450	24	5136913	53735317
25	8035250	93851200	25	4718700	49016617
26	7445493	86405707	26	4333277	44683340
27	6897898	79507809	27	3978363	40704977
28	6390037	73117772	28	3651802	37053175
29	5917400	67200372	29	3350668	33702507
30	5479037	61721335	30	3073568	30628939
31	5071684	56649651	31	2818183	27810756
32	4693600	51956051	32	2583170	25227586
33	4342415	47613636	33	2366703	22860883
34	4016290	43597346	34	2167387	20693496
35	3713226	39884120	35	1983716	18709780
36	3431804	36452316	36	1814636	16895144
37	3170428	33281888	37	1658958	15236186
38	2927634	30354254	38	1515621	13720565
39	2702173	27652081	39	1383675	12336890
40	2492683	25159398	40	1262170	11074720
41	2298024	22861374	41	1150276	99244440
42	2117140	20744234	42	1047243	88772006
43	1949223	18795011	43	9525071	79246935
44	1793191	17001820	44	8653042	70593893
45	1648221	15363599	45	7850650	62743243
46	1513614	13839985	46	7113075	55630168
47	1388529	12451456	47	6434584	49195584
48	1272440	11179016	48	5811617	43383967
49	1164577	10014439	49	5238971	38144996
50	1064441	89499984	50	4713348	33431648
51	9714873	79785111	51	4231118	29200530
52	8852869	70932242	52	3789471	25411059
53	8052846	62879396	53	3384772	22026287
54	7310938	55568458	54	3014519	19011768
55	6623876	48994582	55	2676515	16335253
56	5987791	42956791	56	2368153	13967100
57	5398727	37558064	57	2087634	11879466

TABLE NO. XVII. (Concluded)

Carlisle Table—Makeham's Formula.

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Six Per Cent Nxx	x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx
58	4854190	32703874	58	1832555	10046911
59	4351843	28352031	59	1601522	8445389
60	3888467	24463064	60	1319554	7052835
61	3461998	21001566	61	1204531	5848304
62	3070056	17931510	62	1035622	48126817
63	2710801	15220709	63	8846700	39280117
64	2382418	12838291	64	7504381	31775736
65	2083012	10755279	65	6316526	25459210
66	1810911	8944368	66	5271563	20187647
67	1564613	7379755	67	4358698	15828949
68	1342717	60370380	68	3567598	12261351
69	1143798	48932404	69	2887861	9372490
70	9664231	39268173	70	2309172	7064318
71	8093759	31174414	71	1822148	5242170
72	6712309	24462105	72	1416834	3825336
73	5507909	18954196	73	1084285	27410513
74	4467679	14486517	74	8155302	19255211
75	3577739	10908778	75	6017042	13238169
76	3825286	8083492	76	4347268	8890901
77	2197372	5886120	77	3069948	5820953
78	1680472	4205648	78	2113882	3707071
79	1262211	2943437.5	79	1416706	22903645
80	929026.9	2014410.6	80	9210374	13693271
81	669054.5	1345356.1	81	5795351	7897920
82	470424.4	874931.7	82	3517835	4380085
83	322128.0	552803.7	83	2052277	23278084
84	214376.5	338427.2	84	1147126	11806824
85	138239.3	200187.85	85	6115703	5691121
86	86117.96	114069.89	86	3095940	2595181
87	51670.78	62399.11	87	1481401	11137798
88	29756.37	32642.74	88	6665428	4472370
89	16378.66	16264.08	89	2802389	1669981
90	8580.100	7683.98	90	1093963	5760176
91	4260.396	3423.584	91	3940866.	1819310.
92	1990.776	1432.8083	92	1295995	523314.9
93	873.6162	559.1921	93	387885.6	135429.3
94	356.4633	202.7288	94	104087.3	31341.98
95	134.9851	67.7437	95	24972.24	6369.736
96	46.6738	21.0699	96	5227.470	1142.2656
97	14.8306	6.2393	97	963.9864	178.2792
98	4.8154	1.4240	98	154.5013	23.7779
99	1.1278	.2962	99	21.4279	2.3499
100	.2387	.0575	100	2.1485	.2014
101	.0445	.0130	101	.1779	.0235
102	.0105	.0025	102	.0210	.0025
103	.0025	.0000	103	.0025	.0000

TABLE NO. XVIII.

American Experience Table, Makeham's Formula,

Annuity Commutation Columns

x Age	Two Lives Equal Ages Three & One Half Per Cent Dxx	Two Lives Equal Ages Three & One Half Per Cent Nxx	x Age	Three Lives Equal Ages Three & One Half Per Cent Dxxx	Three Lives Equal Ages Three & One Half Per Cent Nxxx
10	7100669	128026372	10	7188715	111888195
11	6755941	121270431	11	6709661	105178534
12	6427697	114842734	12	6334687	98843847
13	6115297	108727437	13	5980516	92863331
14	5817984	102909453	14	5646004	87217327
15	5534918	97374535	15	5329904	81887423
16	5265432	92109103	16	5031226	76856197
17	5008879	87100224	17	4749019	72107178
18	4764653	82335571	18	4482395	67624783
19	4532074	77803497	19	4230374	63394409
20	4310684	73492813	20	3992296	59402113
21	4099871	69392942	21	3767290	55634823
22	3899137	65493805	22	3554648	52080175
23	3707930	61785875	23	3353600	48726575
24	3525885	58259990	24	3163644	45562931
25	3352500	54907490	25	2984060	42578871
26	3187302	51720188	26	2814227	38764644
27	3029914	48690274	27	2653630	37111014
28	2879982	45810292	28	2501783	34609231
29	2737164	43073128	29	2358231	32251000
30	2600952	40472176	30	2222305	30028695
31	2471172	38001004	31	2093775	27934920
32	2347479	35653525	32	1972190	25962730
33	2229494	33424031	33	1857057	24105673
34	2117021	31307010	34	1748131	22357542
35	2009672	29297338	35	1644916	20712626
36	1907282	27390056	36	1547203	19165423
37	1809584	25580472	37	1454667	17710756
38	1716306	23864166	38	1366969	16343787
39	1627183	22236983	39	1283785	15060002
40	1542093	20694890	40	1204212	13855790
41	1460762	19234128	41	1130178	12725612
42	1383051	17851077	42	1083935	116663471
43	1308683	16542394	43	9919034	106744437
44	1237580	15304814	44	9279996	97464441
45	1169500	14135314	45	8672778	88791663
46	1104287	13031027	46	8095641	80696022
47	1041829	119891983	47	7547321	73148701
48	9819618	110072365	48	7026035	66122666
49	9245145	100827220	49	6529939	59592727
50	8694350	92132870	50	6058470	53534257
51	8165466	83967404	51	5609839	47924418
52	7657489	76309915	52	5182974	42741444
53	7169272	69140643	53	4776743	37964701
54	6700205	62440438	54	4390577	33574124

TABLE NO. XXIII. (Concluded)

American Experience Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Three & One Half Per Cent Dxx	Two Lives Equal Ages Three & One Half Per Cent Nxx	x Age	Three Lives Equal Ages Three & One Half Per Cent Dxxx	Three Lives Equal Ages Three & One Half Per Cent Nxxx
55	6249185	56191253	55	4023411	29550713
6	5815617	50375636	6	3674714	25875999
7	5398665	44976971	7	3343720	22532279
8	4997784	39979187	8	3029957	19502322
9	4612537	35366650	9	2733067	16769255
60	4242434	31124216	60	2452636	14316619
1	3887366	27236850	1	2188587	12128032
2	3546890	23689960	2	1940539	10187493
3	3221295	20468665	3	1708704	8478789
4	2910385	17558280	4	1492854	6985935
5	2614375	14943905	5	1293044	5692891
6	2333362	12610543	6	1109186	45837046
7	2067834	10542709	7	9414020	36423026
8	1818091	8724618	8	7895791	28527235
9	1584450	7140168	9	6535236	21991999
70	1367422	5772746	70	5330499	16661500
1	1167330	46054159	1	4277330	12384170
2	9845444	36208715	2	3370581	9013589
3	8193524	28015191	3	2603328	6410261
4	6717844	21297347	4	1966246	4444015
5	5417067	15880280	5	1448470	2995545
6	4288522	11591758	6	1037994	19575515
7	3326534	8265224	7	7214255	12361260
8	2522044	5743180	8	4845100	7516160
9	1864614	3878566	9	3133487	4382673
80	1340320	25382455	80	1942793	2439880
1	9338504	16043951	1	1149474	12904058
2	6284720	9759231	2	6456294	6447764
3	4069518	5689713	3	3422465	3025299
4	2524653	3165060	4	1701364	13239350
5	1493999	1671060.9	5	7879353	5359997
6	838235.3	832825.6	6	3368868	19911290
7	443477.4	389348.2	7	1318904	6722249
8	219790.3	169557.9	8	4681533	2040716
9	101282.4	68275.47	9	1489865	5508511
90	43079.25	25196.22	90	4204534	1303977
1	16742.08	8454.137	1	1036334	267643.4
2	5905.151	2548.986	2	220852.6	46790.85
3	1867.992	680.994	3	39975.03	6815.821
4	521.1980	159.7956	4	5993.776	822.0448
5	128.0922	31.7034	5	744.6475	77.3973
6	26.8197	4.8837	6	72.4132	4.9841
7	4.3010	.5827	7	4.7311	.2530
8	.5495	.0332	8	.2198	.0332
9	.0332	.0000	99	.0332	.0000

TABLE NO. XIX.

American Experience Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cent Nxxx
10	6149080	88895620	10	6154061	79411583
11	5766951	83128669	11	5727361	73684222
12	5408390	77720279	12	5330130	68354092
13	5072024	72648255	13	4960237	63393855
14	4756497	67891758	14	4615895	58777960
15	4460432	63431326	15	4295217	54482743
16	4182643	59248683	16	3996599	50486144
17	3922009	55326674	17	3718535	46767609
18	3677479	51649195	18	3459625	43307984
19	3447998	48201197	19	3218464	40089520
20	3232721	44968476	20	2993945	37095575
21	3030697	41937779	21	2784846	34310729
22	2841133	39096646	22	2590119	31720610
23	2663212	36433434	23	2408716	29311894
24	2496280	33937154	24	2239813	27082081
25	2339619	31597535	25	2082495	24989586
26	2192555	29404980	26	1935916	23053670
27	2054513	27350467	27	1799362	21254308
28	1924949	25425518	28	1672168	19582140
29	1803355	23622163	29	1553699	18028441
30	1689133	21933030	30	1443229	16585212
31	1581924	20351106	31	1340233	15244979
32	1481274	18869832	32	1244463	14000516
33	1386727	17483105	33	1155074	12845442
34	1297957	16185148	34	1071790	117736517
35	1214537	14970611	35	9941012	107795505
36	1136192	13834419	36	9216904	98578601
37	1062597	127718219	37	8541876	90036725
38	9934239	117783980	38	7912224	82124501
39	9283832	108500148	39	7324573	74799928
40	8672666	99827482	40	6776647	68023281
41	8097906	91729576	41	6265268	61758013
42	7557570	84172006	42	5788268	55969745
43	7049035	77122971	43	5342746	50626999
44	6570821	70552150	44	4927130	45699869
45	6120651	64431499	45	4538952	41160917
46	5696794	58734705	46	4176376	36984541
47	5297801	53436904	47	3837886	33146655
48	4922040	48514864	48	3521760	29624895
49	4567885	43946979	49	3226344	26398551
50	4234369	39712610	50	2950636	23447915
51	3919986	35792624	51	2693109	20754806
52	3623608	32169016	52	2452639	18302167
53	3344112	28824904	53	2228115	16074052
54	3080667	25744237	54	2018730	14055322

TABLE NO XIX. (Concluded)

American Experience Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cent Nxxx
55	2832246	22911991	55	1823485	12231837
56	2598092	20313899	56	1641619	10590218
57	2377367	17936532	57	1472446	9117772
58	2169394	15767138	58	1315217	7802555
59	1973567	13793571	59	1169398	6633157
60	1789279	12004292	60	1034424	55987326
61	1616105	10388187	61	9098670	46888656
62	1453492	8934695	62	7952205	38936451
63	1301208	7633487	63	6902130	32034321
64	1158826	6474661	64	5944088	26090233
65	1026092	54485694	65	5074946	21015287
66	9027165	45458529	66	4291171	16724116
67	7885623	37572906	67	3590009	13134107
68	6834194	30738712	68	2968023	10166084
69	5870864	24867848	69	2421490	7744598
70	4994331	19873517	70	1946890	5797704
71	4202600	15670917	71	1539917	4257787
72	3493904	12177013	72	1196139	3061648
73	2866141	9310872	73	9106590	2150989
74	2316369	6994503	74	6779780	14730113
75	1841166	5153337	75	4923099	9807014
76	1436771	3716566	76	3477559	6329455
77	1098558	26180077	77	2382443	3947012
78	8209840	17970237	78	1577192	2369820
79	5983045	11987192	79	1005451	13643691
80	4239284	7747908	80	6144843	7498848
81	2911470	4836438	81	3583730	3915118
82	1931399	2905039	82	1984126	1930992
83	1232765	1672274.3	83	1036753	8942393
84	753858.6	918415.7	84	5080253	3862140
85	439733.7	478682.0	85	2319156	15429841
86	243195.9	235486.1	86	9774044	5655797
87	126827.7	108658.41	87	3771846	1883951
88	61958.34	46700.07	88	1319716	5642354
89	28143.49	18556.58	89	4139906	1502448
90	11799.47	6757.109	90	1151628	3508199
91	4520.172	2236.937	91	2797986	710212.6
92	1571.549	665.3880	92	587759.3	122453.3
93	490.0300	175.3580	93	104866.6	17586.69
94	134.7628	40.5952	94	15498.84	2087.852
95	32.6496	7.9457	95	1893.654	194.198
96	6.7384	1.2073	96	181.9361	12.262
97	1.0652	.1421	97	11.7170	.5446
98	.1341	.0080	98	.5366	.0080
99	.0080	.0000	99	.0080	.0000

TABLE NO. XX.

American Experience Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Six Per Cent Nxx	x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx
10	5592904	71118805	10	5597526	164268432
11	5195952	65922853	11	5160360	59108072
12	4826910	61095943	12	4757064	54351008
13	4484004	56611939	13	4385177	49965831
14	4165387	52446552	14	4042258	45923573
15	3869265	48577287	15	3725947	42197626
16	3594064	44983223	16	3434200	38763426
17	3338312	41644911	17	3165120	35598306
18	3100645	38544266	18	2916962	32681344
19	2879732	35664534	19	2688029	29993315
20	2674459	32990075	20	2476923	27516392
21	2483673	30506402	21	2282197	25234195
22	2306361	28200041	22	2102593	23131602
23	2141529	26058512	23	1936888	21194714
24	1988363	24070149	24	1784079	19140635
25	1845997	22224152	25	1643122	17767513
26	1713640	20510512	26	1513059	16254454
27	1590601	?18919911	27	1393065	14861389
28	1476235	17443676	28	1282376	13579013
29	1369906	16073770	29	1180283	12398730
30	1271062	14802708	30	1086021	11312709
31	1179159	13623549	31	9990775	103136315
32	1093718	12529831	32	9188652	93947663
33	1014248	115155827	33	8448181	85499482
34	9403679	105752148	34	7765088	77734394
35	8716298	97035850	35	7134275	70600119
36	8077100	88958750	36	6552225	64047894
37	7482636	81476114	37	6015066	58032828
38	6929551	74546563	38	5519110	52513718
39	6414773	68131790	39	5061000	47452118
40	5935947	62195843	40	4638230	42814888
41	5490267	56705577	41	4247765	38566723
42	5075588	51629989	42	3887333	34679390
43	4689401	46940588	43	3554284	31125106
44	4330028	42610560	44	3246872	27878234
45	3995324	38615236	45	2962852	24915382
46	3683565	34931671	46	2700458	22214924
47	3393259	31538412	47	2458179	19756745
48	3122840	28415572	48	2234424	17522321
49	2870810	25544762	49	2027678	15494643
50	2636090	22908672	50	1836907	13657736
51	2417351	20491321	51	1660783	11996953
52	2218605	18272716	52	1498209	10498744
53	2023498	16249218	53	1348216	91505280
54	1846504	14402714	54	1209995	79405330

TABLE NO. XX. (Concluded)

American Experience Table, Makeham's Formula,

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Six Per Cent Nxx	x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx
55	1681550	12721164	55	1082657	68578760
56	1528012	11193152	56	9655051	58923709
57	1383007	98081450	57	8578180	50345529
58	1253365	85547800	58	7589907	42755622
59	1128170	74266100	59	6684746	36070876
60	1013175	64134348	60	5857366	30213510
61	9064817	55069531	61	5103492	25110018
62	8075802	46993729	62	4418353	20691665
63	7161485	39832244	63	3798739	16892926
64	6317681	33514563	64	3240592	13652334
65	5541271	27973292	65	2740658	10911676
66	4829009	23144283	66	2295512	8616164
67	4178555	18965728	67	1902329	6713835
68	3587243	15378485	68	1557903	5155932
69	3052525	12325962	69	1259043	3896889
70	2572277	97536850	70	1002725	28941639
71	2144085	76096000	71	7856355	21085284
72	1765706	58438940	72	6044894	15040390
73	1434789	44091050	73	4558756	10481634
74	1148634	33604705	74	3361939	7119695
75	9043792	24560913	75	2418219	4701476
76	6990827	17570086	76	1692060	3009416
77	5294774	12275312	77	1148276	18611345
78	3919607	8355705	78	7529957	11081438
79	2823017	5532688	79	4755016	6326422
80	1985944	3546744	80	2878626	3447796
81	1351048	2195696.3	81	1663005	17847906
82	887796.7	1307899.6	82	9120335	8727571
83	561313.1	746586.5	83	4720643	4006928
84	340015.1	406571.4	84	2291362	1715566
85	196463.0	210108.4	85	1036148	6794182
86	107629.7	102478.65	86	4325635	2468547
87	55599.59	46879.06	87	1653532	8150146.
88	26905.64	19973.43	88	5730900	2419246.
89	12106.09	7867.344	89	1780806.	638440.3
90	5027.724	2839.620	90	490705.9	147734.4
91	1907.866	931.7543	91	118096.8	29637.61
92	657.0566	274.6977	92	24573.95	5063.663
93	202.9468	71.7509	93	4343.063	720.6004
94	55.2898	16.4611	94	635.8321	84.7683
95	13.2678	3.1933	95	76.9531	7.8151
96	2.7125	.4808	96	7.3237	.4915
97	.4247	.0561	97	.4672	.0243
98	.0530	.0031	98	.0212	.0031
99	.0031	.0000	99	.0031	.0000

TABLE NO. XXI

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Four Per Cent Dxx	Two Lives Equal Ages Four Per Cent Nxx	x Age	Two Lives Equal Ages Four Per Cent Dxxx	Two Lives Equal Ages Four Per Cent Nxxx
10	6744128	113666335	57	3914373	29884788
11	6410221	107256114	58	3581148	26303640
12	6078538	101177576	59	3264994	23038646
13	5763708	95413868	60	2965543	20073103
14	5464865	89949003	61	2682371	17390732
15	5181185	84767818	62	2415288	14975444
16	4902848	79864970	63	2164030	12811414
17	4656223	75208747	64	1928400	10883014
18	4413505	70795242	65	1708235	9174779
19	4183050	66612192	66	1503372	7671407
20	3964229	62647963	67	1313675	6357732
21	3756442	58891521	68	1138984	52187485
22	3559116	55332405	69	9791134	42396351
23	3371710	51960695	70	8338383	34057968
24	3193709	48766986	71	7028753	27029215
25	3024628	45742358	72	5858681	21170534
26	2864000	42878358	73	4823737	16346797
27	2711389	40166969	74	3918533	12428264
28	2566375	37600594	75	3136641	9291623
29	2428564	35172030	76	2470590	6821033
30	2297583	32874447	77	1911900	4909133
31	2173072	30701375	78	1451216	3457917
32	2054694	28646681	79	1078458	23794586
33	1942130	26704551	80	7830811	15963775
34	1835074	24869477	81	5543537	10420238
35	1733240	23136237	82	3816787	6603451
36	1636355	21499882	83	2549145	4054306
37	1544159	19955723	84	1685088	2369218
38	1456407	18499316	85	1025674	13435438
39	1372868	17126448	86	6138344	7297094
40	1293322	15833126	87	3516436	3780658
41	1217535	14615591	88	1920266	1860391.6
42	1145392	13470199	89	995072.7	865318.9
43	1076625	12393574	90	486880.5	378438.4
44	1011094	113824802	91	223716.7	154721.72
45	9486278	104338524	92	95960.80	58760.92
46	8890758	95447766	93	38174.43	20586.49
47	8322879	87124887	94	13983.97	6602.517
48	7781446	79343441	95	4680.243	1922.274
49	7265047	72078394	96	1418.942	503.3322
50	6772563	65305831	97	386.0507	117.28147
51	6302913	59002918	98	93.29245	23.98902
52	5855097	53147821	99	19.70997	4.27905
53	5428205	47719616	100	3.629225	.64982
54	5021400	42698216	101	.565410	.08441
55	4633932	38064284	102	.076024	.00839
56	4265123	33799161	103	.008386	.00000

TABLE NO. XXII.

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Three Lives Equal Ages Four Per Cent Dxxx	Three Lives Equal Ages Four Per Cent Nxxx	x Age	Three Lives Equal Ages Four Per Cent Dxxx	Three Lives Equal Ages Four Per Cent Nxxx
10	6761692	100738977	57	2368336	14469822
11	6367851	94371126	58	2113490	12356332
12	5996512	88374614	59	1876321	10480011
13	5646375	82728239	60	1656342	8823669
14	5316215	77412024	61	1453099	7370570
15	5004880	72407144	62	1266155	6104415
16	4711284	67695860	63	1095078	50093373
17	4434398	63261462	64	9394259	40099114
18	4173261	59088201	65	7987361	32711753
19	3926959	55161242	66	6725117	25986636
20	3694636	51466506	67	5602074	20384562
21	3475482	47991024	68	4612218	15772344
22	3268737	44722287	69	3748864	12023480
23	3073680	41648607	70	3004620	9018860
24	2889636	38758971	71	2371325	6647535
25	2715968	36043003	72	1840348	4807187
26	2552072	33490931	73	1402143	3405044
27	2397383	31093548	74	1046933	23581110
28	2251370	28842178	75	7646230	15934880
29	2113532	26728646	76	5450888	10483992
30	1983384	24745262	76	3784264	6699728
31	1860492	22884770	78	2552103	4147625
32	1744432	21140338	79	1667330	2480295
33	1634811	19505527	80	1052068	14282268
34	1531254	17974273	81	6390440	7891828
35	1433413	16540860	82	3723179	4168649
36	1340961	15199899	83	2072408	2096241
37	1253587	13946312	84	1097322	9989193
38	1171002	12775310	85	5500861	4488332
39	1092932	11682378	86	2597230	1891102
40	1019124	106632544	87	1148429	7426727
41	9493376	97139168	88	4726168	2700559
42	8833476	88305692	89	1797901	9026576
43	8211357	80094335	90	6275288	2751288
44	7619398	72474937	91	1993263	7580247
45	7061451	65413486	92	5710505	1869742
46	6533938	58879548	93	1461198	4085437
47	6035286	52844262	24	3303792	781644.7
48	5564045	47280217	95	652358.3	129286.4
49	5118874	42161343	96	111057.3	18229.09
50	4698546	37462797	97	16072.51	2156.579
51	4301918	33160879	98	1949.168	209.4118
52	3927968	29232911	99	192.8319	16.57994
53	3575753	25657158	100	15.53774	1.04220
54	3244422	22412736	101	.99137	.05082
55	2933198	19479538	102	.04899	.00183
56	2641380	16838158	103	.00183	.00000

TABLE NO. XXIII.

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx	x Age	Two Lives Equal Ages Five Per Cent Dxx	Two Lives Equal Ages Five Per Cent Nxx
10	6142783	89619008	57	2268675	18521737
11	5769756	83849252	58	2055780	14197282
12	5419105	78430147	59	1856439	12340843
13	5089493	73340654	60	1670095	10670748
14	4779649	68561005	61	1496254	9174494
15	4488380	64072625	62	1334441	7840053
16	4214567	59858058	63	1184260	6655793
17	3957155	55900903	64	1045240	56105527
18	3715150	52185753	65	9170840	46934687
19	3487676	48698077	66	7994163	38940524
20	3273705	45424372	67	6918927	32021597
21	3072569	42351803	68	5941722	26079875
22	2883440	39468363	69	5059082	21020793
23	2705534	36762829	70	4267412	16753381
24	2538355	34224474	71	3562912	13190469
25	2381070	31843404	72	2941512	10248957
26	2233151	29610253	73	2398824	7850133
27	2094020	27516233	74	1930112	5920021
28	1963149	25553084	75	1530270	4389751
29	1840039	23713045	76	1193842	31959092
30	1724219	21988826	77	9150744	22808348
31	1615249	20373577	78	6879662	15928686
32	1512713	18860864	79	5063867	10864819
33	1416227	17444637	80	3641917	7222902
34	1325412	16119225	81	2553608	4669294
35	1239938	14879287	82	1741479	2927815
36	1159479	13719808	83	1151992	1775823
37	1083737	12636077	84	737092.0	1038731.4
38	1012410	116236666	85	454728.9	584002.5
39	9452491	106784175	86	269549.6	314452.9
40	8819946	97964229	87	152944.6	161508.26
41	8224265	89739964	88	82725.04	78783.22
42	7663091	82076873	89	42459.45	36323.77
43	7134426	74942447	90	20577.18	15746.586
44	6636348	68306099	91	9364.968	6381.618
45	6167055	62139044	92	3978.743	2402.875
46	5724858	56414186	93	1567.719	835.1560
47	5308185	51106001	94	568.8138	266.3422
48	4915575	46190426	95	188.5611	77.78111
49	4545654	41644772	96	57.94189	19.83922
50	4197156	37447616	97	15.25867	4.58055
51	3868899	33578717	98	3.652267	.92828
52	3559786	30018931	99	.7642677	.16401
53	3268808	26750123	100	.1393856	.02462
54	2995043	23755080	101	.0217582	.00286
55	2737612	21017468	102	.0028645	.00000
56	2495731	18521737	103		

TABLE NO. XXIV.

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cent Nxxx	x Age	Three Lives Equal Ages Five Per Cent Dxxx	Three Lives Equal Ages Five Per Cen Nxxx
10	6144631	80501337	57	1372630	7948018
11	5731620	74769717	58	1213262	6734756
12	5345974	69423743	59	1066851	56679048
13	4985884	64437859	60	9328081	47350967
14	4649638	59788221	61	8105533	39245434
15	4335651	55452570	62	6995479	32249955
16	4042443	51410127	63	5992660	26257295
17	3768630	47641497	64	5091914	21165381
18	3512920	44128577	65	4288110	16877271
19	3274109	40854468	66	3576073	13301198
20	3051072	37803396	62	2950527	10350671
21	2842758	34960638	68	2406050	7944621
22	2648188	32312450	69	1937040	6007581
23	2466446	29846004	70	1537703	4469878
24	2296683	27549321	71	1202064	32678137
25	2138088	25411233	72	9239972	23458165
26	1989930	23421303	73	6972801	16465364
27	1851513	21569790	74	5156770	11308594
28	1722186	19847604	75	3730357	7578237
29	1601346	18246258	76	2633990	4944247
30	1488429	16757829	77	1811222	3133025
31	1382908	15374921	78	1209855	19231699
32	1284292	14090629	79	7828902	11402797
33	1192121	12898508	80	4892907	6509890
34	1105972	11792536	81	2943730	3566160
35	1025443	107670928	82	1698733	18674272
36	9501707	98169221	83	9365491	9308781
37	8798000	89371221	84	4911718	4397063
38	8140124	81231097	85	2438738	1958325
39	7525074	73706023	86	1140507	8178184
40	6950060	66755963	87	4995003	3183181
41	6412484	60343479	88	2036032	11471490
42	5909916	54433563	89	7671589	3799901
43	5440121	48993442	90	2652145	11477561
44	5001017	43992425	91	8343963	3133598
45	4590669	39401756	92	2367699	765899.5
46	4206790	35194966	93	6000742	165825.3
47	3849175	31345791	94	1343855	31439.79
48	3514832	27830959	95	262827.0	5157.086
49	3202819	24628140	96	44317.50	725.3360
50	2911824	21716316	97	6352.662	90.06981
51	2640634	19075682	98	762.2886	13.84095
52	2388131	16687551	99	74.77195	6.36375
53	2153288	14534263	100	5.96749	.39626
54	1935154	12599109	101	.37713	.01913
55	1732861	10866248	102	.01846	.00067
56	1545600	9320648	103	.00067	.00000

TABLE NO. XXV.

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Six Per Cent Nxx	x Age	Two Lives Equal Ages Six Per Cent Dxx	Two Lives Equal Ages Six Per Cent Nxx
10	5587153	71739261	57	1321685	8910336
11	5198467	66540794	58	1186358	7723978
12	4836449	61704345	69	1061216	66627624
13	4499447	57204898	60	9456896	57170728
14	4185661	53019237	61	8392567	48778161
15	3893510	49125727	62	7414336	41363825
16	3623164	45502563	63	6517696	34846129
17	3368227	42134336	64	5698431	29147693
18	3132404	39001932	65	4952590	24195108
19	2912832	36089100	66	4276413	19918695
20	2708372	33380728	67	3666308	16252387
21	2517988	30862740	68	3118787	13133600
22	2340704	28522036	69	2630442	10503158
23	2175615	26346421	70	2197885	8305273
24	2021877	24324544	71	1817728	6487545
25	1878706	22445838	72	1486545	5001000
26	1745369	20700469	73	1200851	38001491
27	1621188	19079281	74	9570988	28430503
28	1505530	17573751	75	7516667	20913836
29	1397805	16175946	76	5808827	15105009
30	1297464	14878482	77	4410418	10694591
31	1203998	13674484	78	3284542	7410049
32	1116931	12557553	79	2394823	5015226
33	1035821	115217320	80	1706099	3309127
34	9602584	105614736	81	1184981	2124145.7
35	8898571	96716165	82	800480.4	1323665.3
36	8242643	88473522	83	524535.4	799129.9
37	7631474	80842048	84	340196.8	458933.1
38	7061983	73780065	85	203162.9	255770.2
39	6531310	67248755	86	119292.8	136477.35
40	6036786	61211969	87	67049.08	69428.27
41	5575937	55636032	88	35923.56	33504.71
42	5146456	50489576	89	18264.19	15240.522
43	4746209	45743367	90	8767.882	6472.640
44	4373210	41370157	91	3952.747	2519.893
45	4025611	37344546	92	1663.495	856.3983
46	3697453	33647093	93	515.7365	340.6618
47	3399911	30247182	94	233.3530	107.30881
48	3118739	27128443	95	76.62632	30.68249
49	2856832	24271611	96	22.79303	7.88946
50	2612934	21658677	97	6.08428	1.80518
51	2385847	19272830	98	1.44258	.36261
52	2174518	17098312	99	.29902	.06358
53	1977937	15120375	100	.05402	.00956
54	1795141	13325234	101	.00835	.00121
55	1625402	11699832	102	.00109	.00012
66	1467811	10232021	103	.00012	.00000

TABLE NO. XXVI.

Actuaries Table, Makeham's Formula

Annuity Commutation Columns.

x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx	x Age	Three Lives Equal Ages Six Per Cent Dxxx	Three Lives Equal Ages Six Per Cent Nxxx
10	5588949	65164416	56	9090115	51971763
11	5164107	60000309	57	7996670	43975093
12	4771209	55229100	58	7001543	36973550
13	4407851	50821249	59	6098572	30874978
14	4071872	46749377	60	5282004	25592974
15	3761022	42988355	61	4546436	21046538
16	3473593	39514762	62	3886783	17159755
17	3207760	36307002	63	3298192	13861563
18	2961898	33345104	64	2776008	11085555
19	2734503	30610601	65	2315737	8769818
20	2524184	28086417	66	1912989	6856829
21	2329657	25756760	67	1563471	5293358
22	2149732	23607028	68	1262926	4030432
23	1983310	21623718	69	1007155	30232765
24	1829374	19794344	70	7919778	22312987
25	1686986	18107358	71	6132699	16180288
26	1555274	16552084	72	4669583	11510705
27	1433440	15118644	73	3490582	8020123
28	1320736	13797908	74	2557128	5462995
29	1216480	12581428	75	1832348	3630647
30	1120034	11461394	76	1281609	23490377
31	1030812	104305822	77	8729656	14760721
32	9482728	94823094	78	5776185	8984536
33	8719149	86103945	79	3702474	5282062
34	8012730	78091215	80	2292141	2989921
35	7359242	70731973	81	1366014	16239069
36	6754688	63977285	82	7808482	8430587
37	6195424	57781861	83	4264377	4166210
38	5678080	52103781	84	2215347	1950863
39	5199539	46904242	85	1089597	8612655
40	4756922	42147320	86	5047467	3565188
41	4347575	37799745	87	2189749	13754388
42	3965040	33834705	88	8841522	4912866
43	3619064	30215641	89	3299981	1612885
44	3295562	26920079	90	1130072	4828132
45	2996610	23923469	91	3521804	1306328.4
46	2717308	21206161	92	989924.5	316403.9
47	2465410	18740751	93	248521.8	67882.11
48	2230023	16510728	94	55130.96	12751.15
49	2012893	14497835	95	10680.60	2070.550
50	1812475	12685090	96	1783.961	286.5886
51	1628408	11056682	97	253.3077	33.28092
52	1458803	9597879	98	30.10894	3.17198
53	1302939	8294940	99	2.92549	.24649
54	1159901	7135039	100	.23128	.01521
55	1028851	6106187S	101	.01448	.00074
			102	.00074	.00000

TABLE NO. XXVII.

Showing the Force of Mortality, μ, at each year of age according to the Makehamized, Carlisle, Actuaries and American Experience Tables of Mortality.

x Age	American	Carlisle	Actuaries	x Age	American	Carlisle	Actuaries
10	.00768	.00495	.00693	59	.02373	.02632	.02671
11	.00769	.00507	.00695	60	.02553	.02804	.02869
12	.00770	.00519	.00698	61	.02752	.02993	.03086
13	.00772	.00533	.00700	62	.02974	.03199	.03323
14	.00773	.00548	.00703	63	.03220	.03425	.03582
15	.00775	.00565	.00707	64	.03494	.03672	.03867
16	.00776	.00583	.00710	65	.03798	.03942	.04179
17	.00778	.00602	.00714	66	.04136	.04238	.04519
18	.00781	.00624	.00719	67	.04512	.04562	.04892
19	.00783	.00648	.00724	68	.04929	.04917	.05301
20	.00786	.00674	.00729	69	.05393	.05305	.05748
21	.00788	.00702	.00735	70	.05908	.05730	.06239
22	.00792	.00733	.00741	71	.06481	.06195	.06775
23	.00793	.00767	.00748	72	.07117	.06703	.07362
24	.00799	.00804	.00755	73	.07824	.07260	.08005
25	.00804	.00894	.00764	74	.08610	.07870	.08708
26	.00809	.00902	.00773	75	.09483	.08537	.09479
27	.00814	.00910	.00783	67	.10453	.09267	.10322
28	.00821	.00920	.00793	77	.11531	.10066	.11046
29	.00827	.00930	.00805	78	.12729	.10941	.12057
30	.00835	.00942	.00818	79	.14060	.11898	.13165
31	.00843	.00955	.00833	80	.15540	.12945	.14374
32	.00853	.00968	.00848	81	.17183	.14092	.15702
33	.00863	.00983	.00865	82	.19010	.15347	.16155
34	.00875	.01000	.00884	83	.21040	.16720	.17746
35	.00888	.01018	.00905	84	.23295	.18223	.19488
36	.00902	.01038	.00927	85	.25801	.19869	.21396
37	.00918	.01060	.00952	86	.28586	.21669	.23485
38	.00935	.01083	.00979	87	.31681	.23640	.25771
39	.00955	.01109	.01008	88	.35120	.25797	.28275
40	.00977	.01137	.01030	89	.38941	.28158	.31016
41	.01001	.01168	.01069	90	.43187	.30742	.34018
42	.01028	.01202	.01108	91	.47905	.33569	.37304
43	.01058	.01239	.01154	92	.53149	.36665	.40902
44	.01091	.01280	.01204	93	.58975	.40053	.44832
45	.01128	.01324	.01260	94	.65449	.43760	.49146
46	.01169	.01373	.01321	95	.72643	.47818	.53868
47	.01215	.01426	.01382	96	.80637	.52258	.59029
48	.01265	.01484	.01449	97	.89521	.57120	.64701
49	.01321	.01548	.01521	98	.99392	.62440	.70900
50	.01384	.01618	.01609	99		.68262	.77687
51	.01458	.01690	.01697	100		.74635	.78018
52	.01531	.01778	.01792	101		.81611	.86154
53	.01617	.01870	.01897	102		.89243	.94062
54	.01712	.01970	.02012	103		.97599	
55	.01818	.02079	.02137	104		1.06742	
56	.01936	.02190	.02275	105		1.16751	
57	.02066	.02331	.02426				
58	.02212	.02475	.02591				

TABLE NO. XXVIII.

Annuities on Two and Three Lives, Equal Ages, American Experience Table—Makehamized

x Age	Equal Ages 3½ % a_{xx} 2 Lives	Equal Ages 3½ % a_{xxx} 3 Lives	Equal Ages 5 % a_{xx} 2 Lives	Equal Ages 5 % a_{xx} 3 Lives	Equal Ages 6 % a_{xx} 2 Lives	Equal Ages 6 % a_{xxx} 3 Lives
10	18.03023	15.74490	14.45673	19.90393	12.71569	11.48156
11	17.95018	15.67560	14.41467	12.86509	12.68735	11.45624
12	17.86687	15.60359	14.37032	12.82409	12.65736	11.42527
13	17.77960	15.52764	14.32333	12.80909	12.62531	11.39410
14	17.68582	15.44761	14.27348	12.73382	12.59104	11.36087
15	17.59277	15.36397	14.22090	12.68452	12.55379	11.32534
16	17.53350	15.27584	14.16537	12.63228	12.51597	11.28774
17	17.38916	15.18358	14.10654	12.57680	12.47185	11.24716
18	17.28050	15.08675	14.04486	12.51811	12.43105	11.20389
19	17.16730	14.98554	13.97948	12.45610	12.38467	11.15811
20	17.04899	14.87918	13.91044	12.39014	12.33523	11.10910
21	16.72565	14.76780	13.83767	12.32054	12.28278	11.05697
22	16.79700	14.65129	13.76100	12.24678	12.22707	11.00144
23	16.66317	14.53996	13.68026	12.16909	12.16828	10.94266
24	16.52351	14.40209	13.59509	12.08721	12.10568	10.87992
25	16.37807	14.26878	13.50542	11.99908	12.03911	10.81326
26	16.22758	14.12986	13.41129	11.90740	11.96869	10.74278
27	16.06985	13.98500	13.31226	11.81213	11.89484	10.66812
28	15.90695	13.83382	13.20857	11.71065	11.81633	10.58895
29	15.73641	13.67593	13.09901	11.60356	11.76026	10.50487
30	15.56053	13.51244	12.98479	11.49146	11.64593	10.41666
31	15.37792	13.34189	12.86478	11.37399	11.55628	10.32315
32	15.18800	13.16442	12.73872	11.25999	11.45618	10.22430
33	14.99176	12.98058	12.60746	11.12073	11.35381	10.12045
34	14.78823	12.78940	12.46969	10.98504	11.24582	10.01075
25	14.57817	12.59190	12.32615	10.84362	11.13269	9.89816
36	14.36078	12.38714	12.17611	10.69545	11.01370	9.77498
37	14.13611	12.17510	12.01947	10.54064	10.88869	9.64791
38	13.90438	11.95622	11.85336	10.37945	10.75280	9.51490
39	13.69744	11.73196	11.68700	10.21219	10.62108	9.37617
40	13.42000	11.49895	11.51059	10.03790	10.47782	9.23077
41	13.16720	11.25983	11.32757	9.85722	10.32838	9.07928
42	12.90632	11.01364	11.13744	9.66952	10.17222	8.92111
43	12.64050	10.76150	10.94092	9.47584	10.00993	8.75707
44	12.35815	10.50264	10.73719	9.27517	9.84071	8.58618
45	12.08683	10.23897	10.52690	9.06848	9.66511	8.40927
46	11.80040	9.96784	10.31013	8.85565	9.50498	8.22635
67	11.50784	9.69201	10.08662	8.63670	9.29442	8.03674
48	11.20943	9.41109	9.85666	8.41155	9.09927	7.84199
49	10.90596	9.12608	9.62086	8.18219	8.89833	7.64159
50	10.59691	8.83627	9.37864	7.94572	8.69039	7.43518
51	10.28323	8.54292	9.13080	7.70662	8.47677	7.22374
52	9.96531	8.24651	8.87762	7.46224	8.25512	7.00753
53	9.64402	7.94782	8.61960	7.21419	8.03024	6.78712
54	9.31918	7.66449	8.35671	6.96245	7.79999	6.56248

TABLE NO. XXVIII.

Annuities on Two and Three Lives, American Experience Table, Makehamized.

x Age	3½ % Equal Ages a_{xx} 2 Lives	3½ % Equal Ages a_{xxx} 3 Lives	5 % Equal Ages a_{xx} 2 Lives	5 % Equal Ages a_{xxx} 3 Lives	6 % Equal Ages a_{xx} 2 Lives	6 % Equal Ages a_{xxx} 3 Lives
55	8.99175	7.34469	8.08967	6.70794	7.56497	6.33430
56	8.66213	7.04164	7.81877	6.43609	7.32530	6.10289
57	8.33113	6.73869	7.54470	6.19084	7.08166	5.86902
58	7.99938	6.43502	7.26706	5.93252	6.82546	5.63321
59	7.66750	6.13569	6.98915	5.67242	6.58288	5.39599
60	7.33641	5.83724	6.70901	5.41245	6.34463	5.15821
61	7.00651	5.54149	6.42791	5.15335	6.07508	4.92016
62	6.67908	5.24983	0.14705	4.90759	5.81908	4.68313
63	6.35417	4.96212	5.86646	4.64122	5.56201	4.44698
64	6.03297	4.67958	5.58726	4.38928	5.30408	4.21291
65	5.71605	4.40277	5.31002	4.14099	5.04817	3.98141
66	5.40445	4.13249	5.03575	3.89735	4.79276	3.75347
67	5.09842	3.86902	4.76487	3.65852	4.53873	3.52927
68	4.79878	3.61297	4.49778	3.42546	4.28699	3.30954
69	4.50639	3.36514	4.23581	3.19091	4.03796	3.09512
70	4.22161	3.12569	3.97922	2.97793	3.79185	2.88629
71	3.94526	2.89530	3.72888	2.76558	3.54911	2.68385
72	3.67771	2.67413	3.48522	2.55961	3.30916	2.48811
73	3.41919	2.46233	3.24857	2.36201	3.07300	2.29923
74	3.17392	2.26015	3.01960	2.17265	2.92562	2.11774
75	2.93153	2.06808	2.79895	1.99204	2.71578	.1.94419
76	2.70297	1.88589	2.58675	1.82009	2.51331	1.77851
77	2.48464	7.71345	2.38312	1.65671	2.31841	1.62081
78	2.27719	1.55129	2.18888	1.50256	2.13177	1.47165
79	2.08007	1.39930	2.00353	1.35697	1.95534	1.33049
80	1.89376	1.25009	1.82765	1.22035	1.78592	1.19772
81	1.71804	1.12260	1.66117	1.09247	1.62514	1.07323
82	1.55285	.99867	1.50411	.97300	1.47319	.95693
83	1.39813	.88305	1.35652	.86254	1.33105	.84881
84	1.25364	.77816	1.21827	.76023	1.19574	.74861
85	1.11851	.68026	1.08855	.66532	1.06945	.65572
86	.99354	.59104	.96827	.57865	.95214	.57068
87	.87794	.51086	.85682	.49948	.84275	.49278
88	.77145	.43591	.75373	.42695	.74235	.42214
89	.67411	.36973	.65936	.36292	.64987	.35851
90	.58219	.31014	.57216	.30533	.56349	.30107
91	.50496	.25826	.49488	.25383	.48838	.25096
92	.43165	.21186	.42340	.20834	.41807	.20606
93	.36456	.17050	.35785	.16771	.35355	.16592
94	.30659	.13715	.30121	.13472	.29772	.13332
95	.24750	.10394	.24337	.10255	.24068	.10156
96	.18210	.06883	.17917	.06740	.17727	.06711
97	.13547	.05359	.13343	.04658	.13210	.05203

TABLE NO. XXIX.

Annuities on Two and Three Lives, Actuaries Table Makehamized

x Age	Equal Ages 4 % axx 2 Lives	Equal Ages 4 % $axxx$ 3 Lives	Equal Ages 5 % axx 2 Lives	Equal Ages 5 % $axxx$ 3 Lives	Equal Ages 6 % axx 2 Lives	Equal Ages 6 % $axxx$ 3 Lives
10	16.84028	14.89848	14.59928	13.10109	12.84003	11.65951
11	16.73230	14.81995	14.53255	13.04513	12.80008	11.61894
12	16.64482	14.73767	14.47189	12.98616	12.75813	11.57594
13	16.55425	14.65156	14.41021	12.92406	12.71376	11.52945
14	16.45951	14.56153	14.34437	12.85868	12.66704	11.48124
15	16.36074	14.46731	14.27555	12.78988	12.61837	11.42996
16	16.28946	14.36888	14.20266	12.71772	12.58774	11.37550
17	16.15229	14.26608	14.12654	12.64160	12.50935	11.32391
18	16.04055	14.15876	14.04674	12.56179	12.47982	11.28387
19	15.92431	14.04648	13.96310	12.47804	12.38970	11.19421
20	15.80331	13.93006	13.87552	12.39020	12.32502	11.12693
21	15.67741	13.80848	13.78384	12.29807	12.25691	11.05603
22	15.54671	13.68182	13.68797	12.20172	12.18532	10.98138
23	15.41078	13.51891	13.58770	12.10081	12.10971	10.90284
24	15.26970	13.41310	13.48293	11.99529	12.03067	10.82088
25	15.12330	13.27078	13.37355	11.88503	11.94750	10.73356
26	14.97129	13.12304	13.25941	11.76991	11.86022	10.64254
27	14.81418	12.96976	13.14041	11.64442	11.76868	10.54711
28	14.65125	12.81084	13.01638	11.52466	11.67280	10.44711
29	14.48231	12.64646	12.88726	11.39433	11.57239	10.34252
30	14.30828	12.47628	12.75292	11.25873	11.46735	10.23309
31	14.12810	12.30038	12.61328	11.11782	11.35756	10.11908
32	13.94207	12.11875	12.46823	10.97152	11.24291	9.99956
33	13.75014	11.93137	12.31772	10.81979	11.12288	9.87529
34	13.55230	11.73818	12.16167	10.66259	10.99832	9.74587
35	13.34855	11.53945	12.00002	10.50020	10.86873	9.61131
36	13.13882	11.33508	11.83274	10.33175	10.73364	9.47337
37	12.92336	11.12512	11.65979	10.15812	10.59324	9.32654
38	12.70203	10.90972	11.48119	9.95613	10.44750	9.17631
39	12.47494	10.68902	11.29691	9.79472	10.29640	9.02083
40	12.24221	10.46312	11.10706	9.60508	10.13981	8.86021
41	12.00425	10.23231	10.91165	9.41031	9.97788	8.69444
42	11.76034	9.99671	10.71061	9.21055	9.81281	8.52466
43	11.51150	9.75409	10.50434	9.00598	9.63788	8.34902
44	11.25759	9.51189	10.29272	8.79669	9.45991	8.16858
45	10.99889	9.26346	10.07597	8.58301	9.27673	7.98351
46	10.73562	9.01124	9.85425	8.38548	9.10007	7.80411
47	10.46805	8.75588	9.62777	8.14351	8.89646	7.60148
48	10.19647	8.49745	9.39675	7.91815	8.69851	7.40385
49	9.92126	8.23645	9.16145	7.68952	8.49599	7.20249
50	9.64271	7.97328	8.92214	7.45797	8.28906	6.99763
51	9.36121	7.70839	8.67921	7.22390	8.07799	6.78987
52	9.07719	7.44225	8.43278	6.98770	8.86304	6.57929
53	8.79105	7.17531	8.18343	6.74980	7.64452	6.36638
54	8.50325	6.90808	7.93147	6.51065	7.42277	6.15142
55	8.21425	6.64106	7.67729	6.27070	7.19812	5.93495

TABLE NO. XXIX.

Annuities on Two and Three Lives, Actuaries Table—Makehamized.

x Age	4 % Equal Ages a_{xx} 2 Lives	4 % Equal Ages a_{xxx} 3 Lives	5 % Equal Ages a_{xx} 2 Lives	5 % Equal Ages a_{xxx} 3 Lives	6 % Equal Ages a_{xx} 2 Lives	6 % Equal Ages a_{xxx} 3 Lives
56	7.92454	6.37476	7.42137	6.03044	6.97106	5.71739
57	7.63462	6.10970	7.16412	5.79036	6.74165	5.49919
58	7.34503	5.84641	6.90603	5.55095	6.51068	5.28077
59	7.05026	5.58542	6.64759	5.31272	6.27843	5.06266
60	6.76885	5.32720	6.38928	5.07616	6.04540	4.85648
61	6.48334	5.07231	6.13164	4.84962	5.81207	4.62924
62	6.20027	4.82122	5.87512	4.61011	5.57889	4.41490
63	5.92016	4.57441	5.62033	4.38158	5.34627	4.20278
64	5.64357	4.33234	5.36772	4.15665	5.11504	3.99334
65	5.36610	4.09544	5.11782	3.93583	4.88537	3.78705
66	5.10280	3.86411	4.87112	3.71950	4.65780	3.58435
67	4.83966	3.63875	4.62823	3.50807	4.42780	3.38564
68	4.58193	3.41969	4.39096	3.30194	4.21112	3.19870
69	4.33008	3.20723	4.15739	3.10142	3.99293	3.00181
70	4.08447	3.00166	3.92865	2.90686	3.77873	2.87138
71	3.84552	2.80324	3.70548	2.71840	3.56904	2.63836
72	3.61353	2.61211	3.48825	2.53661	3.36418	2.47129
73	3.38883	2.42846	3.27743	2.36131	3.16454	2.29765
74	3.17166	2.25240	3.07330	2.19296	2.97049	2.13638
75	2.96228	2.08401	2.87632	2.03151	2.78233	1.98059
76	2.76089	1.92335	2.68996	1.87709	2.60035	1.83288
77	2.56767	1.77042	2.50538	1.72978	2.42484	1.69087
78	2.38332	1.62518	2.33245	1.59325	2.25602	1.55544
79	2.20128	1.48759	2.16882	1.45650	2.09414	1.42663
80	2.03863	1.35754	1.98327	1.33047	1.93955	1.30442
81	1.87975	1.23494	1.82851	1.21144	1.79255	1.18879
82	1.73015	1.11965	1.68126	1.09931	1.65359	1.07969
83	1.59046	1.01150	1.54508	.99394	1.52350	.97698
84	1.40599	.91032	1.40955	.89522	1.34902	.88061
85	1.30991	.81593	1.28459	.80299	1.25896	.79044
86	1.11877	.72812	1.16659	.71706	1.14405	.70617
87	1.07514	.64667	1.05842	.63727	1.03558	.62813
88	.96882	.57246	.95235	.56342	.93267	.55566
89	.86961	.50206	.85549	.49532	.83445	.48876
90	.77727	.43843	.76525	.43277	.73822	.42724
91	.69160	.38029	.68144	.37555	.63751	.37093
92	.64120	.32668	.60393	.32348	.55164	.31962
93	.53927	.27959	.53266	.27635	.48629	.27314
94	.47215	.23659	.46824	.23395	.42817	.23129
95	.41072	.19188	.41155	.19622	.38328	.19386
96	.35472	.16414	.34240	.16368	.34613	.16065
97	.30310	.13418	.30019	.14100	.29670	.13445
98	.25714	.10755	.25416	.11456	.25136	.10535
99	.21710	.08598	.20989	.08511	.21214	.08622
100	.16710	.06708	.17663	.03252	.18117	.05224

TABLE NO. XXX.

Annuities on Two and Three Lives, Carlisle Table, Makehamized

x Age	Two Lives Equal Ages a_{xx} 5 %	Two Lives Equal Ages a_{xx} 6 %	Three Lives Equal Ages a_{xxx} 5 %	Three Lives Equal Ages a_{xxx} 6 %	x Age	Two Lives Equal Ages a_{xx} 5 %	Two Lives Equal Ages a_{xx} 6 %	Three Lives Equal Ages a_{xxx} 5 %	Three Lives Equal Ages a_{xxx} 6 %
10	14.77563	13.01341	13.36781	11.90719	57	7.40845	6.95684	6.00285	5.69040
11	14.67123	12.98360	13.24954	11.81489	58	7.16179	6.73709	5.77408	5.48261
12	14.56368	12.85107	13.12781	11.71807	59	6.91318	6.51495	5.54453	5.27335
13	14.45337	12.76599	13.00316	11.61843	60	6.66396	6.29131	5.31594	5.06437
14	14.34104	12.67907	12.87658	11.51692	61	6.41426	6.066.33	5.08818	4.85525
15	14.22609	12.58976	12.74729	11.41280	62	6.16490	5.84078	4.86224	4.64714
16	14.10981	12.49917	12.61705	11.30762	63	5.91605	5.61484	4.63820	4.44009
17	13.98808	12.40681	12.48511	11.20070	64	5.66798	5.38876	4.41625	4.23428
18	13.87241	12.31343	12.35265	11.09302	65	5.42153	5.16333	4.20693	4.03057
19	13.75303	12.21481	12.22086	10.98584	66	5.17732	4.93915	3.96352	3.82954
20	13.63256	12.12509	12.08850	10.87783	67	4.95379	4.71664	3.77032	3.63149
21	13.51140	12.02970	11.95613	10.76961	68	4.69721	4.49614	3.56291	3.43671
22	13.38856	11.93566	11.82696	10.66401	69	4.46209	4.27806	3.36002	3.24579
23	13.27303	11.84234	11.69990	10.56023	70	4.23123	4.06325	3.16241	3.05924
24	13.15727	11.75155	11.57769	10.46062	71	4.00454	3.85166	2.96985	2.87692
25	13.06260	11.67994	11.48487	10.38773	72	3.78319	3.64436	2.78341	2.69992
26	12.96475	11.60510	11.38643	10.31167	73	3.56696	3.44127	2.60276	2.52798
27	12.86170	11.52638	11.28738	10.23159	74	3.35599	3.24028	2.42785	2.36107
28	12.75050	11.44247	11.18075	10.14654	75	3.15123	3.04901	2.25958	2.20011
29	12.64165	11.35640	11.07064	10.05842	76	2.95285	2.86112	2.09797	2.04517
30	12.52427	11.26498	10.95488	9.96527	77	2.76083	2.67871	1.94285	1.89611
31	12.40254	11.16979	10.83489	9.86833	78	2.57595	2.50261	1.79495	1.75368
32	12.27517	11.06955	10.70912	9.76613	79	2.39720	2.33197	1.65299	1.61668
33	12.14272	10.96478	10.57835	9.65938	80	2.22620	2.16830	1.48405	1.48672
34	12.00488	10.85538	10.44245	9.54765	81	2.06210	2.10183	1.39068	1.36280
35	11.86025	10.74110	10.30137	9.43168	82	1.90512	1.85988	1.26942	1.24511
36	11.71384	10.62193	10.15474	9.37049	83	1.76809	1.71610	1.15540	1.13426
37	11.55994	10.49759	10.00313	9.18419	84	1.61360	1.57866	1.04757	1.02925
38	11.40053	10.36821	9.84599	9.05285	85	1.47871	1.44813	.94641	.93057
39	11.23523	10.23328	9.68301	8.91603	86	1.35128	1.32458	.85189	.83825
40	11.06532	10.09330	9.51502	8.77435	87	1.23089	1.20763	.76355	.75184
41	10.88858	9.94828	9.34212	8.62788	88	1.11722	1.09700	.68099	.67098
42	10.70755	9.79823	9.16442	8.47673	89	1.01060	.99301	.60444	.59571
43	10.52007	9.64237	8.98086	8.31983	90	.91094	.89556	.53378	.52654
44	10.32757	9.48132	8.79266	8.15828	91	.81739	.80358	.46778	.46165
45	10.12991	9.31522	8.59991	7.99214	92	.73249	.71972	.40984	.40379
46	9.92673	9.14367	8.40211	7.82083	93	.65343	.64009	.35344	.34915
47	9.71888	8.96737	8.20044	7.64549	94	.59467	.56873	.30470	.30111
48	9.50552	8.78547	7.99392	7.46504	95	.50508	.50186	.25802	.25507
49	9.28794	8.59835	7.78404	7.28101	96	.44326	.45142	.22097	.21857
50	9.06582	8.40816	7.57048	7.09297	97	.38763	.42071	.18739	.18494
51	9.83956	8.21268	7.35375	6.90137	98	.35331	.38531	.15199	.15390
52	8.60876	8.01234	7.13319	6.70576	99	.29004	.36201	.11081	.10967
53	8.37472	7.80834	6.91089	6.50746	100	.23723	.34122	.09475	.09374
54	8.13755	7.60073	6.68650	6.30673					
55	7.89680	7.38912	6.45970	6.10304					
56	7.65400	7.17466	6.23213	5.89789					

TABLE NO. XXXI

Expectation of Life By Tables Named

Age x	Northampton	Carlisle	Actuaries	American Experience	x Age	Northampton	Carlisle	Actuaries	American Experience
0	25.18	38.72			53	16.54	18.97	18.16	18.79
1	32.74	44.68			54	16.60	18.28	17.50	18.09
2	37.79	47.55			55	15.58	17.58	16.86	17.40
3	39.55	49.82			56	15.10	16.89	16.22	16.72
4	40.58	50.76			57	14.63	16.21	15.59	16.05
5	40.84	51.25			58	14.15	15.55	14.97	15.39
6	41.07	51.17			59	13.68	14.92	14.37	14.74
7	41.03	50.80			60	13.21	14.34	13.77	14.10
8	40.79	50.24			61	12.75	13.82	13.18	13.47
9	40.36	49.57			62	12.28	13.31	12.61	12.86
10	39.78	48.82	48.36	48.72	63	11.81	12.81	12.05	12.26
11	39.14	48.04	47.68	48.08	64	11.35	12.30	11.51	11.67
12	38.49	47.27	47.01	47.45	65	10.88	11.79	10.97	11.10
13	37.83	46.51	46.33	46.80	66	10.42	11.27	10.46	10.54
14	37.17	45.75	45.64	46.16	67	9.96	10.75	9.96	10.00
15	36.51	45.00	44.96	45.50	68	9.50	10.23	9.47	9.47
16	35.85	44.27	44.27	44.85	69	9.05	9.70	9.00	8.97
17	35.20	43.57	43.58	44.19	70	8.60	9.18	8.54	8.48
18	34.58	42.87	42.88	43.53	71	8.17	8.65	8.10	8.00
19	33.99	42.17	42.19	42.87	72	7.74	8.16	7.67	7.55
20	33.43	41.46	41.49	42.20	73	7.33	7.72	7.26	7.11
21	32.90	40.75	40.79	41.53	74	6.92	7.33	6.86	6.68
22	32.39	40.04	40.09	40.85	75	6.54	7.01	6.48	6.27
23	31.88	39.31	39.39	40.17	76	6.18	6.69	6.11	5.88
24	31.36	38.59	38.68	39.49	77	5.83	6.40	5.76	5.49
25	30.85	37.86	37.98	38.81	78	5.48	6.12	5.42	5.11
26	30.33	37.14	37.27	38.12	79	5.11	5.80	5.09	4.74
27	29.82	36.41	36.56	37.43	80	4.75	5.51	4.78	4.39
28	29.30	35.69	35.86	36.73	81	4.41	5.21	4.48	4.05
29	28.79	35.00	35.15	36.03	82	4.09	4.93	4.18	3.71
30	28.27	34.34	34.43	35.33	83	3.80	4.65	3.90	3.39
31	27.76	33.68	33.72	34.63	84	3.58	4.39	3.63	3.08
32	27.24	33.03	33.01	33.92	85	3.37	4.12	3.36	2.77
33	26.72	32.36	32.30	33.21	86	3.19	3.90	3.10	2.47
34	26.20	31.68	31.58	32.50	87	3.01	3.71	2.84	2.18
35	25.68	31.00	30.87	31.78	88	2.86	3.59	2.59	1.91
36	25.16	30.32	30.15	31.07	89	2.66	3.47	2.35	1.66
37	24.64	29.64	29.44	30.35	90	2.41	3.28	2.11	1.42
38	24.12	28.96	28.72	29.62	91	2.09	3.26	1.89	1.19
39	23.60	28.28	28.00	28.90	92	1.75	3.37	1.67	.98
40	23.08	27.61	27.28	28.18	93	1.37	3.48	1.47	.80
41	22.56	26.97	26.56	27.45	94	1.05	3.53	1.28	.64
42	22.04	26.34	25.84	26.72	75	.75	3.53	1.12	.50
43	21.54	25.71	25.12	26.00	96	.50	3.46	.99	
44	21.03	25.09	24.40	25.27	97		3.28	.89	
45	20.52	24.46	23.69	24.54	98		3.07	.75	
46	20.22	23.82	22.97	23.81	99		2.77	.50	
47	19.51	23.17	22.27	23.08	100		2.28		
48	19.00	22.50	21.56	22.36	101		1.79		
49	18.49	21.81	20.87	21.63	102		1.30		
50	17.99	21.11	20.18	20.91	103		.83		
51	17.50	20.39	19.50	20.20	104		.50		
52	17.02	19.68	18.82	19.49					

TABLE NO. XXXII.

Actuaries or Combined Experience Commutation Columns at Three Per Cent.

x Age	Dx	Nx	Mx	Rx
10	74409.391	1737895.578	21623.808	743735.36
11	71753.769	1666141.818	21135.452	722111.55
12	69191.125	1596950.693	20662.722	700976.10
13	66718.250	1530232.443	20205.122	680313.37
14	64331.391	1465901.052	19761.512	660108.25
15	62026.971	1403874.081	19330.923	640346.74
16	59802.215	1344071.866	18912.678	621015.92
17	57653.833	1286418.033	18506.107	602103.24
18	55579.278	1230838.755	18110.790	583597.13
19	53575.521	1177263.234	17725.847	565486.34
20	51640.230	1125623.004	17351.009	547760.50
21	49770.613	1075852.391	16985.475	530409.49
22	47964.530	1027887.861	16629.022	513424.01
23	46219.915	981667.946	16281.432	495794.99
24	44534.270	937133.676	15941.998	480513.56
25	42905.695	894227.981	15610.539	464571.56
26	41332.357	852895.624	15286.880	448961.02
27	39812.019	813083.605	14970.398	433674.14
28	38342.995	774740.610	14660.947	418703.74
29	36923.226	737817.384	14357.964	404042.80
30	35551.161	702266.223	14061.333	389684.83
31	34224.900	668041.323	13770.543	375623.50
32	32943.018	635098.305	13485.503	361852.96
33	31703.760	603394.545	13205.750	348367.45
34	30505.816	572888.729	12931.216	335161.70
35	29347.917	543540.812	12661.835	322230.49
36	28228.483	515312.329	12397.196	309568.65
37	27146.347	488165.982	12137.249	297171.46
38	26100.374	462065.608	11881.946	285034.21
39	25089.146	436976.462	11630.922	273152.26
40	24111.615	412864.847	11384.144	261521.34
41	23166.768	389698.079	11141.577	250137.19
42	22253.327	367444.752	10902.897	238995.62
43	21369.797	346074.955	10667.521	228092.72
44	20513.953	325561.002	10434.099	217425.20
45	19683.489	305877.513	10201.128	206991.10
46	18876.809	287000.704	9967.755	196789.97
47	18091.700	268909.004	9732.454	186822.22
48	17327.356	251581.648	9495.054	177089.76
49	16582.791	234998.857	9255.169	167594.71
50	15857.320	219141.537	9012.692	158339.54
51	15150.075	203991.462	8767.311	149326.85
52	14460.256	189531.206	8518.756	140559.54
53	13787|122	175744.084	8266.794	132040.78
54	13129.988	162614.096	8011.227	123773.99

TABLE NO. XXXII.

Actuaries or Combined Experience Commutation Columns at Three Per Cent -Concluded.

x Age	Dx	Nx	Mx	Rx
55	12488.615	150125.481	7752.2815	115762.7605
56	11862.195	138263.286	7489.6069	108010.4790
57	11250.356	127012.930	7223.2693	100520.8721
58	10653.112	116359.818	6953.7048	93297.6028
59	10069.925	106289.893	6680.8029	86343.8980
60	9500.470	96789.423	6404.6472	79663.0951
61	8943.945	87845.478	6124.8348	73258.4479
62	8400.260	79445.218	5841.6530	67133.6131
63	7869.164	71576.054	5555.2249	61291.9601
64	7350.871	64225.283	5266.1305	55736.7352
65	6845.405	57379.778	4974.7681	50470.6047
66	6353.056	51026.723	4681.7995	45495.8366
67	5874.333	45152.389	4388.1174	40814.0371
68	5409.666	39742.723	4094.5478	36425.9197
69	4959.930	34782.794	3802.3741	32331.3719
70	4926.118	30256.675	3513.0269	28528.9978
71	4108.956	26147.719	3227.6930	25015.9709
72	3709.397	22438.322	2947.8127	21788.2779
73	3328.356	19109.966	2674.8128	18840.4652
74	2966.814	16143.151	2410.2132	16165.6524
75	2625.580	13517.572	2155.3903	13755.4392
76	2305.513	11212.058	1911.7973	11600.0489
77	2007.410	9204.648	1680.8446	9688.2516
78	1731.695	7472.954	1463.5977	8007.4070
79	1478.759	5994.195	1261.0997	6543.8093
80	1248.956	4745.240	1074.3672	5282.7096
81	1042.325	3702.915	904.1136	4208.3424
82	858.718	2844.197	750.8660	3304.2288
83	697.651	2146.546	614.8103	2553.3628
84	558.180	1588.365	495.6595	1938.5525
85	439.132	1149.234	392.8685	1442.8930
86	338.901	810.333	305.4280	1050.0245
87	255.827	554.506	232.2254	744.5965
88	188.211	366.295	172.0602	512.3711
89	134.256	232.039	123.5870	340.3109
90	92.2347	139.804	85.4763	216.7239
91	60.5588	79.2455	56.4868	131.2476
92	37.5708	41.6746	35.2626	74.7608
93	21.6939	19.9808	20.4801	39.4982
94	11.4319	8.5489	10.8499	19.0181
95	5.3685	3.1804	5.1195	8.1682
96	2.1668	1.0135	2.0742	3.0487
97	0.7392	0.2744	0.7096	0.9745
98	0.2208	0.0536	0.2128	0.2648
99	0.0536	0.0000	0.0520	0.0520

TABLE NO. XXXIII.

Actuaries or Combined Experience Commutation Columns at Three and One-Half Per Cent.

x Age	Dx	Nx	Mx	Rx
10	70891.881	1506695.178	17543.524	561018.40
11	68031.548	1438663.630	17080.501	543474.87
12	65284.922	1373378.708	16634.459	526394.37
13	62647.540	1310731.168	16204.779	509759.91
14	60114.491	1250616.677	15790.248	493555.13
15	57681.122	1192935.555	15389.734	477764.89
16	55343.582	1137591.973	15002.764	462375.13
17	53097.620	1084494.353	14628.324	447372.39
18	50939.731	1033554.622	14266.007	432744.06
19	48866.026	984688.596	13914.901	418478.06
20	46873.313	937815.283	13574.664	404563.16
21	44958.039	892857.244	13244.476	390988.49
22	43117.289	849739.955	12924.046	377744.02
23	41348.262	808391.693	12613.092	364819.97
24	39647.821	768743.872	12310.902	352206.88
25	38013.408	730730.464	12017.238	339895.98
26	36442.563	694287.901	11731.869	327878.74
27	34932.512	659355.389	11454.176	316146.87
28	33481.028	625874.381	11183.964	304692.69
29	32085.513	593788.868	10920.678	293508.73
30	30743.976	563044.892	10664.158	282588.05
31	29454.070	533590.822	10413.902	271923.89
32	28213.917	505376.905	10169.781	261509.99
33	27021.387	478355.518	9931.345	251340.21
34	25874.763	452480.755	9698.488	241408.86
35	24772.389	427708.366	9471.106	231710.38
36	23712.374	403995.992	9248.804	222239.27
37	22693.202	381302.790	9031.499	212990.47
38	21713.407	359589.383	8819.108	203958.97
39	20771.315	338818.058	8611.285	195139.86
40	19865.580	318952.488	8407.965	186528.57
41	18994.913	299957.575	8209.079	178120.61
42	18157.820	281799.755	8014.325	169911.53
43	17352.658	264447.097	7823.196	161897.20
44	16577.227	247869.870	7634.568	154074.01
45	15829.291	232040.579	7447.216	146439.44
46	15107.229	216933.350	7260.445	138992.22
47	14408.955	202524.395	7073.043	131731.78
48	13733.534	188790.861	6884.881	124658.74
49	13079.903	175710.958	6695.669	117773.86
50	12447.253	163263.705	6505.335	111078.19
51	11834.648	151429.057	6313.653	104572.85
52	11241.219	140187.838	6120.430	98259.20
53	10666.156	129521.682	5925.504	92138.77
54	10108.704	119412.978	5728.745	86213.26

TABLE NO. XXXIII.

Actuaries or Combined Experience Commutation Columns at Three and One-Half Per Cent Concluded.

x Age	Dx	Nx	Mx	Rx
55	9568.466	109844.512	5530.3468	80484.520
56	9044.613	100799.899	5330.0643	74954.173
57	8536.663	92263.236	5127.9699	69624.109
58	8044.429	84218.807	4924.4150	64496.139
59	7567.316	76651.491	4719.3355	59571.724
60	7104.894	69546.597	4512.8134	54852.388
61	6656.385	62890.212	4304.5676	50339.575
62	6221.555	56668.657	4094.8323	46035.007
63	5800.049	50868.608	3883.7174	41940.175
64	5391.860	45476.748	3671.6667	38056.458
65	4996.846	40479.902	3458.9849	34384.791
66	4615.050	35864.852	3246.1637	30925.806
67	4246.677	31618.175	3033.8549	27679.642
68	3891.866	27726.309	2822.6526	24645.788
69	3551.075	24175.234	2613.4700	21823.135
70	3224.832	20950.402	2407.3118	19209.665
71	2913.463	18036.939	2204.9952	16802.353
72	2617.449	15419.490	2007.5041	14579.358
73	2337.229	13082.261	1815.7987	12589.854
74	2073.285	11008.976	1630.8898	10774.055
75	1825.959	9183.017	1453.6734	9143.165
76	1595.621	7587.396	1285.0851	7689.492
77	1382.596	6204.800	1126.0173	6404.407
78	1186.937	5017.863	977.1121	5278.389
79	1008.672	4009.191	838.9866	4301.277
80	847.806	3161.385	712.2303	3462.291
81	704.125	2457.260	597.2182	2750.061
82	577.290	1879.970	494.1945	2152.842
83	466.744	1413.225	403.1702	1658.648
84	371.631	1041.594	323.8407	1255.478
85	290.957	750.637	255.7341	931.6369
86	223.462	527.175	198.0782	675.9028
87	167.871	359.304	150.0435	477.8246
88	122.905	236.399	110.7546	327.7810
89	87.2478	149.1515	79.2537	217.0264
90	59.6504	89.5011	54.6066	137.7727
91	38.9756	50.5255	35.9490	83.1661
92	24.0637	26.4618	22.3551	47.2171
93	13.8276	12.6342	12.9328	24.8620
94	7.2515	5.3827	6.8242	11.9292
95	3.3889	1.9939	3.2069	5.1050
96	1.3612	0.6326	1.2938	1.8982
97	0.4621	0.1706	0.4407	0.6044
98	0.1374	0.0332	0.1316	0.1637
99	0.0332	0.0000	0.0321	0.0321

TABLE NO. XXXIV.

Actuaries or Combined Experience Commutation Columns at Four Per Cent.

x Age	Dx	Nx	Mx	Rx
10	67556.41	1314214.95	14411.365	427355.116
11	64518.98	1249695.97	13972.249	412943.751
12	61616.50	1188079.47	13551.270	398971.502
13	58843.05	1129236.42	13147.684	385420.232
14	56192.37	1073044.05	12760.199	372272.548
15	53658.54	1019385.51	12387.616	359512.349
16	51236.50	968149.01	12029.364	347124.733
17	48920.88	919228.13	11684.377	335095.369
18	46707.09	872521.04	11352.165	323410.992
19	44590.28	827930.76	11031.782	312058.827
20	42566.30	785364.46	10722.808	301027.045
21	40630.73	744733.73	10424.401	290304.237
22	38779.81	705953.92	10136.205	279879.836
23	37009.95	668943.97	9857.877	269743.631
24	35317.31	633626.66	9588.693	259885.754
25	33698.62	599928.04	9328.362	250297.061
26	32150.76	567777.28	9076.601	240968.699
27	30670.38	537106.90	8832.789	231892.098
28	29254.64	507852.26	8596.687	223059.309
29	27900.52	479951.74	8367.742	214462.622
30	26605.43	453346.31	8145.752	206094.880
31	25366.62	427979.69	7930.226	197949.128
32	24181.75	403797.94	7720.993	190018.902
33	23048.30	380749.64	7517.615	182297.909
34	21964.17	358785.47	7319.951	174780.293
35	20927.30	337858.17	7127.862	167460.342
36	19935.51	317922.66	6940.969	160332.480
37	18986.95	298935.71	6759.154	153391.511
38	18079.83	280855.88	6582.305	146632.357
39	17212.24	263643.64	6410.092	140050.052
40	16382.56	247261.08	6242.419	133639.960
41	15589.23	231671.85	6079.193	127397.541
42	14830.58	216841.27	5920.126	121318.438
43	14104.82	202736.45	5764.770	115398.223
44	13409.74	189326.71	5612.184	109633.453
45	12743.15	176583.56	5461.358	104021.269
46	12103.40	164480.16	5311.724	98559.911
47	11488.46	152991.70	5162.305	93248.187
48	10897.30	142094.40	5013.002	88085.882
49	10328.76	131765.64	4863.588	83072.880
50	9781.92	121983.72	4714.010	78209.292
51	9255.78	112727.94	4564.097	73495.280
52	8749.39	103978.54	4413.706	68931.184
53	8261.89	95716.65	4262.718	64517.480
54	7792.45	87924.20	4111.043	60254.760

TABLE NO. XXXIV.

Actuaries or Combined Experience Commutation Columns at Four Per-Cent—Concluded.

x Age	Dx	Nx	Mx	Rx
55	7340.540	80583.661	3958.840	56143.718
56	6905.301	73678.360	3805.930	52184.877
57	6486.161	67192.179	3652.379	48378.947
58	6082.776	61109.403	3498.461	44726.567
59	5694.498	55414.905	3344.137	41228.106
60	5320.816	50094.089	3089.473	37883.970
61	4960.965	45133.124	3034.269	34694.497
62	4614.595	40518.529	2878.706	31660.228
63	4281.277	36237.252	2722.872	28781.522
64	3960.841	32276.411	2567.101	26058.650
65	3653.017	28623.394	2411.617	23491.549
66	3357.679	25265.715	2256.779	21079.932
67	3074.814	22190.901	2103.056	18823.154
68	2804.366	19386.535	1950.870	16720.098
69	2546.500	16840.035	1800.864	14769.228
70	2301.431	14538.604	1653.737	12968.364
71	2069.223	12469.381	1510.046	11314.627
72	1850.048	10619.333	1370.457	9804.581
73	1644.044	8975.289	1235.608	8434.125
74	1451.369	7523.920	1106.166	7198.517
75	1272.086	6251.834	982.7048	6092.3507
76	1106.275	5145.559	865.8194	5109.6461
77	953.971	4191.588	756.0649	4243.8267
78	815.031	3376.557	653.8165	3487.7618
79	689.294	2687.263	559.4261	2833.9453
80	576.578	2110.685	473.2214	2274.5192
81	476.560	1634.125	395.3799	1801.2978
82	388.838	1245.287	325.9874	1405.9179
83	312.868	932.419	264.9721	1079.9304
84	247.914	684.505	212.0516	814.9583
85	193.163	491.342	166.8363	602.9067
86	147.640	343.702	128.7432	436.0704
87	110.379	233.3228	97.1593	307.3272
88	80.4242	152.8986	71.4502	210.1678
89	56.8171	96.0815	50.9363	138.7176
90	38.6584	57.4231	34.9630	87.7813
91	25.1380	32.2851	22.9294	52.8183
92	15.4457	16.8394	14.2040	29.8889
93	8.8328	8.0066	8.1851	15.6849
94	4.6098	3.3968	4.3019	7.4998
95	2.1440	1.2528	2.0133	3.1979
96	.8570	.3958	.8089	1.1846
97	.2895	.1063	.2743	.3757
98	.0857	.0206	.0816	.1014
99	.0206	.0000	.0198	.0198

TABLE NO. XXXV.

American Experience Commutation Columns,
Three Per Cent.

x Age	Dx	Nx	Mx	Rx
10	74409.40	1736936.32	21651.7690	734216.0588
11	71701.05	1665235.27	21110.6754	712564.2898
12	69089.43	1596145.84	20587.4460	691453.6144
13	66571.17	1529574.67	20081.4991	670866.1684
14	64142.87	1465431.80	19592.2719	650784.6693
15	61801.66	1403630.14	19119.2196	631192.3974
16	59543.60	1344086.54	18661.1719	612073.1778
17	57366.45	1286720.09	18218.2999	593412.0059
18	55267.37	1231452.72	17790.0893	575193.7060
19	53243.04	1178209.68	17375.4904	557403.6167
20	51290.86	1126918.82	16974.0755	540028.1263
21	49408.31	1077510.51	16585.4274	523054.0508
22	47592.42	1029918.09	16208.6209	506468.6234
23	45840.91	984077.18	15843.2961	490260.0025
24	44151.55	939925.63	15489.1038	474416.7064
25	42522.27	897403.36	15145.7054	458927.6026
26	40950.73	856452.63	14812.7728	443781.8972
27	39434.76	817017.87	14489.5371	428969.1244
28	37972.35	779045.52	14175.7160	414479.5873
29	36561.69	742483.83	13871.0353	400303.8713
30	35200.56	707283.27	13574.8168	386432.8360
31	33887.31	673395.96	13286.8261	372858.0192
32	32620.32	640775.44	13006.8350	359571.1931
33	31397.61	609377.83	12734.2450	346564.3581
34	30217.37	579160.46	12468.4965	333830.1131
35	29078.18	550082.28	12209.4220	321361.6166
36	27978.68	522103.60	11956.8583	309152.1946
37	26916.89	495186.71	11709.9759	297195.3363
38	25891.58	469295.13	11468.6581	285485.3604
39	24900.82	444394.31	11232.1587	274016.7023
40	23943.93	420450.38	11000.4017	262784.5436
41	23018.85	397431.53	10772.7163	251784.1419
42	22124.74	375306.79	10549.0619	241011.4256
43	21260.11	354046.68	10328.8357	230462.3637
44	20423.80	333622.88	10111.7554	220133.5280
45	19614.20	314008.68	9897.0318	210021.7726
46	18830.34	295178.34	9684.4540	200124.7408
47	18070.51	277107.83	9473.0825	190440.2868
48	17333.65	259774.18	9262.5435	180967.2043
49	16618.26	243155.92	9052.0281	171704.6608
50	15922.79	227233.13	8840.5728	162652.6327
51	15245.97	211987.16	8627.5252	153812.0599
52	14586.69	197400.47	8412.2973	145184.5347

TABLE NO. XXXV.

American Experience Commutation Columns, Three Per Cent—Concluded.

x Age	Dx	Nx	Mx	Rx
53	13943.89	183456.58	8194.3620	136772.2374
54	13316.65	170139.93	7973.2488	128577.8754
55	12703.88	157436.05	7748.3440	120604.6266
56	12104.71	145331.34	7519.2917	112856.2826
57	11518.55	133812.79	7285.5971	105336.9909
58	10944.46	122868.33	1047.0045	98051.3938
59	10382.00	112486.33	6803.2983	91004.3893
60	9830.43	102655.90	6554.1301	84201.0910
61	9289.34	93366.56	6299.3656	77646.9609
62	8758.32	84608.24	6038.9024	71347.5953
63	8237.14	76371.10	5772.8224	65308.6929
64	7727.78	68645.32	5501.3722	59535.8705
65	7224.18	61421.14	5224.7976	54034.4983
66	6732.31	54688.33	4943.3430	48809.7007
67	6250.54	48438.29	4657.6654	43866.3577
68	5779.34	42658.95	4368.5175	39208.6923
69	5319.23	37339.72	4076.7341	34840.1748
70	4871.16	32468.56	3783.5980	30763.4407
71	4436.10	28032.46	3490.4164	26979.8427
72	4015.47	24016.99	3198.9885	23489.4363
73	3611.06	20405.93	2911.5411	20290.4478
74	3224.79	17181.14	2630.4461	17378.9067
75	2858.39	14322.75	2357.9742	14748.4606
76	2513.25	11809.50	2096.0826	12390.4864
77	2190.41	9619.09	1846.4400	10294.4038
78	1890.42	7728.67	1610.2499	8447.9638
79	1613.60	6115.07	1388.4893	6837.7139
80	1360.22	4754.85	1182.1156	5449.2246
81	1129.82	3625.026	991.3330	4267.1090
82	922.940	2702.086	817.3571	3275.7760
83	739.878	1962.208	661.1768	2458.4189
84	580.725	1381.483	523.5731	1797.2421
85	444.644	936.839	404.4069	1273.6690
86	330.007	606.832	302.7209	869.2621
87	235.274	371.558	217.5979	566.5412
88	159.204	212.354	148.3820	348.9433
89	100.980	111.374	94.7949	200.5613
90	59.229	52.145	55.9850	105.7664
91	31.366	20.779	29.8470	49.7814
92	14.237	6.452	13.6322	19.9344
93	5.056	1.486	4.8650	6.3022
94	1.305	.181	1.2615	1.4372
95	.181	.000	0.1757	0.1757

TABLE NO. XXXVI.

American Experience Commutation Columns, at Three and One-Half Per Cent.

x Age	Dx	Nx	Mx	Rx
10	70891.89	1504643.35	17612.9031	552495.7869
11	67981.56	1436661.79	17099.8787	534882.8838
12	65188.97	1371472.82	16606.1884	517783.0051
13	62509.35	1308963.47	16131.1112	501176.8167
14	59938.42	1249025.05	15673.9527	485045.7055
15	57471.61	1191553.44	15234.0442	469371.7528
16	55104.25	1136449.19	14810.1742	454137.7086
17	52832.94	1083616.25	14402.3011	439327.5344
18	50653.87	1032962.38	14009.8359	424925.2333
19	48562.77	984399.61	13631.6827	410915.3974
20	46556.19	937843.42	13267.3224	397283.7147
21	44630.76	893212.66	12916.2547	384016.3923
22	42782.78	850429.88	12577.5278	371100.1376
23	41009.20	809420.68	12250.7089	358522.6098
24	39307.09	770113.59	11935.3798	346271.9009
25	37673.62	732439.97	11631.1371	334336.5211
26	36106.08	696333.89	11337.5917	322705.3840
27	34601.50	661732.39	11053.9729	311367.7923
28	33157.37	628575.02	10779.9451	300313.8194
29	31771.35	596803.67	10515.1840	289533.8743
30	30440.78	566362.89	10259.0198	279018.6903
31	29163.54	537199.35	10011.1740	268759.6715
32	27937.53	509261.82	9771.3768	258748.4975
33	26760.46	482501.36	9539.0460	248977.1207
34	25630.11	456871.25	9313.6404	239438.0747
35	24544.71	432326.54	9094.9573	230124.4343
36	23502.54	408824.00	8882.7998	221029.4770
37	22501.38	386322.62	8676.4165	212146.6772
38	21539.71	364782.91	8475.6595	203470.2607
39	20615.52	344167.39	8279.8615	194994.6012
40	19727.43	324439.96	8088.9171	186714.7397
41	18873.63	305566.33	7902.2332	178625.8226
42	18052.89	287513.44	7719.7402	170723.5894
43	17263.59	270249.85	7540.9125	163003.8492
44	16504.41	253745.44	7365.4910	155462.9367
45	15773.57	237971.87	7192.8117	148097.4457
46	15070.02	222901.85	7022.6843	140904.6340
47	14392.08	208509.77	6854.3396	133881.9497
48	13738.52	194771.25	6687.4680	127027.6101
49	13107.89	181663.36	6521.4211	120340.1421
50	12498.64	169164.72	6355.4387	113818.7210
51	11909.54	157255.18	6189.0142	107463.2823
52	11339.50	145915.68	6021.6988	101274.2681

TABLE NO. XXXVI.

American Experience Commutation Columns at Three and One-Half Per Cent—Concluded.

x Age	Dx	Nx	Mx	Rx
53	10787.47	135128.21	5853.0971	95252.5693
54	10252.41	124875.796	5682.8633	89399.4722
55	9733.398	115142.398	5510.5468	83716.6089
56	9229.602	105912.796	5335.9005	78206.0621
57	8740.164	97172.632	5158.5732	72870.1616
58	8264.438	88908.194	4978.4061	67711.5884
59	7801.823	81106.371	4795.2666	62733.1823
60	7351.656	73754.715	4608.9270	57937.9157
61	6913.441	66841.274	4419.3227	53328.9887
62	6486.746	60354.528	4226.4136	48909.6660
63	6071.271	54283.257	4030.2965	44683.2524
64	5666.851	48616.406	3831.1879	40652.9559
65	5273.332	43343.074	3629.3005	36821.7680
66	4890.551	38452.523	3424.8435	33192.4675
67	4518.648	33933.875	3218.3212	29767.6240
68	4157.822	29776.053	3010.3000	26549.3028
69	3808.316	25967.737	2801.3968	23539.0028
70	3470.674	22497.063	2592.5388	20737.6060
71	3145.429	19351.634	2384.6578	18145.0672
72	2833.421	16518.213	2179.0185	15760.4094
73	2535.754	13982.459	1977.1678	13581.3909
74	2253.567	11728.892	1780.7314	11604.2231
75	1987.870	9741.022	1591.2404	9823.4917
76	1739.394	8001.628	1409.9869	8232.2513
77	1508.634	6492.994	1238.0465	6822.2644
78	1295.725	5197.269	1076.1584	5584.2179
79	1100.647	4096.622	924.8937	4508.0595
80	923.338	3173.285	784.8046	3583.1658
81	763.234	2410.051	655.9245	2798.3612
82	620.465	1789.586	538.9657	2142.4367
83	494.994	1294.592	434.4776	1603.4710
84	386.641	907.951	342.8623	1168.9934
85	294.610	613.341	263.9059	826.1311
86	217.598	395.744	196.8569	562.2252
87	154.383	241.361	141.0003	365.3683
88	103.963	137.398	95.8011	224.3680
89	65.623	71.775	60.9768	128.5669
90	38.305	33.470	35.8775	67.5901
91	20.187	13.283	19.0551	31.7126
92	9.119	4.164	8.6697	12.6575
93	3.222	.942	3.0816	3.9878
94	.828	.114	.7958	.9062
95	.114	.000	.1104	.1104

TABLE NO. XXXVII.

American Experience Commutation Columns at Four Per Cent.

x Age	Dx	Nx	Mx	Rx
10	67556.41	1311526.80	14514.759	420042.335
11	64471.56	1247055.24	14028.223	405527.576
12	61525.93	1185529.31	13562.274	391499.353
13	58713.33	1126815.98	13116.047	377937.079
14	56027.78	1070788.20	12688.716	364821.032
15	53463.64	1017324.56	12279.486	352132.316
16	51014.92	966309.64	11887.063	339852.830
17	48677.02	917632.62	11511.274	327965.767
18	46444.97	871187.65	11151.419	316454.493
19	44313.56	826874.09	10806.354	305303.074
29	42278.32	784595.77	10475.473	294496.720
21	40334.96	744260.81	10158.196	284021.247
22	38478.94	705871.87	9853.544	273863.051
23	36706.47	669075.40	9561.015	264009.507
24	35013.80	634061.60	9280.128	254448.492
25	33397.39	600664.21	9010.419	245168.364
26	31853.91	568810.30	8751.444	236157.945
27	30379.74	538430.56	8502.430	227406.501
28	28971.85	509458.71	8262.993	218904.071
29	27627.33	481831.38	8032.765	210641.078
30	26343.05	455488.33	7811.084	202608.313
31	25116.41	430371.92	7597.633	194797.229
32	23944.87	406427.05	7392.106	187199.596
33	22825.74	383601.31	7193.936	179807.490
34	21756.49	361844.82	7002.597	172613.554
35	20734.96	341109.86	6817.857	165610.957
36	19759.10	321350.76	6639.491	158793.100
37	18826.45	302524.31	6466.814	152153.609
38	17935.20	284589.11	6299.652	145686.795
39	17083.14	267505.97	6137.403	139387.143
40	16268.62	251237.35	5979.936	133249.740
41	15489.70	235747.65	5826.723	127269.804
42	14744.92	221002.73	5677.670	121443.081
43	14032.42	206970.31	5532.313	115765.411
44	13350.81	193619.50	5390.410	110233.098
45	12698.30	180921.20	5251.397	104842.688
46	12073.60	168847.60	5115.097	99591.291
47	11475.01	157372.59	4980.873	94476.194
48	10901.26	146471.33	4848.464	89495.321
49	10350.85	136120.48	4737.342	84646.857
50	9822.30	126298.17	4586.901	79929.515
51	9314.36	116983.81	4456.742	75342.614
52	8825.89	108157.92	4326.515	70885.872

TABLE NO. XXXVII.

American Experience Commutation Columns at Four Per Cent—Concluded.

x Age	Dx	Nx	Mx	Rx
53	8355.838	99802.079	4195.918	66559.357
54	7903.230	91898.849	4064.691	62363.439
55	7467.065	84431.784	3932.497	58298.748
56	7046.535	77385.249	3799.160	64366.251
57	6640.873	70744.466	3664.428	50567.091
58	6249.134	64495.332	3528.195	46902.663
59	5870.969	58624.363	3390.380	43374.468
60	5505.613	53118.750	3250.831	39984.088
61	5152.547	47966.203	3109.520	36733.257
62	4811.291	43154.912	2966.437	33623.737
63	4481.477	38673.435	2821.674	30657.300
64	4162.850	34510.585	2675.410	27835.626
65	3855.146	30655.439	2527.817	25160.216
66	3558.120	27097.319	2379.064	22632.399
67	3271.735	23825.584	2229.531	20253.335
68	2996.005	20829.579	2079.637	18023.804
69	2730.969	18098.610	1929.831	15944.167
70	2476.878	15621.732	1780.778	14014.336
71	2233.970	13387.762	1633.135	12233.558
72	2002.700	11385.062	1487.786	10600.423
73	1783.688	9601.374	1345.801	9112.637
74	1577.573	8023.801	1208.289	7766.836
75	1384.886	6628.915	1076.277	6558.547
76	1205.954	5432.961	950.611	5482.270
77	1040.936	4392.025	831.975	4531.659
78	889.735	3502.290	720.811	3699.684
79	752.145	2750.145	617.442	2978.873
80	627.945	2122.200	522.170	2361.431
81	516.566	1605.634	434.942	1839.261
82	417.919	1187.715	356.164	1404.319
83	331.805	855.910	286.124	1048.155
84	257.927	597.983	225.007	762.031
85	195.588	402.395	172.589	537.024
86	143.767	258.629	128.290	364.435
87	101.510	157.118	91.563	236.145
88	68.029	89.089	61.986	144.582
89	42.735	46.354	39.308	82.596
90	24.825	21.530	23.042	43.288
91	13.020	8.510	12.192	20.246
92	5.853	2.657	5.526	8.055
93	2.058	0.598	1.956	2.529
94	0.526	0.072	0.503	0.573
95	0.072	0.000	0.069	0.069

TABLE NO. XXXVIII.

American Experience Commutation Columns at Four and One-Half Per Cent.

x Age	Dx	Nx	Mx	Rx
10	64392.77	1149751.31	12109.051	322708.032
11	61158.34	1088592.97	11647.519	310598.980
12	58084.84	1030508.13	11207.629	298951.462
13	55164.32	975343.81	10788.375	287743.833
14	52389.25	922954.56	10388.796	276955.457
15	49752.43	873202.13	10007.973	266566.661
16	47246.54	825955.59	9644.538	256558.689
17	44865.64	781089.95	9298.173	246914.152
18	42603.53	738486.42	8968.081	237615.979
19	38411.27	659621.23	8352.455	219994.827
20	40453.92	698032.50	8653.071	228647.898
21	36470.32	623150.91	8065.577	211642.373
22	34625.68	588525.23	7791.433	203576.796
23	32872.65	555652.58	7529.458	195785.363
24	31206.73	524445.85	7279.111	188255.905
25	29623.67	494822.18	7039.878	180976.794
26	28119.40	466702.78	6811.265	173936.916
27	26689.75	440013.03	6592.497	167125.651
28	25331.08	414681.95	6383.149	160533.154
29	24039.93	390642.02	6182.816	154150.006
30	22812.75	367829.27	5990.843	147967.189
31	21646.42	346182.85	5806.881	141976.347
32	20537.99	325644.86	5630.597	136169.465
33	19484.42	306160.44	5461.436	130538.867
34	18482.83	287677.61	5298.887	125077.431
35	17530.73	270146.88	5142.696	119778.544
36	16625.73	253521.15	4992.615	114635.848
37	15765.20	237755.95	4848.016	109643.233
38	14947.00	222808.95	4708.706	104795.216
39	14168.78	208640.17	4574.136	100086.511
40	13428.66	195211.51	4444.158	95512.379
41	12724.53	182486.98	4318.296	91068.216
42	12054.73	170432.25	4196.438	86749.920
43	11417.36	159014.89	4078.169	82553.483
44	10810.79	148294.10	3963.263	78475.314
45	10233.23	137970.87	3851.236	74512.050
46	9683.25	128287.62	3741.921	70660.814
47	9159.13	119128.49	3634.786	66918.893
48	8659.54	110468.95	3529.605	63284.107
49	8182.981	102285.967	3425.946	59754.502
50	7727.976	94557.992	3323.318	56328.556
51	7293.276	87264.716	3221.401	53005.239
52	6877.730	80386.986	3119.920	49783.837

TABLE NO. XXXVIII.

American Experience Commutation Columns at Four and One Half Per Cent—Concluded.

x Age	Dx	Nx	Mx	Rx
53	6480.277	73906.709	3018.637	46663.918
54	6099.937	67806.772	2917.352	43645.281
55	5735.717	62071.055	2815.808	40727.929
56	5386.794	56684.261	2713.877	37912.121
57	5052.322	51631.939	2611.737	35198.243
58	4731.609	46900.330	2508.223	32586.870
59	4424.005	42476.325	2404.374	30078.647
60	4128.846	38347.479	2299.722	27674.274
61	3845.582	34501.897	2194.255	25374.552
62	3573.704	30928.193	2087.977	23180.297
63	3312.801	27615.392	1980.965	21092.320
64	3062.540	24552.852	1873.361	19111.356
65	2822.598	21730.254	1765.299	17237.995
66	2592.661	19137.593	1656.908	15472.697
67	2372.578	16705.015	1548.471	13815.788
68	2162.230	14602.785	1440.292	12267.317
69	1961.522	12641.263	1332.694	10827.025
70	1770.509	10870.754	1226.148	9494.332
71	1589.234	9281.520	1121.116	8268.184
72	1417.893	7863.626	1018.210	7147.068
73	1256.792	6606.834	918.167	6128.858
74	1106.244	5500.590	821.739	5210.691
75	966.479	4534.111	729.611	4388.951
76	837.581	3696.530	642.332	3659.339
77	719.510	2977.021	560.329	3017.008
78	612.055	2364.966	483.858	2456.679
79	514.931	1850.035	413.090	1972.821
80	427.844	1422.191	348.177	1559.731
81	350.273	1071.919	289.030	1211.554
82	282.027	789.892	235.868	922.524
83	222.842	567.049	188.828	686.656
84	172.397	394.653	147.978	497.828
85	130.104	264.548	113.110	349.850
86	95.175	169.373	83.783	236.740
87	66.879	102.494	59.586	152.957
88	44.606	57.887	40.193	93.371
89	27.887	30.001	25.394	53.178
90	16.122	13.879	14.830	27.784
91	8.415	5.464	7.817	12.954
92	3.765	1.699	3.530	5.137
93	1.318	0.381	1.245	1.607
94	0.335	0.046	0.319	0.363
95	0.046	0.000	0.044	0.044

TABLE NO. XXXIX.

National Fraternal Congress Table
Commutation Columns at
Four Per Cent.

x Age	Dx	Nx	Sx	Mx	Rx
20	45638.69	889569.75	15263684.02	9669.139	312174.121
21	43663.94	845905.81	14374114.27	9449.722	3025.04.982
22	41773.16	804132.65	13528208.46	9238.323	293055.260
23	399*2.83	764169.82	12724075.81	9034.648	283816.938
24	38229.56	725940.26	11959905.99	8838.417	274782.290
25	36569.76	689370.50	11233965.73	8648.983	265943.873
26	34980.36	654390.13	10544595.26	8466.113	257294.890
27	33458.09	620932.04	9890205.11	8289.237	248828.777
28	32000.16	588931.88	9269273.06	8118.163	240539.540
29	30603.61	558328.27	8680341.18	7952.386	232421.377
30	29265.61	529062.66	8122012.92	7791.444	224468.991
31	27983.77	501078.89	7592950.26	7635.209	216677.547
32	26755.54	474323.35	7091871.37	7483.273	209042.337
33	25578.47	448744.88	6617548.02	7335.263	201559.064
34	24450.25	424294.62	6168803.15	7190.836	194223.803
35	23368.71	400925.92	5744508.52	7049.684	187032.965
36	22331.75	378594.17	5343582.60	6911.523	179983.282
37	21337.41	357256.75	4964988.44	6776.100	173071.758
38	20383.60	336873.15	4607731.69	6642.956	166295.658
39	19468.34	317404.81	4270858.53	6511.684	159652.702
40	18590.01	298814.80	3953453.73	6382.128	153141.018
41	17746.83	281067.97	3654638.92	6253.950	146758.890
42	16937.16	264130.82	3373570.95	6126.851	140504.939
43	16159.26	247971.56	3109440.13	6000.381	134378.089
44	15411.69	232559.87	2861468.57	5874.324	128377.707
45	14693.27	217866.59	2628908.71	5748.665	122503.383
46	14002.88	203863.71	2411042.11	5623.393	116754.719
47	13339.26	190524.45	2207178.40	5498.350	111131.325
48	12701.11	177823.34	2016653.95	5373.246	105632.975
49	12087.19	165736.15	1838830.61	5247.832	100259.729
50	11496.50	154239.65	1673094.46	5122.035	95011.897
51	10927.82	143311.83	1518854.81	4995.529	89889.862
52	10379.90	132931.93	1375542.98	4868.904	84894.333
53	9851.950	123079.98	1242611.05	4739.183	80026.429
54	9342.763	113737.22	1119531.07	4608.918	75287.246
55	8851.579	104885.64	1005793.86	4477.071	70678.328
56	8377.463	96508.174	900908.221	4343.400	66201.258
57	7919.451	88588.722	804400.047	4207.599	61857.858
58	7476.876	81111.847	715811.324	4069.618	57650.259
59	7049.117	74062.730	634699.478	3929.430	53580.642

TABLE NO. XXXIX.

National Fraternal Congress Table
Commutation Columns at Four Per Cent—Concluded.

x Age	Dx	Nx	Sx	Mx	Rx
60	6635.311	67427.419	560636.748	3786.745	49651.212
1	6234.957	61192.462	493209.329	3641.595	45864.467
2	5847.410	55345.052	432016.867	3493.854	42222.873
3	5472.254	49872.798	376671.815	3343.598	38729.019
4	5109.017	44763.781	326799.016	3190.833	35385.421
5	4757.423	40006.358	282035.235	3035.739	32194.588
6	4417.128	35589.230	242028.878	2878.422	29158.849
7	4087.881	31501.349	206439.648	2719.065	26280.427
8	3769.647	27731.701	174938.300	2558.057	23561.362
9	3462.366	24269.335	147206.599	2395.762	21003.305
70	3166.145	21103.190	122937.264	2232.709	18607.543
1	2881.043	18222.147	101834.074	2069.382	16374.834
2	2607.310	15614.837	83611.927	1906.458	14305.452
3	2345.348	13269.489	67997.090	1744.777	12398.994
4	2095.452	11174.037	54727.601	1585.087	10654.217
5	1858.143	9315.893	43553.564	1428.373	9069.129
6	1633.959	7681.934	34237.671	1275.655	7640.756
7	1423.441	6258.493	26555.736	1127.982	6365.1010
8	1227.169	5031.324	20297.243	986.4579	5237.1188
9	1045.649	3985.675	15265.919	852.1366	4250.6609
80	879.4003	3106.2748	11280.244	726.1051	3398.5243
1	879.4003	3106.2748	11280.244	726.1051	3398.5243
2	594.1278	1783.3320	5796.5093	502.6870	2063.0764
3	475.4340	1307.8980	4013.1773	406.8443	1560.3893
4	372.5940	935.3040	2705.2793	322.2903	1153.5450
5	285.1631	650.1409	1769.9754	249.1898	831.2547
6	212.4782	437.6627	1119.8345	187.4728	582.0649
7	153.5674	284.0954	682.1718	136.7342	394.5922
8	107.1794	176.9160	398.0764	96.2527	257.8580
9	71.8748	105.0412	221.1605	65.0703	161.6053
90	46.0150	59.0262	116.1193	41.9749	96.5350
1	27.9280	31.0982	57.0931	25.6578	54.5601
2	15.9064	15.1918	25.9949	14.7103	28.9023
3	8.4159	6.7759	10.8031	7.8316	14.1920
4	4.0586	2.7173	4.0272	3.7980	6.3604
5	1.7586	.9587	1.3099	1.6540	2.5624
6	.6717	.2870	.3512	.6349	.9083
7	.2227	.0643	.0643	.2117	.2735
8	.0643	.0000	.0000	.0618	.0618

TABLE NO. XL.

Life Annuities by Actuaries or Combined Experience at the Rates Stated.

x Age	3 %	3½ %	4 %	5 %	6 %	x Age	3 %	3½ %	4 %	5 %	6 %
10	23.256	21.253	19.454	16.556	14.347	55	12.021	11.479	10.978	10.077	9.295
11	23.220	21.149	19.369	16.502	14.312	56	11.656	11.145	10.670	9.816	9.071
12	23.080	21.037	19.282	16.445	14.274	57	11.290	10.808	10.359	9.550	8.843
13	22.936	20.922	19.191	16.386	14.234	58	10.923	10.469	10.046	9.282	8.611
14	22.787	20.804	19.096	16.324	14.193	59	10.555	10.129	9.731	9.010	8.375
15	22.633	20.681	18.998	16.259	14.149	60	10.188	9.789	9.415	8.735	8.136
16	22.475	20.555	18.896	16.192	14.102	61	9.822	9.448	9.098	8.459	7.893
17	22.313	20.424	18.790	16.121	14.054	62	9.457	9.108	8.781	8.182	7.649
18	22.146	20.290	18.681	16.048	14.003	63	9.096	8.771	8.464	7.903	7.403
19	21.974	20.151	18.568	15.971	13.950	64	8.737	8.434	8.149	7.625	7.156
20	21.797	20.008	18.450	15.891	13.894	65	8.382	8.101	7.836	7.349	6.908
21	21.616	19.860	18.329	15.808	13.836	66	8.032	7.771	7.526	7.070	6.660
22	21.430	19.708	18.204	15.722	13.775	67	7.686	7.443	7.217	6.795	6.413
23	21.239	19.551	18.075	15.632	13.712	68	7.347	7.124	6.913	6.521	6.167
24	21.043	19.389	17.941	15.539	13.645	69	7.013	6.808	6.613	6.251	5.922
25	20.842	19.223	17.803	15.442	13.576	70	6.685	6.497	6.317	5.983	5.678
26	20.635	19.054	17.660	15.341	13.503	71	6.364	6.191	6.026	5.718	5.437
27	20.423	18.875	17.512	15.236	13.427	72	6.049	5.891	5.740	5.457	5.198
28	20.205	18.693	17.360	15.127	13.347	73	5.742	5.597	5.459	5.200	4.962
29	19.982	18.505	17.202	15.014	13.264	74	5.441	5.310	5.184	4.947	4.729
30	19.754	18.314	17.040	14.896	13.177	75	5.148	4.929	4.915	4.699	4.449
31	19.519	18.116	16.872	14.774	13.087	76	4.863	4.755	4.651	4.459	4.273
32	19.279	17.912	16.698	14.647	12.992	77	4.585	4.488	4.394	4.216	4.050
33	19.032	17.703	16.520	14.515	17.893	78	4.315	4.228	4.143	3.982	3.832
34	18.780	17.487	16.335	14.378	12.789	79	4.053	3.975	3.899	3.754	3.618
35	18.521	17.266	16.144	14.235	12.681	80	3.799	3.729	3.661	3.531	3.409
36	18.255	17.038	15.948	14.087	12.568	81	3.553	3.490	3.429	3.313	3.204
37	17.983	16.803	15.744	13.933	12.450	82	3.312	3.257	3.203	3.099	3.002
38	17.703	16.561	15.534	13.773	12.326	83	3.077	3.028	2.980	2.889	2.803
39	17.417	16.312	15.317	13.606	12.196	84	2.846	2.803	2.761	2.681	2.605
40	17.123	16.055	15.093	13.433	12.060	85	2.617	2.580	2.554	2.474	2.408
41	16.821	15.792	14.861	13.252	11.918	86	2.391	2.359	2.328	2.268	2.210
42	16.512	15.519	14.621	13.064	11.768	87	2.167	2.140	2.114	2.063	2.013
43	16.195	15.240	14.374	12.868	11.612	88	1.946	1.923	1.901	1.858	1.817
44	15.870	14.953	14.119	12.666	11.448	89	1.728	1.709	1.691	1.655	1.621
45	15.540	14.659	13.857	12.456	11.279	90	1.516	1.501	1.485	1.456	1.428
46	15.204	14.360	13.590	12.241	11.104	91	1.309	1.296	1.284	1.261	1.238
47	14.864	14.056	13.317	12.020	10.923	92	1.109	1.100	1.090	1.072	1.054
48	14.519	13.746	13.039	11.794	10.737	93	.921	.914	.906	.892	.879
49	14.171	13.433	12.757	11.563	10.546	94	.748	.742	.737	.726	.716
50	13.820	13.116	12.470	11.326	10.349	95	.592	.588	.584	.576	.569
51	13.465	12.795	12.170	11.085	10.148	96	.468	.465	.462	.456	.450
52	13.107	12.471	11.884	10.840	9.942	97	.371	.369	.367	.363	.359
53	12.747	12.143	11.558	10.590	9.731	98	.243	.242	.240	.238	.236
54	12.385	11.812	11.283	10.336	9.515						

TABLE NO. XLI.

Life Annuities by American Experience at the Rates Stated.

x Age	3 %	3½ %	4 %	4½ %	5 %	6 %	x Age
10	23.343	21.225	19.414	17.855	16.504	14.293	10
11	23.225	21.133	10.343	17.800	16.461	14.265	11
12	23.103	21.038	19.269	17.741	16.415	14.236	12
13	22.977	20.940	19.192	17.681	16.366	14.204	13
14	22.846	20.839	19.112	17.617	16.316	14.171	14
15	22.712	20.733	19.028	17.551	16.263	14.137	15
16	22.573	20.624	18.942	17.482	16.207	14.100	16
17	22.430	20.510	18.851	17.409	16.149	14.062	17
18	22.282	20.393	18.757	17.334	16.088	14.021	18
19	22.130	20.271	18.660	17.255	16.024	13.978	19
20	21.971	20.144	18.568	17.173	15.957	13.932	20
21	21.808	20.013	18.452	17.086	15.886	13.885	21
22	21.640	19.878	18.342	16.997	15.813	13.834	22
23	21.467	19.737	18.228	16.903	15.735	13.781	23
24	21.289	19.592	18.109	16.805	15.655	13.725	24
25	21.104	19.442	17.985	16.704	15.570	13.666	25
26	20.914	19.286	17.857	16.597	15.482	13.604	26
27	20.718	19.124	17.723	16.486	15.389	13.538	27
28	20.516	18.957	17.585	16.370	15.292	13.469	28
29	20.308	18.784	17.440	16.250	15.191	13.396	29
30	20.093	18.605	17.291	16.124	15.084	13.320	30
31	19.872	18.420	17.135	15.993	14.973	13.239	31
32	19.643	18.229	16.973	15.856	14.857	13.154	32
33	19.408	18.030	16.806	15.713	14.735	13.064	33
34	19.166	17.826	16.632	15.565	14.608	12.969	34
35	18.917	17.614	16.451	15.410	14.475	12.870	35
36	18.661	17.395	16.263	15.249	14.336	12.765	36
37	18.397	17.169	16.069	15.081	14.191	12.655	37
38	18.125	16.935	15.868	14.907	14.039	12.540	38
39	17.847	16.695	15.659	14.725	13.881	12.418	39
40	17.560	16.446	15.443	14.537	13.716	12.291	40
41	17.266	16.190	15.220	14.341	13.544	12.157	41
42	16.963	15.926	14.988	14.138	13.365	12.017	42
43	16.653	15.654	14.749	13.927	13.179	11.870	43
44	16.335	15.374	14.502	13.709	12.984	11.716	44
45	16.009	15.087	14.248	13.483	12.783	11.555	45
46	15.676	14.791	13.985	13.248	12.574	11.386	46
47	15.335	14.488	13.714	13.006	12.357	11.211	47
48	14.987	14.177	13.436	12.757	12.133	11.028	48
49	14.632	13.859	13.151	12.500	11.901	10.838	49
50	14.271	13.535	12.858	12.236	11.662	10.640	50
51	13.905	13.204	12.559	11.965	11.416	10.436	51
52	13.533	12.868	12.255	11.688	11.164	10.226	52

TABLE NO. XLI.

Life Annuities by American Experience at the Rates Stated.

x Age	3 %	3½ %	4 %	4½ %	5 %	6 %	x Age
53	13.157	12.526	11.944	11.405	10.905	10.009	53
54	12.776	12.180	11.628	11.116	10.640	9.786	54
55	12.393	11.830	11.307	10.822	10.370	9.556	55
56	12.006	11.475	10.982	10.523	10.095	9.321	56
57	11.617	11.118	10.653	10.219	9.814	9.081	57
58	11.226	10.758	10.321	9.912	9.530	8.836	58
59	10.835	10.396	9.985	9.601	9.241	8.586	59
60	10.443	10.032	9.648	9.288	8.949	8.332	60
61	10.051	9.668	9.309	8.972	8.654	8.074	61
62	9.660	9.304	8.969	8.654	8.357	7.813	62
63	9.272	8.941	8.630	8.336	8.059	7.549	63
64	8.885	8.579	8.290	8.017	7.759	7.283	64
65	8.502	8.219	7.952	7.699	7.459	7.016	65
66	8.123	7.863	7.616	7.381	7.159	6.747	66
67	7.749	7.510	7.282	7.066	6.861	6.479	67
68	7.381	7.161	6.952	6.754	6.564	6.212	68
69	7.020	6.819	6.627	6.445	6.270	5.945	69
70	6.666	6.482	6.307	6.140	5.980	5.681	70
71	6.319	6.152	5.993	5.840	5.694	5.420	71
72	5.981	5.829	5.685	5.546	5.413	5.163	72
73	5.651	5.514	5.383	5.257	5.136	4.908	73
74	5.328	5.205	5.086	4.972	4.863	4.656	74
75	5.011	4.900	4.794	4.691	4.592	4.406	75
76	4.699	4.600	4.505	4.413	4.325	4.157	76
77	4.392	4.304	4.219	4.138	4.059	3.908	77
78	4.088	4.011	3.936	3.864	3.794	3.660	78
79	3.790	3.722	3.656	3.593	3.531	3.413	79
80	3.496	3.437	3.380	3.324	3.270	3.167	80
81	3.209	3.158	3.108	3.060	3.013	2.924	81
82	2.928	2.884	2.842	2.801	2.761	2.683	82
83	2.652	2.615	2.580	2.545	2.511	2.445	83
84	2.379	2.348	2.318	2.289	2.261	2.205	84
85	2.107	2.082	2.057	2.033	2.010	1.964	85
86	1.839	1.819	1.799	1.780	1.761	1.724	86
87	1.579	1.563	1.548	1.532	1.517	1.488	87
88	1.334	1.322	1.310	1.298	1.286	1.263	88
89	1.103	1.094	1.085	1.076	1.067	1.050	89
90	.880	.874	.867	.861	.854	.842	90
91	.663	.658	.654	.649	.645	.637	91
92	.459	.457	.454	.451	.449	.443	92
93	.294	.292	.291	.289	.288	.285	93
94	.139	.138	.137	137	.136	.135	94

TABLE NO. XLII.

Life Annuities by Carlisle Experience at the Rates Stated.

x Age	4 %	5 %	6 %	x Age	4 %	5 %	6 %
0	14.282	12.083	10.439	52	12.258	11.154	10.208
1	16.555	13.995	12.078	53	11.945	10.892	9.988
2	17.726	14.983	12.925	54	11.627	10.624	9.761
3	18.715	15.824	13.652	55	11.300	10.347	9.524
4	19.231	16.271	14.042	56	10.966	10.063	9.280
5	19.592	16.590	14.325	57	10.626	9.771	9.027
6	19.745	16.735	14.460	58	10.286	9.478	8.772
7	19.790	16.790	14.518	59	9.963	9.199	8.529
8	18.704	16.786	14.526	60	9.663	8.940	8.304
9	19.691	16.742	14.500	61	9.398	8.712	8.108
10	19.583	16.669	14.448	62	9.136	8.487	7.913
1	19.459	16.581	14.384	63	8.872	8.258	7.714
2	19.335	16.494	14.321	64	8.593	8.016	7.502
3	19.209	16.406	14.257	65	8.307	7.765	7.281
4	19.082	16.316	14.191	66	8.010	7.503	7.049
5	18.955	16.227	14.126	67	7.700	7.227	6.803
6	18.836	16.144	14.067	68	7.380	6.941	6.546
7	18.721	16.066	14.012	69	7.049	6.643	6.277
8	18.606	15.987	13.956	70	6.709	6.336	5.998
9	18.486	15.904	13.897	71	6.358	6.015	5.704
20	18.362	15.817	13.835	72	6.025	5.711	5.424
1	18.232	15.726	13.769	73	5.724	5.435	5.170
2	18.094	15.628	13.697	74	5.458	5.190	4.944
3	17.950	15.525	13.621	75	5.239	4.989	4.760
4	17.801	15.417	13.541	76	5.024	4.792	4.579
5	17.645	15.303	13.456	77	4.825	4.609	4.410
6	17.486	15.187	13.368	78	4.622	4.422	4.238
7	17.320	15.065	13.275	79	4.393	4.210	4.040
8	17.154	14.942	13.182	80	4.183	4.015	3.858
9	16.997	14.827	13.096	81	3.953	3.799	3.656
30	16.852	14.723	13.020	82	3.746	3.606	3.474
1	16.705	14.617	12.942	83	3.534	3.406	3.286
2	16.552	14.506	12.860	84	3.329	3.211	3.102
3	16.391	14.387	12.771	85	3.115	3.009	2.909
4	16.219	14.260	12.675	86	2.928	2.830	2.739
5	16.041	14.127	12.573	87	2.776	2.685	2.599
6	15.856	13.987	12.465	88	2.683	2.597	2.515
7	15.666	13.843	12.345	89	2.577	2.495	.2417
8	15.471	13.695	12.239	90	2.416	2.339	2.266
9	15.272	13.542	12.120	91	2.398	2.321	2.248
40	15.074	13.390	12.002	92	2.492	2.412	2.337
1	14.883	13.245	11.890	93	2.600	2.518	2.440
2	14.695	13.101	11.779	94	2.650	2.569	2.492
3	14.505	12.957	11.668	95	2.674	2.596	2.522
4	14.309	12.806	11.551	96	2.628	2.555	2.486
5	14.105	12.648	11.428	97	2.492	2.428	2.368
6	13.889	12.480	11.296	98	2.332	2.278	2.227
7	13.662	12.301	11.154	99	2.087	2.045	2.004
8	13.419	12.107	10.998	100	1.653	1.624	1.598
9	13.153	11.892	10.823	101	1.210	1.192	1.175
50	12.869	11.660	10.631	102	.762	.753	.744
51	12.566	11.410	10.422	103	.321	.317	.314

TABLE NO. XLIII.

Single Premiums for an Insurance of 1 at Rates and by the Tables Named.

x Age	American Experience 3 Per Cent	American Experience 3 1-2 Per Cent	American Experience 4 Per Cent	American Experience 4 1-2 Per Cent	Actuaries or Combined Experience 3 Per Cent	Actuaries or Combined Experience 3 1-2 Per Cent	Actuaries or Combined Experience 4 Per Cent	x Age
10	.29098	.24844	.21485	.18803	.29064	.24747	.21332	10
11	.29443	.25154	.21759	.19045	.29456	.25164	.21671	11
12	.29798	.25474	.22043	.19295	.29863	.25484	.21993	12
13	.30165	.25806	.22339	.19557	.30285	.25866	.22344	13
14	.30545	.26150	.22647	.19830	.30718	.26267	.22708	14
15	.30937	.26507	.22968	.20115	.31165	.26681	.23086	15
16	.31340	.26877	.23301	.20413	.31625	.27108	.23478	16
17	.31758	.27260	.23648	.20724	.32099	.27543	.23884	17
18	.32189	.27658	.24010	.21050	.32586	.28006	.24305	18
19	.32634	.28070	.24386	.21389	.33086	.28476	.24740	19
20	.33094	.28497	.24778	.21745	.33600	.28901	.25191	20
21	.33568	.28941	.25185	.22115	.34128	.29460	.25655	21
22	.34057	.29399	.25608	.22502	.34669	.29974	.26138	22
23	.34562	.29874	.26047	.22905	.35226	.30505	.26636	23
24	.35081	.30365	.26504	.23325	.35707	.31050	.27156	24
25	.35618	.30874	.26979	.23764	.36383	.31614	.27682	25
26	.36172	.31401	.27474	.24223	.36985	.32192	.28231	26
27	.36743	.31947	.27989	.24700	.37603	.32789	.28799	27
28	.37332	.32512	.28521	.25199	.38236	.33404	.29386	28
29	.37939	.33096	.29075	.25719	.38886	.34036	.29992	29
30	.38564	.33702	.29651	.26261	.39552	.34687	.30617	30
31	.39209	.34328	.30250	.26825	.40235	.35356	.31262	31
32	.39873	.34976	.30871	.27416	.40936	.36045	.31929	32
33	.40558	.35647	.31517	.28030	.41654	.36754	.32617	33
34	.41263	.36339	.32186	.28669	.42389	.37482	.33327	34
35	.41988	.37055	.32881	.29335	.43144	.38235	.34060	35
36	.42736	.37795	.33602	.30029	.43917	.38996	.34817	36
37	.43504	.38560	.34350	.30751	.44710	.39817	.35599	37
38	.44295	.39259	.35125	.31503	.45524	.40617	.36407	38
39	.45107	.40164	.35927	.32283	.46358	.41458	.37241	39
40	.45942	.41004	.36757	.33095	.47214	.42323	.38104	40
41	.46800	.41869	.37617	.33937	.48093	.43217	.38996	41
42	.47680	.42762	.38566	.34812	.48995	.44137	.39919	42
43	.48583	.43681	.39425	.35719	.49919	.45083	.40871	43
44	.49510	.44628	.40375	.36660	.50863	.46055	.41852	44
45	.50458	.45601	.41355	.37635	.51826	.47048	.42857	45
46	.51430	.46600	.42366	.38643	.52804	.48060	.43886	46
47	.52423	.47626	.43406	.39685	.53795	.49088	.44935	47
48	.53437	.48677	.44476	.40759	.54798	.50130	.46002	48
49	.54470	.49752	.45574	.41867	.55812	.51190	.47088	49
50	.55522	.50849	.46699	.43004	.56836	.52264	.48191	50
51	.56589	.51967	.47848	.44169	.57870	.53347	.49311	51
52	.57671	.53104	.49021	.45363	.58912	.54447	.50446	52
53	.58767	.54258	.50215	.46582	.59960	.55555	.51595	53
54	.59874	.55430	.51431	.47826	.61015	.56670	.52757	54
55	.60992	.56615	.52665	.49093	.62075	.57797	.53931	55
56	.62118	.57813	.53915	.50380	.63139	.58931	.55116	56
57	.63251	.59022	.55181	.51687	.64205	.60070	.56310	57
58	.64389	.60239	.56459	.53010	.65274	.61215	.57514	58
59	.65530	.61464	.57748	.54348	.66344	.62350	.58726	59
60	.66672	.62693	.59046	.55699	.67414	.63517	.59942	60
61	.67813	.63924	.60349	.57059	.68480	.64668	.61163	61
62	.68950	.65155	.61656	.58426	.69541	.65816	.62383	62
63	.70083	.66383	.62963	.59797	.70595	.66959	.63600	63
64	.71208	.67607	.64269	.61170	.71640	.68096	.64812	64
65	.72324	.68824	.65570	.62541	.72673	.69224	.66017	65
66	.73427	.70030	.66863	.63908	.73694	.70338	.67212	66
67	.74516	.71223	.68145	.65265	.74700	.71440	.68396	67
68	.75588	.72401	.69414	.66611	.75689	.72526	.69565	68
69	.76641	.73560	.70665	.67943	.76662	.73766	.70719	69
70	.77673	.74698	.71896	.69254	.77617	.74650	.71857	70

CHAPTER VIII

Life Contingencies and Money

If a certain sum, say $1000.00, be promised and be made payable at a certain time, say in twenty years, its present value at a given rate per cent, say 4%, would be $456.39. This result is reached by the process of discounting illustrated in the Chapter on Interest (Chapter V) Article 25 et seq. If the same sum be promised and be made payable upon the happening of the death of A, now aged thirty-five years, its present value computed on the American experience Table of Mortality at the rate of four per cent would be $328.81. This benefit is called a life insurance. The result last stated is arrived at by a process of discounting, but one of the factors, that of time, differs from the case first instanced in that instead of employing a time certain during which the rate of interest is applied, we must use also the probability of living, or rather of dying, of A. This probability is determined by means of a mortality table, of which a number of examples are given in this book. A process which may be used in finding the present value of the $1000.00 insurance promised is as follows:

The group of lives treated in the American Experience table of mortality numbers 100000, called the *radix* of the table, beginning at age ten. At the age given, 35, there remain living of this 100000 group, 81822, of whom 732 will die during the current year. Hence, $732000 must be paid their beneficiaries at the end of the year. It follows that for this first year 81822 persons must pay in advance a sum which improved at 4% interest, will amount to $732000 at the end of the year. Discounting $732000 at 4% one year, we have $703846.24, which, divided by 81822, gives $8.602, the sum to be contributed by each person for the first year's insurance. Between the ages 36 and 37, there will die of the group, 737, making it necessary to pay beneficiaries $737000 at the end of that year. This sum discounted two years gives $681393.40 or $8.327 per member which also must be paid down at the beginning. For the third year, $742000 must be provided, making it necessary to pay down $659560.80 or $8.061 per member and so on to the end of the table. The total of these discounted sums has been computed and is $328.809 for each $1000 of insurance. This latter sum is the present value and also the net cost of $1000 life insurance at age 35 according to the rate of mortality shown by the American Experience table and 4% compound interest. It is hence called a **Single Premium.** This premium is usually represented by the symbol Ax. Life Annuities are closely related to life insurances and their values may be computed by a very similar process. We may take the same group and age. At the end of the first year there will survive of the original 81822 only 81090, 732 having died as already stated. If the annuity be one dollar, then there must have been paid down at the beginning of the year an

amount which at 4% interest would pay 81090 dollars at the end of the year. This would be the last mentioned sum discounted one year at 4% and is $77971 or $.9524 for each of the members of the group. The number living at the end of the second year and to each of whom a dollar must be paid, is 80353, requiring $80353. To provide for this, the latter sum discounted two years should have also been paid down at the beginning. This process is continued to the end of the table by which means it is found that the total amount required to be paid at the beginning by each member of the original group is $16.451. This then is the present value and also the net cost of a life annuity of one dollar at age 35 by the table and rate of interest used in the computation. The value is usually represented by the symbol a_x. If instead of proceeding as above, we had divided the number living at the end of the year by the number of the original group at the beginning of the year, we should have had the probability of living one year at age 35, which is .991054. Also if we divide the number dying during the year, 732, by the original group 81822, we will have the probability of dying during the year, which is .008946. The sum of these probabilities it will be observed, is unity or certainty, since man must either live or die and the two factors cover all of the contingencies. The probabilities for the succeeding years may be found in the same way. These probabilities have been tabulated and are given in connection with the mortality tables published in this book. If instead of dealing with the whole group of lives, we had taken a single life, we might have found the values of the insurance and annuity as well. For instance, the probability of a given person dying between the ages 35 and 36 is, .008946. As the insured must pay for his insurance, he, on a $1000.00 policy, should pay at the beginning of the year the value of the chance or probability that the company will have to pay his beneficiary $1000. That is, $1000 multiplied by the probability of its becoming due, .008946, which gives $8.946. This sum must be discounted or divided by $1.04, which gives $8.602, as the sum which must have been paid at the beginning of the year, as before. The contribution to be made for the subsequent years may be found in the same way, having due regard to the probability of dying and the number of years discount to be used. In the case of annuities, we must use the probability of living instead of that of dying, because the value of each of the one dollar payments depends not only on the number of years which will intervene before it is to be received, but also upon the probability of the nominee living to receive it.

While the foregoing discussion illustrates the principles underlying life insurance and life annuities, it is apparent that the processes employed are too cumbersome for practical purposes. They may be employed, however, in deriving other functions which may be used with great facility in computing many of the benefits depending upon life contingencies.

If the process suggested for finding the present value of the life insurance be represented in the form of an equation, using 1 for the number living, d for the number dying, x for the age of the insured, px for the probability of living and qx for the probability of dying at a given age, and n for a number of years, it would appear as follows: The net single premium, A_{35} or

$$\$328.809 = \frac{\$732.000}{81822 \times 1.04} + \frac{\$737000}{81822 \times (1.04)^2} + \frac{\$742000}{81822 \times (1.04)^3} \text{ and so on}$$

to the end of the table. Each of the numbers on the right hand might have been written in this form, viz:

$\frac{732}{81822} \times \frac{1}{1.04} \times (\$1000)$, performing the indicated operation the factors become respectively $.008946 \times .961538 \times 1000$ which read in the light of the notation above suggested, will be

$\frac{dx \times vx}{lx} \times 1000$, or if the policy be assumed to be 1, the equation finally becomes, $Ax = .3288 =$

$$\frac{v^x dx}{lx} + \frac{v^{x+1} dx + 1}{lx} + \frac{v^{x+2} dx + 2}{lx} + \text{ years to ends of the table,}$$

This series will be adequate to compute the single premium of a policy of 1, for age 35, but it would not be available for computing the premium at any other age, for instance, at age 34 or 36,

A formula may be derived, however, which is general in its application, as follows:

If we begin with the first year of the table, it will appear in the form

$$Ax = \frac{vdx + v^2 dx + 1 + v^3 dx + 2 + v^4 dx + 3}{lx} + \text{ \& etc,}$$

By multiplying both the numerator and the denominator of the right hand member of said equation by v, it will appear as follows:

$$Ax = \frac{v^{x+1} dx + v^{x+2} dx + 1 + v^{x+3} dx + 2}{v^x lx} \text{ \& etc.}$$

The denominator of the above equation has been called D, adding the subscript x to denote the age. In the above case, the x represents 10 that being the youngest age of the table and the number living is then 100000. The quantities in the numerator $v^{x+1} dx, + v^{x+2} dx + 1$ have been called Cx, Cx+1, Cx+2 and so on and the equation at age 10 would be written:

$$Ax = \frac{Cx + (Cx+1) + (Cx+2)}{Dx} + \&\ \text{etc.}$$

At age 11, it would be $Ax+1 = \frac{(Cx+1)+(Cx+2)+(Cx+3)\ \text{etc.}}{Dx+1}$

and so on for older ages throughout the table.

Since all the years of the life after ten are contained in the first equation it is apparent that if the quantities C be summed from the oldest age down to ten and the numbers recorded at each addition we will have the numerators of all the equations which we would have had we extended those above given to all the years to 95, the end of the table. The numerator columns thus formed have been called M columns by the actuaries and they are so marked in the tables here published. The D column has been formed for each year by computing and recording the quantities $v^x 1x$, $v^{x+1} 1x+1$, $v^{x+2} 1x+2$ and so on to the end of the table. Now, to solve the first equation above, we would simply divide M, the equivalent of $v^{x+1}dx + v^{x+2}dx+1 + v^{x+3}dx+2$ and etc. by Dx which is the equivalent of $v^x 1x$. The quotient will be Ax or $213.99. For age eleven, $Mx+1$ divided by $Dx+1$ would give the net single premium for that age and so on for all the ages given in the table from which we may derive the general

formula, $Ax = \frac{Mx.}{Dx}$

By a similar process, a table Nx is derived for findng the value of annuities but instead of using dx, the number dying, as a factor, the number living at the end of each year is employed. That is, beginning at the first year, $1x+1$, $1x+2$ and so on, are substituted in the original equation. Thus, the formula for ax, a life annuity, is derived and is as follows:

$$ax = \frac{Nx}{Dx}$$

The tables which have just been described are called "Commutation" Tables. In some cases, two other columns marked respectively S and R have been computed. These may be explained as follows: If at a given age an annuity of 1 were purchased to be increased by 1 each year, to find its present value, we should have the following equation:

An increasing annuity or $(I ax) = \frac{Nx}{Dx} + \frac{Nx+1}{Dx} + \frac{Nx+2}{Dx} + \text{and etc.}$

For convenience, the quantities Nx have been summed and are called S, and therefore we may use the formula for an annuity of 1, increasing by 1 each year,

$$I ax = \frac{Sx}{Dx}$$

For increasing insurances, the M columns have been summed in the same manner forming the new column R. Examples of these columns will be found in the insurance commutation columns. Other tables have been computed and published here for further reducing the labor of computing values based on life contingencies, and their uses will presently be explained, and illustrated, In books such as Principles and Practice of Life Insurance, Various Derived Tables and other works, many of the values have been computed, tabulated and published for the convenience of the insurance profession. We will now state, without demonstration, a number of formulas and rules for solving problems involving life contingencies and illustrate them by examples.

CHAPTER IX

Of Life Annuities

1. The present value of a life annuity of 1 is found by dividing the quantity N of the given age by the quantity D at the same age,

thus, $$a_x = \frac{N_x}{D_x}. \qquad (1)$$

2. What is the present value of an annuity of $250.00 at age 35, computing on the American Experience table of Mortality, at 4% interest?

SOLUTION: By the commutation table (No. 37) in the N column opposite age 35, we find the figures 341109.86 and D35 = 20734.96, and by formula (1) $a_{35} = \frac{341109.86}{20734.96} = 16.451$ which, multiplied by 250 gives 4112.75. Answer, $4112.75.

3. **A Temporary Annuity** is found by dividing the difference between the N functions of the age at the beginning and the end of the given term by the D function of the given age, thus:

$$|_n a_x = \frac{N_x - N_{x+n}}{D_x} \qquad (2)$$

4. EXAMPLE: What is the present value of an annuity of $250 for the term of ten years at age 35, computed upon American Experience table at 4%?

SOLUTION: From the 4% commutation table, we obtain the following values: N35 = 341109.86; N45 = 180921.20, D35 = 20734.96, and by formula (2) $|_{10} a_{35} = \frac{341109.86 - 180921.20}{20734.96} = 7.7255$

$250 \times 7.725 = $1931.25, Answer.

5. **A Deferred Annuity** is found by dividing the function N at the age when the annuity is to be entered upon by the function D at the age when the annuity is contracted. Thus:

$$_n|a_x = \frac{N_{x+n}}{D_x} \qquad (3)$$

6. What is the present value of a life annuity of $250, deferred ten years at age 35, computed upon American Experience table at 4%?

SOLUTION: $N_{45}=180921.20$, $D_{35}=20734.96$, and by formula (3), $10|a_{35}=\frac{180921.20}{20734.96}=8.7254$ which, multiplied by 250 gives 2181.35. Answer, $2181.35.

7. NOTE—By adding the temporary and deferred annuities above, we find their sum to be the same as the whole life annuity, thus $7.7255+8.7254=16.451$. It thus appears that the term and deferred annuities supplement each other and that the following equations hold:

$ax-|n ax=n| ax.$

$ax-n|ax=|\bar{n}ax$ and $|\bar{n}ax+n|ax=ax.$

8. **A Deferred Temporary** Annuity is found by dividing the difference between the N functions for the age at the beginning and the end of the deferred term by the function D for the given age, thus:

$$n|m a_x=\frac{N_{x+n}-N_{x+\overline{m+n}}}{Dx} \qquad (4)$$

9. EXAMPLE: What is the value of annuity of $250 for term of fifteen years entered upon ten years from the date of the contract, the annuitant being 35 years of age, computed on the American Experience table at 4%?

SOLUTION: $N_{45}=180921.20$, $N_{60}=53118.75$ and $D_{35}=20734.96$, and by the formula,

$$10|15 a_{35}=\frac{180921.20-53118.75}{20734.96}\times 250=(6.163\times 250)=1540.75,$$

Answer, $1540.75

10. Ordinary life annuities are payable at the end of each year survived by the life on which it is based. Such are the annuities already considered. But the contract or law governing may provide for payment in the same proportion for the fractional part of the last year of the life. Such an annuity is called a **Complete Annuity** and is sometimes represented by the symbol $\overset{\circ}{a}_x$.

11. On the assumption that the death rate is uniform throughout the year, the value of a complete annuity is that of an ordinary annuity plus an insurance equal to one-half the annual payment, payable at the moment of death. The value of an insurance has already been assigned the symbol A. But an insurance so marked is payable at the end of the year of the death which, it is assumed in insurance calculations, will occur on the average at the middle of the year, so that the half single premium must be increased so that it equals the first payment of a sum payable semi-annually at the nominal rate

This would be represented by the equation

$$\overset{\circ}{a}_x = a_x + \frac{A(1+i)^{\frac{1}{2}}}{2}. \qquad (5)$$

12. As an example, let it be required to find the value of a complete annuity at age 35, using American Experience table and $3^1/_2\%$. We have, $a_{35} = 17.614$. $A_{35} = .3706$. And,

$^1/_2 \times .3706 \times \sqrt{1.035} = {}^1/_2 \times .3706 \times 1.017344 = .377026 \div 2 = .1885$.

And $\overset{\circ}{a}_{35} = 17.614 + .1885 = 17.8125$.

13. In the last example, it will be noticed that the amount of 1 at the actual rate for the half year, 1.0173, does not differ much from the amount of 1 at the nominal rate for one-half year, 1.0175, and it is considered sufficiently accurate in practice to multiply by 1 plus one-half the nominal rate for yearly annuities.

14. The formula for a complete half-yearly annuity is sometimes written, $\overset{\circ}{a}{}_x^{(2)} = a_x^{(2)} + {}^1/_4 A_x (1+i)^{(\frac{1}{2})}$. This formula will be better understood after reading the next section.

15. An annuity may be made payable in 2, 4 or m installments and the calculation of the exact value of such annuities presents an interesting problem, but one of little practical value here. A useful approximate formula is stated as follows:

$$a_x^{(m)} = a_x + \frac{m-1}{2m} \qquad (6)$$

In this expression, m represents the number of installments into which the year's payment is divided. Thus, if the annuity is 1 payable in semi-annual payments, that is in 2 payments. The formula becomes

$$a_x{}^{(2)} = a_x \left(\frac{2-1}{2 \times 2} \right) = {}^1/_4 =$$

$ax + {}^1/_4$. For 4 installments, we have

$a_x^{(4)} = a_x + \frac{4-1}{2 \times 4} = {}^3/_8 = a_x + {}^3/_8$. For monthly installments, we have

$$ax + \left(\frac{12-1}{2 \times 12} = \frac{11}{24} \right) = a_x + \frac{11}{24} \text{ and so on.}$$

16. The value of an annuity of 1 at age 35, payable in half yearly installments would by the above formula, be $17.614 + .25 = 17.864$; and payable quarterly, the value would be $17.614 + .375 = 17.989$.

17. When m in the above formula is supposed to become infinite the correction $\frac{m-1}{2m}$ approaches $^1/_2$ as its limit and the value of the

annuity becomes $a_x + 1/2$. Such annuities are called **Continuous Annuities,** by which it is meant that the installments are payable momently. The reader will have little occasion for the use of the latter formula. The symbol assigned by actuaries to the continuous annuity is $\bar{a}_x$.

18. An annuity on two or more lives is called a **joint life annuity.** Such an annuity terminates with the first death. It is possible to compute commutation tables for such annuities but the numerous combinations of ages which have to be considered are such that so far as the writer knows it has not been undertaken for more than two lives. Annuities on two joint lives have been computed, but only for ages taken at intervals and not year by year. Tables on the Northampton, Actuaries and some others have been so treated.

19. Single annuities on joint lives may be computed directly from the mortality table by means of formulas for approximate summation. These will be noticed further on.

20. The calculation of annuities on joint lives has been greatly facilitated by regraduating the fundamental table by what is known as Makeham's law. Not all tables of mortality are susceptible of regraduation by this process, but some of the most important ones have been so treated. The English Twenty Office healthy males table was so treated by George King in 1880. Mr. Arthur Hunter, a New York actuary and author, regraduated the American Experience table in 1902. The Carlisle table was also regraduated in 1880 by Mr. King. Mr. Abb Landis, an author and actuary of Nashville, Tenn., has "Makehamized" the National Fraternal Congress table. The Actuaries or Combined Experience table was also regraduated by Makeham's formula in 1915 by the author of this work.

The effect of this process is to convert the table into a series with decrements conforming to certain constants and the force of mortality derived from the original table. This latter function may be used to find two or more equal ages having a force of mortality equivalent to that of two or more lives of different ages, so that it is only necessary to compute commutation columns, annuities and premiums for equal ages. This method saves an enormous amount of labor and space and while annuity and insurance values derived from such tables do not exactly represent the values which the original data would produce they are sufficiently exact for all practical purposes and the labor saved warrants their employment where joint lives are involved.

We have computed and published here D and N commutation tables and annuities at the rates of five and six per cent on two and three lives from the regraduated Carlisle table; and at the rate of three and one-half, five and six per cent on two and three lives from the regraduated American Experience table. Also at the rates of four, five and six per cent on two and three lives from the regraduated Actuaries table. One or the other, sometimes two, of these tables in their original form, have been adopted as standard tables in nearly

all the states of the Union. They, with the single life tables may therefore be employed in computing all sorts of values depending on one, two or three lives. And by means of the formulas for differencing approximate values for four or even five lives may be derived.

21. Where joint life commutation columns are available, whole life, temporary, deferred and deferred temporary annuities are computed by the same forms of formulas as single life annuities, thus:

$$a_{xy} = \frac{N_{xy}}{D_{xy}}; \quad n|a_{xy} = \frac{N_{x+n;y+n}}{D_{xy}}; \quad |na_{xy} = \frac{Nxy - Nx+n;\ y+n}{D_{xy}} \text{ and}$$

$$n|m\ axy = \frac{Nx+n:y+n - Nx+\overline{n+m}:y+\overline{n+m}}{Dxy}$$

22. **A Revisionary Annuity,** is one which begins at the death of a person now aged y and continues during the life of a person now aged x. Its value is the difference between the value of a whole life annuity on x and the value of a joint life annuity on x and y The formula is $a_y|x = a_x - a_{xy}$. (7)

24. An annuity on three lives and to the survivors successively is represented by the symbol $a_{\overline{xyz}}$ and its value is
$a_x + a_y + a_z - a_{xy} - a_{xz} - a_{yz} + a_{xyz}$. (8)

25. An annuity on the survivor of x an y but not to begin until the first death is equal to the sum of a reversionary annuity on the life of each, that is, $(a_x - a_{xy}) + (a_y - a_{xy}) =$
$a_x + a_y - 2a_{xy}$. (9)

26. An annuity on the life of x provided he shall survive both y and z, that is, to begin at the death of both, is,
$a_x - a_{xy} - a_{xz} + a_{xyz}$. (10)

27. The value of an annuity to be shared by x and y during their joint lives and by the survivor of them during his life is equal to the sum of the values of two annuities on their separate lives less the value of a joint annuity on their lives, thus:
$a_{\overline{xy}} = a_x + a_y - a_{xy}$. (11)

28. If the payments are postponed until the first death the value of the annuity of the last article would become:
$a_x + a_y - 2a_{xy}$.

29. The value of the interest of x or y in the annuity of Article 27 is that of an annuity on his life less one half the value of the joint life annuity, thus, for x it is: $a_x - {}^1/_2 a_{xy}$. (12)

30. The value of an annuity payable during the joint lives of y and z after the death of x, is the value of the joint life annuity on y and z less a joint life annuity on the three lives, thus:
$a_x|_{yz} = a_{yz} - a_{xyz}$. (13)

31. The value of an annuity on the life of x to begin after the death of either y or z is the value of an annuity for the life of x less the value of a joint life annuity on the lives of x, y and z, thus:

$$a_{yz}|_x = a_x - a_{xyz}. \qquad (14)$$

32. The value of an annuity on the life of x to begin after the death of *both* y and z is the value of an annuity for the life of x less an annuity which is to fail either on the death of x or of both y and z, thus

$$a_{\overline{yz}}|_x = a_x - a_{x:\overline{yz}} \qquad (15)$$

33. The value of an annuity on the life of x after an annuity for a term certain on the life of y is the value of $n|ax$ less a deferred joint life annuity on x and y, thus:

$$n|a_y|_x = na|_x - n|a_{xy}. \qquad (16)$$

34. The value of a joint life annuity on two of three lives, to begin only after the first death of the three will be understood by the formula: $a_{xy} + a_{xz} + a_{yz} - 3a_{xyz}$. (13)

35. A little study of the principles involved in the foregoing formulas will enable the reader to form other combinations necessary to solve the problems which will arise in practice. Nearly all of those given are of great practical value to those who have occasion to value properties and interests involving life contingencies. A number of examples will be given illustrating the use of formulas and principles on which they are based, in the succeeding articles.

36. Before employing the foregoing formulas in the solution of problems, it will be helpful to explain a method of deriving joint life annuities by the Carlisle, American Experience and Actuaries regraduated tables. As stated, these tables have been graduated by Makeham's law of mortality and they constitute a series in which the force of mortality is the only variable, the other functions involved in the process being constants derived from the original data. The force of mortality for each year of life is given in tables (Table 27). These regraduated tables have the property that enables us to take the average force for two, three or more different ages and use it to find an equal age at which two, three or more lives will have the same value. Thus, $\mu 20 + \mu 30 + \mu 40 = \mu 33 + \mu 33 + \mu 33$, nearly for the force of mortality by the table, American Experience, for age 20 is .00786, for age 30, .00835 and for age 40, .00977. Their sum is .02598, which sum divided by 3 gives .00866, corresponding nearly to age 33 at which μ is .00863. By interpolating in a manner similar to that employed with logarithms a closer approximate age may be found. Thus, .00866 falls between ages 33 and 34 which have $\mu = .00863$ and .00875 respectively and their difference is .00012. The difference between .00866 and .00863 is .00003, which is one-fourth of that of a whole year which as we have seen is .00012, hence the approximate equal age is $33\frac{1}{4}$ years.

37. Now let it be required to find the value of an annuity on the joint lives of A, B and C, aged respectively, 20, 30 and 40 years. We

have found the equivalent equal age from the American Experience table to be $33\frac{1}{4}$. We will therefore use annuities derived from that table. Using the six per cent table (Table No. XXVIII) we find $a_{33:33:33} = 10.12045$ and $a_{34:34:34} = 10.01075$. The difference in the annuity for a difference of a whole year is $10.12045 - 10.01075 = .10970$. For one-fourth year, it is $.10970 \div 4 = .02742$. Now since the annuity decreases as the age increases, this is to be deducted from the younger age and we have $10.12045 - .02742 = 10.09303$, which is the required annuity.

38. Let us now compute the values by the Carlisle table, omitting the explanatory matter so as to show up the figures more clearly.

$\mu 20 = .00674$	$a_{27:27:27} = 10.23159$
$\mu 30 = .00942$	$a_{28:28:28} = 10.14654$
$\mu 40 = .01137$	Difference for one year ..... $= .08505$
$3) .02753$	$.8$
Average $= .00918$	Difference for $^8/_{10}$ year $= .068040$ which subtracted from $10.231590 =$
$\mu 27 = .00910$	
$\mu 28 = .00920$	$a_{27.8:27.8:27.8} = 10.16355$
$.00010$	Hence, $a_{20:30:40} = 10.16355$

$\therefore \quad \mu .00918 = \text{Age } 27.8 \text{ years.}$

39. Let it be required to compute the present value of a term annuity on the same lives, using American Experience at 6%. Let the time be ten years. The average equal age we have already found to be 33 years. The formula is

$$\frac{N_{xxx} - {}^{n}N_{xxx}}{D_{xxx}}$$

From the commutation table (Table No. XX) for three lives 6%, we have $N_{33:33:33} = 85499482$, $N_{43:43:43} = 31125106$, and $D_{33:33:33} = 8448181$. Putting these values in the formula and solving we have $(85499482 - 31125106) \div 8448181 = 6.4362$, the required annuity.

For a deferred annuity, same ages, we have,

$$\frac{N_{43:43:43}}{D_{33:33:33}} = 31125106 \div 8448181 = 3.6842,$$ which is the value of the deferred annuity. The sum of these should equal the whole life annuity at age 33, and so it does, 10.1204.

We might have found the annuities for $33\frac{1}{4}$ years, by first finding them for the equal ages 33 and 34 separately and interpolating as illustrated in the two preceding articles.

40. The value of a joint life annuity on the lives of A and B is found as follows: $\mu 20 = .00674$, $\mu 30 = .00942$. Their sum is .01616 and their average equal age is μ .00808 age = 25.8 years.

41. The value of A and C, we find in the same way to be 11.18926, it being the equal age 34.5:34.5, and the value of B and C, having the equal age $36^1/_4$:$36^1/_4$ is 10.98245.

42. From the table of single life annuities, American Experience 6% (Table XLI), we have values as follows: A, or $a_{20} = 13.932$; B or $a_{30} = 13.320$ and C or $a_{40} = 12.291$.

We may use the values now found in solving a variety of practical problems and for convenience of reference, they may be collected as follows:

$a_{20} = 13.932$	$a_{xy} = 11.973$	a_{xyz}	= 10.093
$a_{30} = 13.320$	$a_{xz} = 11.189$	$\|10 a_{xyz}$	= 6.436
$a_{40} = 12.291$	$a_{yz} = 10.982$	$10\|a_{xyz}$	= 3.684

43 What is the present value of the dower interest of a widow aged 30 in an estate valued at $100000 producng a net annual income of 6% on the value?

Solution: The dower is one-third of the estate during the life of the widow. It would produce one-third of the annual income or $2000 per year. This may be valued as an annuity. The present value of an annuity of 1 at age 30 is 13.320 and the present value of an annuity of 2000 is, of course, $13.320 \times 2000 = 26640$. Answer, $26640.

44. What is the present value of the remainder of the estate mentioned in Article 43?

Solution: The present value of the whole estate, by the problem, is $100000. The present value of the remainder is therefore $100000 - 26640 = 73360$. Answer, $73360.

45. What is the present value of the inchoate dower right of a woman in an estate the value of which is 100000 owned by her husband, aged 40, her age being 30, the income of the estate being 6%?

Solution: The interest or right of the wife is a reversionary annuity of $2000, dependent upon her surviving her husband and is the same as an annuity on her life less the value of a joint life annuity on the lives of herself and husband. A life annuity of 1 at age 30, is 13.320. A joint annuity of 1 on the lives of two persons aged 30 and 40 is 10.982. Their difference, $13.320 - 10.982$, is 2.338, which, multiplied by $2000 = 4676$. Answer, $4676.00.

46. What is the present value of the curtesy interest of the husband of the woman of the preceding problem assuming that he has such interest, her real estate being worth $100000, earning an income of 6% on its value?

Solution: Here, again we have to find the value of a reversionary

annuity. The annuity on his life is 12.291. The joint life annuity is 10.982. The difference is 1.304, which, multiplied by 6000 gives 7824. Answer, $7824.00.

NOTE: The value of the courtesy initiate is not considered in this problem. If considered, his interest, of course, would be a life annuity.

47. What is the present value of the interest of A and B in an annuity of $5000 bequeathed to them to be enjoyed by them equally during their joint lives and by the survivor during his life?

SOLUTION: The whole value is the same as if each of the nominees was to receive an annuity for his life less the value of the joint life annuity on their lives; but here we are to find the value of the interest of each. First A, whose age is 20, receives an annuity for life less one-half the joint annuity which goes to B, that is,

$13.932-\frac{11.973}{2}=7.945$. And $7.945\times5000=\$39725$. B's interest is

$13.320-\frac{11.973}{2}=7.33250$, and the latter sum multiplied by $5000=$

36662.50 We have, therefore, A's share = $39727.50. B's share = $36662.50 and the whole value $a_{20}+a_{30}-a_{20:30}=\$76390.00$, the sum of the interests of A and B.

48. What would be the present value of the interests of A and B in the annuity of the preceding problem on the supposition that the payments are not to begin until the death of one of the nominees.

SOLUTION: The values of the separate interests of A and B are found by the method illustrated in Article 46. A's interest being $(13.932-11.973)\times5000$ $9795.00 and B's being $(13.320-11.973)\times5000$, $6735.00. The whole being $(a_x+a_y-2a_{xy})\times5000=\$16530.00$.

49. Let it be required to find the present value of an annuity of $1500.00 bequeathed to A and B jointly subject to a life interest in C, that is, to be entered upon at the death of C.

SOLUTION: So long as C lives he receives the annuity and A and B get nothing. Also when either A or B dies, the annuity expires. Hence, the value of the interest of A and B is $a_{xy}-a_{xyz}$. That is $(11.973-10.093)\times1500=2820$. Answer, $2820.00.

50. Let the preceding problem be so modified that after the death of C the annuity goes to A and B and to the survivor of them, what then is the present value of their shares of the whole?

SOLUTION: First to find the value of A's share, it is obvious that he has the value of an annuity on his life less the value of the interest of C, the first taker, and of that of his co-nominee, B. This is represented by $a_x-a_{xz}-{}^1/_2a_{xy}+{}^1/_2a_{xyz}$.

From the figures derived above, we have, $(13.932-11.189-{}^1/_2(11.973)+{}^1/_2(10.093))\times 1500=2704.50$. B's share is $13.320-10.982-{}^1/_2(11.973)+{}^1/_2(10.093)\times 1500=2097.00$. The whole value is $2704.50+2097.00=4801.50$. Answer, $2704.50 $2097.00 and $4801.50.

51. Let the problem be so modified that A and B enjoy the annuity until the death of both, when it shall go to C. What then would be the present value of the interest of C?

Solution: So long as A or B live, C gets nothing. His annuity is therefore one on his life less the value of the annuity during that part of his life passed in connection with the lives of A and B and the survivor of them. That is, $a_z-a_{z:\overline{xy}}$, which is equivalent to $a_z-a_{xz}+a_{yz}-a_{xyz}$ for which we have the values $(12.291-11.189+10.982-10.093)\times 1500=2986.50$. Answer, $2986.50.

52. Again let us now suppose that the annuity goes to all and to the survivors and to the last survivor, what is its present value?

Solution: This value is expressed by the formula $a_{\overline{xyz}}=a_x+a_y+a_z-a_{xy}-a_{xz}-a_{yz}+a_{xyz}$. For these, we have derived the values $13.932+13.320+12.291-11.973-11.189-10.982+10.093=15.492$ which, multiplied by 1500, gives 23238. Answer, $23238.00.

53. What would be the present values of the several shares of the three nominees in the annuity of the last problem?

Solution: First take A's interest. During the period preceding the first death he receives one-third of the annual payments, which is $a_{xyz}\times 500$. If he be one of the survivors, he will then receive $a_{xy}\times 750$ or $a_{xz}\times 750$. The value of which is $750(a_z|_{xy}+a_y|_{xz})$. If he again survives, he will receive the whole annuity during his life. The value of this contingent benefit we have seen is $a_{yz}|_x$. These several values may be reduced to the form $a_x-{}^1/_2(a_{xy}+a_{xz})+{}^1/_3a_{xyz}$, the values of which are $13.932-{}^1/_2(11.973+11.189)+\frac{10.093}{3}\times 1500=8573$.

In the same way we find B's share to be $13.320+{}^1/_2(11.973+10.982)+\frac{10.093}{3}\times 1500=7810.25$ and C's, $12.291-{}^1/_2(11.189+10.982+\frac{10.093}{3})\times 1500=6854.75$. The sum of these values is 23238, as by Article 52, we have found it should be.

54: Again, let us suppose the annuity of the preceding sections to be the income of an estate valued at $25000 which is enjoyed by all and the survivors, the whole estate to go to the last survivor. What is the present value of the annuity and of the remainder?

Solution: The present value of the whole of course, is $25000. We may find the required values by regarding the estate as an in

surance payable to the last survivor on the death of two of the beneficiaries and subtracting its present value from $25000, or we may treat the income as an annuity terminating on the second death and subtract its value from $25000. The remainder in the first instance would be the value of the annuity, and in the second instance, it would be the value of the insurance. Pursuing the latter course, we find the value of the annuity to be $a_{xy}+a_{yz}+a_{xz}-2a_{xyz}$. The values of which are $11.973+11.189+10.982-2\times10.093=13.958$, which multiplied by 1500 gives $20937 as the value of the annuity. The value of the remainder would then be $25000-20937=4063.00$.

55. Let it now be required to find the present value of the share or interest of A in the estate mentioned in the preceding problem.

Solution: The problem may be divided so as to present two branches. First we may find the value of the interests of each in the annuity (income) in accordance with the principles employed in Articles 51 and 54. First to take A's interest. This will be one-half of an annuity on his life in connection with both B and C for the whole period of the annuity less the shares of B and C in the joint annuity on the lives of all three, that is, less two-thirds of the latter. That is, A receives $\frac{1}{2}(a_{xy}+a_{xz})-\frac{2}{3}a_{xyz}$. The values are respectively 11.973, 11.189 and 10.093. Solving, we get for his share in the annuity $7278.00 on an annuity of $1500.00. If we add to the value of the annuity thus found the value of his interest in the insurance as found at Art. 118 in the Chapter on Insurance, $2965.00, we will have $10243.00 as the value of A's interest. The other interests may be valued in the same way.

CHAPTER X

Of Life Insurance

The following sections will be devoted to the discussion of methods and rules for computing the values of premiums, benefits, and so forth, of various forms of life insurance contracts.

1. The **Premium** as applied to life insurance is the consideration paid or promised for the insurance. It may be paid in a gross sum called a **single premium,** at the beginning of the transaction, or, as is more common, in annual installments, continued through life, or through a specified term of years, called the **Annual Premium.**

2. There is a distinction to be noted in premiums at the beginning of this discussion. The premiums written in the insurance contract are called the **Gross or Office Premiums** and may be greater or less than the mathematical value of the benefit promised in the policy though in practice they should be more.

3. The premium which at the outset is of the same mathematical value as the insurance promised in the policy, the value of the premium and benefit being both computed on the same mortality table and at the same rate of interest, is called the **Net Premium.** Examples of net single premiums on an insurance of 1 at various rates of interest are given in Table No. XLIII.

4. The premiums discussed here, unless otherwise stated, are net premiums and are based upon the value of the insurance contracted for. In an introduction to this part (Chapter 8), it was shown how the present value of an insurance is found. This value is the smallest sum for which the insurance can be sold without loss and from it the net annual premiums are derived, indeed it is called the **net single premium.**

5. The net value of an insurance of $1000 on the life of a person aged 35, computed on Actuaries table and 4%, is $340.60. That is assuming the mortality rate shown by that table, $340.60, improved at 4% compound interest, it will amount to exactly $1000 *on the average*, at the end of the year in which the death of a person aged 35 will occur. An insurance company may therefore sell an insurance of $1000 at that price and meet its obligation (leaving out of consideration the question of expenses). But it is seldom desirable to pay all this premium at once, and the companies are willing to accept a whole life or term annuity of equal value in lieu of the single premium. If we divide $340.60 by the value of an annuity due of $1.00 at the same age, and rate, and on the same table we will have a life annuity of value equal to the single premium. The value of an annuity due at age 35 is 17.144 and the annual premium, computed as last suggested, is

$19.87. This latter annuity improved at 4% and accumulated during the life of a person aged 35 will also, *on the average*, provide the $1000 insurance at death. From the foregoing, is derived the formula for **net annual whole life premiums:**

$$P_x = \frac{A_x}{1 + a_x} \qquad (1)$$

P being the symbol of an annual premium.

6. As an example, let it be required to find the net annual premium of an Ordinary Life policy of $1000 issued to A, aged 40 years, computed on the Actuaries table, and 4%.

First Method: If we have tables of single premiums and life annuities such as tables 43 and 42, we simply apply the formula of Article 5, $P_{40} = \frac{A_{40}}{1 + a_{40}} = 381.03 \div 16.093 = \$23.68$, which is the required premium.

Second Method: If we have a table of whole life annuities only, we may proceed on the following principles: A life annuity of 1 pays 1 at the end of each whole year the life survives, but pays nothing the year of the death. Its value, as we have seen is a_x. An insurance of 1 pays 1 at the end of the year of the death. If we had an annuity which paid 1 each year the life *begins* it would include 1 for the year of the death, that is, the insurance. The value of such an annuity is obviously $(1 + a_x)$, discounted one year or $v(1 + a_x)$. Now if we deduct a_x from the latter value, all will be cancelled except the year of the death, which is the insurance. As a formula, this is,

$$A_x = v(1 + a_x) - a_x. \qquad (2)$$

$A_{40} = (16.093 \times .961538) - 15.093 = .38103$, which, multiplied by 1000 gives $381.03. Again, this, if divided by $(1 + a_x)$ gives $23.68, as before.

Third Method: If the insurance were 1 and payable at the beginning of the transaction its value of course, would be 1 but it is not payable until the end of the year of A's death. Hence, if A should pay his 1 down at the beginning, it would pay his insurance at the end, but he would lose the annual interest upon it during his whole life The value of this interest making it payable in advance is $d(1 + a_x)$. Now if this value be deducted from the 1, the remainder will represent the value of the 1 payable at death, that is, of the insurance. Hence we have the formula $A_x = 1 - d(1 + a_x)$ (3)

$A_{40} = 1 - (.038462 \times 16.093) = .381032$. Again this multiplied by $1000 gives $381.03, which, being divided by 16.093, gives $P = \$23.68$, as before. In this formula the character d is used to represent the rate of discount the interest rate being given.

Fourth Method: Suppose we have only a table of single premiums from which to find the annual premium. In this case, we may suppose

that A contracts his insurance of 1 but defers paying the premium until his death, when it is to be deducted from his insurance. He, of course, must pay interest on the single premium each year in advance, which is dA_x. For this annual payment his beneficiary receives $1-A_x$. Hence, dA_x is the annual premium for an insurance of $1-A_x$. And by proportion, $P_x:dA_x::1:1-A_x$, hence

$$P_x = \frac{dA_x}{1-A_x} \tag{4}$$

$P_x = .038462 \times .381032 \div (1 - .381032) = .02368$. This multiplied by $1000 gives the annual premium, $23.68, as before.

7. The four formulas just illustrated provide methods for finding the annual premium in any case where an annuity table or single premium table or both are available. They may be transformed algebraically so as to develop others almost as useful but the ones given are sufficient.

8. Commutation columns afford the most convenient instruments for computing premiums, annuities and etc., and those required in this country are published in this book at those rates which are most usual. See tables 33 to 39, inclusive.

9. The formula for single premiums has already been given, thus:

$$A_x = \frac{M_x}{D_x} \tag{5}$$

10. By Article 5, $P_x = \frac{A_x}{1+a_x} = \frac{A_x}{1} \times \frac{1}{1+a_x} = \frac{M_x}{D_x} \times \frac{D_x}{N_x - 1} = \frac{M_x}{N_x - 1}$

That is, $$P_x = \frac{M_x}{N_x - 1} \tag{6}$$

11. Solving the problem of Article 6 by this formula we have, $6242.419 \div 263643.62 = .02368$ from which P_{40} for $1000 can be found as before.

12. If we have only the D and N columns, we may still find the annual premium by the formula

$$\frac{D_x}{N_x - 1} - d. \tag{7}$$

13. Solving the same problem by this formula, we have $(16382.558 \div 263643.62) - .038462 = .062139 - .038462 = .023677$, which, multiplied by 1000, gives $23.68, as before.

14. If it should happen that the computer has no data at all except a mortality table, or if the premium is desired for an insurance at a rate for which no auxiliary tables are available, resort may be had to Lubbock's formula, (Articles 16-18, Chapter III, title "Of Series") for finding the single premium. From this, the annual premium may be found by formula 4, Article 6, of this chapter.

15. The annual premiums just considered all relate to **Ordinary Life Policies,** that is, policies payable at death whenever it may occur with equal annual premiums payable during the whole period of the life. It often happens, however, that all of the premiums are contracted to be paid during a limited term of years. Since the benefit is the same, its present value or net single premium is the same as in the ordinary life policy and the annual premiums for the limited premium-paying term must therefore be larger in order to be of equal value. It can not be impressed too strongly that whatever the form of the policy the present value of the benefit and the present value of the premiums contracted must balance at the inception of the contract.

16. The **net annual premium of a limited payment policy** is found by dividing the net single premium by the value of a term annuity due on the same life, mortality table and rate of interest for the premium-paying period. As a formula this may be expressed thus:

$$nP_x = \frac{A_x}{1 + a_{x\overline{n-1}|}} \tag{8}$$

17. As an example, let us suppose that our policy already discussed provides for annual premiums limited to ten years. Here, the single premium 381.03 must be converted into a ten-year term annuity with payments in advance. The 10-year annuity due at age 40 is 1 plus a nine-year term annuity on the life of A. $381.03 \div 8.0499 =$ \$47.33 which is the annual premium required.

18. By the commutation columns, the method is equally simple We have for the benefit side

$$A_x = \frac{M_x}{D_x}$$

and for the payment side,

$\frac{N_{x-1} - N_{x+n-1}}{D_x}$. Dividing the former by the latter, we have,

$$nP_x = \frac{M_x}{N_{x-1} - N_{x+n-1}}. \tag{9}$$

19. Solving our problem by this formula, $nP_x = 6242.419 \div (263643.62 - 131765.62) = .04733$. Which, multiplied by 1000 gives \$47.33, as before.

The methods of computing single premiums and term annuities already presented in this and the preceding chapter are sufficient to enable the reader to compute the annual premium for any level premium limited payment life policy. Such policies are often called "Ten pay" or "Twenty pay", etc. "Life Policies."

20. A policy in which the benefit promised is payable only in case of death within a limited period of time is called a **Term Insurance Policy.** The insurance guaranteed is called **Term** or **Temporary Insurance.**

21. In the case of term insurance also, the benefit side and payment side must, at the inception of the contract, exactly balance each other. This gives us the clue for finding the annual premiums. We first find the present value of the benefit and equate it against an annuity of equal value.

22. First, to find the value of the benefit, the net single premium for term insurance, $|nA_x$: If the term was for only one year, the value of an insurance of 1 would be $\frac{d_x v}{l_x}$. If the insurance covered the second year only, its value would be $\frac{d_{x+1} v^2}{l_x}$ and so on for the term and the value of all the n years, that is, $|nA_x$ would be:

$$\frac{vd_x + v^2 d_{x+1} + \ldots v^n d_x + n}{l_x} \qquad (10)$$

23. If both numerator and denominator of this series be multiplied by v^x, the value will not be changed and the numerators will become $d_x v^{x+1} = C_x$; $d_{x+1} v^{x+2} = C_{x+1}$; $d_{x+2} v^{x+3} = C_{x+2}$, etc., and the denominator D_x, as we have already learned, and the series may be written:

$\frac{C_x + C_{x+1} + C_{x+2} + C_{x+n}}{D_x}$. The numerator is that portion of the summation of C which produces M, which extends to M_{x+n}. Hence we have the formula,

$$|nA_x = \frac{M_x - M_{x+n}}{D_x} \qquad (11)$$

24. Formula 10 of Article 22 will enable the computer to get $|nA_x$ by means of the mortality table alone with little labor if the term is short, but if he has a long term an approximate value may be found by the formula for finding the nth term, which has been sufficiently explained. Or, he may employ Lubbock's Formula computing a sufficient number of equidistant values of one year insurances to give at least second differences of both u_0 and u_m. The formula for this purpose is as follows:

$$|nA_x = n(u_0 + u_1 + \ldots + u_{m-1}) + \frac{n-1}{2}(u_m - u_0) - \frac{n^2-1}{12n}(\triangle u_m - \triangle u_0) + \frac{n^2-1(\triangle^2 u_m - \triangle^2 u_0)}{24n} \div \text{ etc.} \qquad (12)$$

25. Having found the net single premium for a temporary insurance, we have but to find as the annual premium, a term annuity of equal value, which is done by dividing the single premium by the

value of a term annuity due of 1 for the term during which the annual premiums are payable. The formula is,

$$|nP_x = \frac{|nA_x}{1 + a_{x\ \overline{n-1}|}} \tag{13}$$

26. By substituting for the numerator and denominator of the last number of equation 13, their equivalents from the commutation columns, and simplifying the result, we have:

$$|nP_x = \frac{M_x - M_{x+n}}{N_{x-1} - N_{x+n} - 1} \tag{14}$$

27. **A Deferred Insurance** is one which does not begin until after the lapse of some period of time. Its value at the time the risk begins would be computed as an ordinary life policy at the deferred age, but this value would not represent the *present* value or net single premium for the insurance, because it is not now due and must be discounted to the present date and because further, the insured may die during the intervening term, and liability never attach. The value must be reduced also by this probability. From this we see that the present value or single premium may be found by first finding the value of the insurance at the time it begins, A_{x+n}, and multiplying this by

$$\frac{v^n l_{x+n}}{l_x} \text{ or } v^n np_x. \quad \text{That is, } n|A_x = A_{x+n} \cdot v^n np_x. \tag{15}$$

28. By the commutation columns, the equivalent formula is

$$n|A_x = \frac{M_{x+n}}{D_x} \tag{16}$$

29. If the annual premiums are payable during the whole life, on principles already explained, the annual premium would be,

$$n|P_x = \frac{n|A_x}{1 + a_x} \tag{17}$$

30. The equivalent of the last member of equation 17 may be stated in commutation symbols by substituting the formulas for the numerator and denominator and simplifying the result whence we have

$$n|P_x = \frac{M_{x+n}}{N_{x-1}} \tag{18}$$

31. If the premiums are limited to any number of, say, n years, we have:

$$nPn|A_x = \frac{n|A_x}{1 + ax\overline{n-1}|} \tag{19}$$

32. The equivalent of the last formula stated in commutation symbols is

$$ {}_{n}P_{n}|A_x = \frac{M_{x+n}}{N_{x-1} - N_{x+\overline{n-1}|}} \qquad (20) $$

33. It may be remarked also that the value of a deferred insurance may be found be deducting from the value of a whole life insurance the value of a temporary insurance for the term intervening between the date of the insurance and the beginning of the risk. The following equations hold:

$$ A_x = {}_{n}|A_{x+}|{}_{n}A_x; $$
$$ A_x - {}_{n}|A_x = |{}_{n}A_x \text{ and } A_x - |{}_{n}A_x = {}_{n}|A_x. $$

Also that the approximate formulas applicable to whole life insurances may be used in computing the values of deferred insurances, making the proper discounts for present value and probability of living as explained in Article 27.

34. It may be helpful to give in this section, numerical examples illustrating the use of each of the last eleven formulas. Using our same $1000 policy on A.

(10). $ {}_{n}A_x = \frac{vd_x + v^2d_{x+1} + \ldots v^nd_{x+n}}{l_x} $ Let n = 5 years.

	v.	dx.	lx	Ax
1.	.961538	× 815 = 783.647	÷ 78653 =	.00996.
2.	.924556	× 826 = 763.683	÷ 78653 =	.00971.
3.	.888996	× 839 = 746.868	÷ 78653 =	.00949
4.	.854804	× 857 = 732.567	÷ 78653 =	.00932.
5.	.821927	× 881 = 724.118	÷ 78653 =	.00921.
		3750.883	÷ 78653	.04769

The last sum multiplied by 1000 gives $47.69 as the net single premium for an insurance on A, aged 40, for a term of five years, by Actuaries table and 4 per cent. The first line by the figures at the right shows the present value of an insurance for one year. The second line the present value of an insurance for one year deferred one year. The third line the present value of an insurance for one year deferred two years, and so on. The method is of not much practical value but shows lucidly the principle underlying life insurance, for by it the value of an insurance for a whole life, or any part of a life, may be computed. If a term insurance is desired, the sum of the insurances for the years of the desired term will be the value; if a deferred insurance is desired, the sum of the years from the time the risk begins is the value; if a deferred term insurance is desired, it is only necessary to follow along the years until those comprising the term are reached and their sum will be the value.

(11) $nA_x = \frac{M_x - M_{x+n}}{D_x}$ Let n = 20 years.

Then, $(6242.42 - 3189.47) \div 16382.56 = \$186.32$.
The last value is the net single premium for a 20-year term insurance on the same life, table and rate.

(12) Equation 12 will be illustrated in connection with joint life insurances, which see.

(13) $|nP_x = \frac{|nA_x}{1 + a_{x:\overline{n-1}|}}$ Let n = 20 years.

Then $|nA_x = \$186.32$. $a_{x\overline{19}|} = 11.710$ and we, by substitution, have $|20P_{40} = \frac{186.32}{12.710} = \$14.66$.

That is, $14.66 is the net annual premium for a 20-year term policy on the life of our subject A, by Actuaries table and 4%, the premiums to be paid annually during the term.

(14) $|nP_x = \frac{M_x - M_x + n}{N_{x-1} - N_{x+n-1}}$. Let n again be 20 years and from the Commutation table, we have as the second member of the equation $(6242.42 - 3189.47) \div (16382.56 - 5320.82) = \$14.66$.

(15) $n|A_x = A_{x+n} \cdot v^n np_x$. Let n again be 20 years. $A_{x+n} = A_{60} = 599.43$; $v^{20} = .456387$ and $20p^{40} = l_{60} = 55973 \div l^{40} = .78653$ and we have $599.43 \times .456387 \times .71160 = \$194.68$. The latter sum then is the net single premium for $1000 insurance on the life of A deferred 20 years.

NOTE: By article 33 *supra* the sum of this and the 20-year term found above, $186.32, should equal the whole life premium on A which we have found is $381.03, and so it does.

(16) $n|A_x = \frac{M_{x+n}}{D_x}$. Let n again be 20 years. $M_{60} = 3189.47$, $D_{40} = 16382.56$, then $\frac{3189.47}{16382.56} = .194687$, which multiplied by 1000 gives $194.69.

(17) $n|P_x = \frac{n|A_x}{1 + a_x}$. Let n again be 20 years. $n|P_x = \frac{194.69}{16.093} = \$12.10$.

The latter sum is the annual premium for an insurance on the life of A, deferred twenty years, the premiums being payable during life.

(18) $n|P_x = \frac{M_{x+n}}{N_{x-1}}$. Let n be 20 years, as before, then

$20P_{40} = \frac{3189.47}{263643.62} = .0120976$, which multiplied by 1000 gives $12.10 as before.

(19) $nPn|A_x = \frac{n|A_x}{1 + a_{x\ n-1}}$. Let n be 20 years as before, then

$20P_{20}|A_{40} = \frac{194.69}{12 \cdot 71} = \$15.32$, which is the annual premium for the deferred insurance of the last problem, the premium-paying period being limited to twenty years.

(20) $nPn|A_x = \frac{M_{x+n}}{N_{x-1} - N_{x+n-1}}$. Let n again be 20 years, then from the commutation table, we have,

$\frac{3189.47}{263643.62 - 55414.91} = .01532$, from which the premium for 1000 may be found by simple multiplication as before.

(35.) We may again assume that cases may arise when auxiliary tables or some of them are not at hand, and show how the premiums may be found. Thus, suppose we have only the N and D columns. For a *Term Insurance* the following formula has been developed for the annual premium:

$$|nP_x = \frac{D_x - D_{x+n}}{N_{x-1} - N_{x+n-1}} - d. \qquad (21)$$

Taking the example already used, we have

$$20P_{40} = \frac{16382.56 - 5320.82}{263643.62 - 55414.91} - .038462 = .014661,$$

which multiplied by 1000, gives the annual premium, $14.66.

(36) Employing the N column alone, this formula may be used:

$$|nP_x = v - \frac{N_x - N_{x+n}}{N_{x-1} - N_{x+n-1}} \qquad (22)$$

These values taken from the tables are

$$.961538 - \left(\frac{247261.06 - 50094.09}{263643.62 - 55414.91}\right) = .014661.$$

From which the annual premium for 1000 is derived as before.

37. In some forms of policy, the consideration is made what is called a **Natural Premium.** That is, the annual cost of each year's insurance is charged. This cost increases with the age of the policy holder. In the example supposed, the simplest way of finding these premiums would be to look into that column of the mortality table showing the probability of dying each year and follow down to age 40. It will then appear that the probability of a person aged 40 dying during one year is .010362. From which we learn that 10.36 persons out of 1000 will die during the year of that age, requiring \$10362.00 to pay each of the beneficiaries of the deceased persons \$1000, which will be \$10.36 for each of the 1000 insured persons. The latter sum is based on the further supposition that it is either paid at the end of the year or that money bears no interest. If paid in advance as is usual, and it bears interest at 4%, the premium would be $10.36 \div 1.04$, or $10.36 \times .961538\ (v) = \$9.96$. That is, it would only be necessary to pay the present value of the annual cost. By the same table, we find the next year's cost to be \$10.61, the 10th year, \$15.94, and so on.

38. Mr. Abb Landis, in his recent book, Life Insurance, has taken the pains to demonstrate that the natural premium plan of insurance has many advantages for those who desire insurance, not as an investment, but for the protection of dependents.

39. It is sometimes desirable to make premiums payable in semi-annual, quarterly, monthly or other fractional parts of a year. Indeed, in the case of what is called **Industrial Insurance,** the payments are often due weekly or bi-weekly. And in the case of **Fraternal Insurance,** the monthly payment is almost universal. The value of these payments are of course susceptible of accurate computation, but in practice, less intricate methods, producing approximately correct results are employed. Some of these will now be discussed and illustrated.

40. We have seen that the value of the premiums, however paid, must equal the value of the insurance. In other words, the payment side of the contract must equal the benefit side. Ordinarily, the whole annual premium is payable in advance, and consequently may be at once placed at interest by the company. If only an installment (fraction) of the annual premium is paid in advance, the interest on the remainder is lost until it falls due, the amount depending on the number of installments. The formula for the annual premium has already been given and is $P_x = \frac{A_x}{1 + a_x}$. Also $P_x(1 + a_x) = A_x$. But in case of fractional premiums instead of multiplying P_x by an annuity due of 1, we have for the factor a fraction of 1 due. Thus $P_x^{(m)} = \left(\frac{1}{m} + a_x^{(m)}\right)$ and for the value of the premiums for the year, we have

$P^{(m)}_x = \frac{A_x}{1/m + a^{(m)}_x}$, in which m represents the number of installments into which the annual premium is divided.

We have seen (Annuities, Art. 50) that an approximate formula for $a^{(m)}_x$ is $a^{(m)}_x = a_x + \frac{m-1}{2m}$, and the above formula may be written:

$P^{(m)}_x = \frac{A_x}{1/m + a_x + \frac{m-1}{2m}}$. Simplifying the right member of this equation, it becomes,

$$P^{(m)}_x = \frac{A_x}{\frac{m+1}{2m} + a_x} \tag{23}$$

41. Taking our familiar example, but assuming the premiums for the year to be made payable in four installments, we have the following result: $381.03 \div [\,(5/8 + 15.093) = (.625 + 15.093)\,] = 24.40$, the quarterly payment being $6.10.

42. Suppose that the installments are payable monthly. The factors then become $381.03 \div (13/24 + 15.903)$ and the value of the annual premium is 24.36. Hence, the monthly payment is 2.03.

43. It is said to be customary to add 4% to the annual rate and divide the result by 2 to obtain the semi-annual premium, and to add 6% and divide by 4 for the quarterly premium. By this method, the quarterly premium would be $6.35 instead of $6.10. The excess charge is probably warranted by the fact that in practice, there is considerable expense incident to collecting, two, four or twelve payments per year instead of one, besides the loss of interest on the deferred installments.

44. When commutation columns computed on the monthly basis are available, the monthly payments for whole life, term and deferred insurances may be found by the formulas already given, dividing the result by 12. The formulas assuming the following form:

$$P^{(12)}_x = \frac{M^{(12)}_x}{12N^{(12)}_{x-1}} \tag{24}$$

$$|_n P^{(12)}_x = \frac{M^{(12)}_x - M^{(12)}_{x+n}}{12(N^{(12)}_{x-1} - N^{(12)}_{x+n-1})} \tag{25}$$

$$_n|P^{(12)}_x = \frac{M^{(12)}_{x+n}}{12N^{(12)}_{x-1}} \tag{26}$$

45. Ordinary commutation columns may be converted into columns on the monthly basis, to which the foregoing monthly premium formulas and others are applicable. Mr Landis suggests the following approximate formulas for making the conversion:

$$D_x^{(12)} = {}^1/_2(D_x + D_{x+1}) \tag{27}$$

$$N_x^{(12)} = \Sigma D_x^{(12)} \tag{28}$$

$$C_x^{(12)} = (1 + {}^1/_2 i)C_x \tag{29}$$

$$M_x^{(12)} = \Sigma C_x^{(12)} \tag{30}$$

46. The Greek letter Σ will be recognized as the symbol for summation, indicating that $D_x^{(12)}$ and $C_x^{(12)}$ are to be summed in the manner with which the reader is already familiar.

47. In the foregoing sections, except 43, the word installment has not been quite accurately used to describe the premiums for fractional parts of a year as they are not installments of the regular annual premium, but are approximate net premiums for the shorter periods, derived from the value of the benefit.

48. A form of insurance called Installment Insurance, Pension Policies, and other names, has come into use. Some of these are mere life or temporary life annuities, or annuities certain engrafted upon an endowment insurance or limited pay life insurance, by the terms of which the value of the policy at the settlement period is employed to purchase the annuity. That is, at settlement the value is paid in annual or other installments instead of at once. In other cases, the contract consists of a deferred annuity. In many cases, the benefit payable at the death of the insured is made payable in installments at the option of the insured. In such case, the face of the policy is usually converted into an annuity certain or a temporary life annuity on the life of the insured, of equal value. The sum which at the settlement period is employed to purchase the annuity with any intervening benefit may be treated as the benefit side of the policy, and the premium or payment side determined therefrom in the usual way.

49. An **Endowment Insurance** is one which combines a pure endowment with an insurance covering the term intervening between the date of the contract and the maturity of the endowment.

An **Endowment** or **Pure Endowment** within the meaning of this section is a sum to be paid to a certain person called the nominee, in case he shall be alive at a certain time.

50. As an example of a pure endowment, let us assume that our familiar subject is to receive his $1000 if he be alive at the end of ten years. What would be the present value, or single premium, for this benefit?

First, it is not due in any event for ten years and hence must be discounted at the rate of four per cent for that time. That is v^{10}.

Second, he may not survive the term of ten years and stands this chance of losing the benefit. This chance we have learned is np_x, in this case, $10p_{40}$, or the probability of living ten years at age 40. This value is the same as the present value of the 10th payment of an annuity and is $\frac{v^{10}l_{40+10}}{l_{40}}$ or in general terms, $\frac{v^n l_{x+n}}{l_x}$. Its symbol is E_x and the formula for a pure endowment is therefore,

$$E_{x\overline{n}|} = v^n {}_np_x \text{ or } E_{x\overline{n}|} = \frac{D_{x+n}}{D_x} \tag{31}$$

51. Numerically, the solution is as follows: $E_{40\ \overline{10}|} = (69804 \times .675564 \div 78106) \times 1000 = 603.76$. Or by formula 31, $(9822.30 \div 16268.62) \times 1000 = 603.76$.

52. The value, single premium, for an endowment insurance is $|nAE_x$ and must equal the present value of two benefits. First, of a pure endowment, payable at the end of the term if the insured survives, $\frac{D_{x+n}}{D_x}$ and second of a temporary insurance for n years, $\frac{M_x - M_{x+n}}{D_x}$

The whole present value may then be expressed in the formula

$$|nAE_x = \frac{M_x - M_{x+n} + Dx+n}{D_x}. \tag{32}$$

The symbol for this insurance is also written, $A_{x\overline{n}|}$.

53. Solving our same problem by the last formula, the numerical work is as follows: $(5979.94 - 4586.90 + 9822.30) \div 16268.62 \times 1000 = 689.39$.

As illustrating the explanation given in Art. 52, we may find the value of the insurance alone, thus, $(5979.94 - 4586.90) \div 16268.62 = 85.63$, the value of the insurance feature. This deducted from 689.39, gives 603.76, the endowment feature as found in Article 51. The principle of formula 2 also applies to endowment insurances and we have, $A_{x\overline{n}|} = v(1 + a_{x\overline{n-1}|}) - a_{x\overline{n-1}|}$. Also Formula (3) may be employed taking the form, $A_{x\overline{n}|} = 1 - d(1 + a_{x\overline{n-1}|})$.

54. The formula for the annual premium then on principles already elucidated is,

$$P_{x\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_{x-1} - N_{x+n-1}} \tag{33}$$

55. As an example of the last article, we may find the annual premium of an endowment insurance on the life of our familiar example, making the term ten years, when we have the figures: $(5979.94 - 45869 + 9822.30) \div (267505.97 - 136120.05) \times 1000 = 85.36$.

56. If we have the single premium and term annuity computed, the annual premium is readily found by dividing the single premium by the value of an annuity due of 1 for the payment term of the endowment, the formula being

$$P_{x\,\overline{n}|} = \frac{|nAE_x}{1 + a_{x\,\overline{n-1}|}} \tag{34}$$

57. To find the annual premium in the problem of Article 55 by the last formula, we have $P_{40\,\overline{10}|} = \frac{689.39}{1+7.077} = 85.36.$

58. The endowment feature of an endowment insurance policy does not always coincide in amount with the insurance feature. Thus it is not unusual in this country, and is said to be of frequent occurrence in England, to write policies called **semi-endowment policies.** In such a policy, the temporary insurance is twice the amount of the endowment. In such cases, the two features of formulas 32 and 33 would be treated separately and the results combined or the endowment branch could be divided by two before combining the parts. The formula will then assume the form,

$$\frac{2(M_x - M_{x+n}) + D_{x+n}}{2D_x} \tag{35}$$

If the problem of Article 53 be so changed that the endowment to be paid is only 500, the temporary insurance remaining 1000, the solution is as follows: $[2(5979.94 - 4586.90) + 9822.30] \div (2 \times 16268.62) \times 1000 = 387.51$. If to one-half the value of the pure endowment found in Article 51, that is, of 603.76, we add the single premium of the temporary insurance found in Article 53, that is, 85.63, we have 387.51 as above, proving the correctness of the computations and also illustrating the nature of such contracts. It may be remarked that the benefits of endowment and insurance contracts may be, and in practice often are, of various natures, with premiums payable in various ways. The problems thus presented need only to be analyzed and understood, and the solution, usually will be obvious enough. For instance, the policy may provide for a pure endowment with clauses insuring only the annual premiums, or the temporary insurance may be one-half or some other proportion of the endowment. Again, the premiums may vary in amount, temporarily or throughout the term. Some of these contracts will now be considered.

60. Suppose the endowment is 1000 and the temporary insurance 500, that is, one-half the endowment. The insurance is $\frac{M_x - M_{x+n}}{2D_x}$ and

the endowment $\frac{D_{x+n}}{D_x}$. Reducing to a common denominator and adding, we have the whole benefit, $\frac{M_x - M_{x+n} + 2D_{x+n}}{2D_x}$

61. If the annual premium for the last benefit be required, we may substitute $N_{x-1} - N_{x+n-1}$ for D_x and the form of the equation becomes:

$$ {}_nP_x : \overline{\underline{n}} | = \frac{M_x - M_{x+n} + 2D_{x+n}}{2(N_{x-1} - N_{x+n-1})} \qquad (35a) $$

62. Formula 35 may be converted into a formula for the annual premium in the same way.

63. Suppose the annual premiums are a certain sum, say for five years and twice said amount for the remainder of life. What is the net annual premium? In this, as in all cases, the benefit side and payment side of the contract must be equal. The value of the insurance is $A_x = P[(1 + a_x) + 4 | a_x]$ or in commutation columns, we may write, the benefit side, M_x and the payment side, $P_x(N_{x-1} + N_{x+5-1})$, from which we have the formula, (using π as the symbol for a special premium

$$ \pi = \frac{M_x}{N_{x-1} + N_{x+n-1}} \qquad (36) $$

64. Supposing our usual subject A to contract such an insurance, we have the numerical result, $5979.94 \div 267505.97 + 193619.95 = .01297 \times 1000 = 12.97 = \pi$ and $2\pi, 25.94$. The level net annual premium for age 40 by American Experience table and 4% is 22.35 per 1000 and at age 45, by the same standard the net annual premium is 27.12.

65. As another example illustrating this class of insurance, let it be required to find the net annual premium of a policy of $1000 issued to A aged 43 years, the gross annual premium being $18.27 for five years and 35.40 for the rest of life, computed on Actuaries table and 4%. The ratio between the initial and final premium instead of being 1 to 2 is 1 to 1.938. Turning now to the commutation columns, we have $M_{43} = 5764.77$, $N_{42} = 216841.25$, $N_{47} = 152991.67$. The latter multiplied by .938 gives 143506.19. Adding the last number to N_{42}, we have for the divisor, 360347.44. Performing the division and multiplying by 1000, we have $\pi = 16.00$ and for the life premium $16.00 \times 1.9376 = 31.00$. The annual net level premium at age 43, by this table, is 26.58 and at age 48, 32.77. The single premium at 43 is 408.71 and the value of these premiums must equal the latter sum. A life annuity due $(1 + a_x)$ at age 43 is 15.374). This multiplied by $16.00 = 245.984$. A life annuity due, deferred five years at age 43, is 10.847 and this multiplied by $15.00 = 162.705$.

$245.984 + 162.705$ gives the single premium, 408.69.

66. The principle illustrated in the last article will enable the computer to solve problems when several increases in the premium are to be made at stated intervals. The annual premiums begin with a sum which may be regarded as unity and to this is added deferred annuities due entered upon at the periods of increase. The sum of these divided into the value of the insurance gives the initial premium from which the others may be derived. The same principle is employed in the solution by means of the commutation columns.

A more general formula for the special net premium discussed in the last article is as follows:

$$\pi = \frac{M_x}{N_{x-1} \mp k(N_{x+t-1} + N_{x+2t-1} \&c = N_{x+mt-1}} \qquad (37)$$

in which k represents the ratio of the increase (or decrease) in the premiums, t represents the periods between the changes in the premiums and m represents the whole number of variations which take place in the premiums.

67. If the premiums are unknown, that is, it if is desired to find an appropriate **step rate** as compensation for a given insurance benefit, the following formula has been developed:

$$\pi = M_x \mp \frac{k(N_{x+t-1} + N_{x+2t-1} + N_{x+3t-1} \&c + N_{x+mt-1})}{N_{x-1}} \qquad (38)$$

68. As a numerical example of the last formula, let us take one from Mr. Landis' book, Life Insurance: Assume that the insurance is 1000 for whole life and that the benefit is uniform and taken at age 35, and that the contribution is P′, and increased by 2 for three years and then continues uniform for the rest of life.
The benefit side $1000M_{35}$,

The payment side $= P'N_{35} + 2N_{36} + 2N_{37} + 2N_{38}$,
Transposing so as to let $P'N_{35}$ stand alone, and dividing both members by N_{35} we have $P'_{35} = \frac{1000M_{35} - (2N_{36} + 2N_{37} + 2N_{38})}{N_{35}} =$

$$\frac{7049683.60 - 2(400925.92 + 378594.17 + 357256.75)}{424294.62} =$$

$$\frac{7049683.60 - 1136776.84}{424294.62} = 11.25$$

The values used are taken from N. F. C. tables at 4% published in this book, where, according to American usage $N_x = N_{x-1}$ of tables arranged in accordance with the recommendation of the International Congress

of Actuaries. The N columns of that table have been rearranged in the present work to conform to the Congress rule followed in the other tables. The initial rate above found pays the first premium. The second, by the problem is 13.25, the third 15.25 and the last 17.25, which remains level throughout the life.

69. It occasionally happens that an insurance contract provides for a **Varying Benefit** in one respect or another. That is, the amount paid at death differs at different periods in the life of the policy. More frequently this occurs where the annual premiums as well as the face of the policy are paid in case of death during the premium-paying period. In some instances, annual additions, or additions to be made at a settlement period or at death are provided for. Some of these features may be treated by means of the R. commutation columns. Others have to be treated by special methods which are usually indicated by the nature of the problem. Some examples will best develop the methods of solving these problems.

70. An **Increasing Insurance,** the symbol of which is IA_x is one which increases by some law usually by its own amount. Thus, by commencing at 1 and increasing by 1 each year. In the case of insured annual premiums, the initial insurance would be P increased by P each year. In the first case, the insurance provided expressed in commutation columns would be

$$\frac{M_x}{D_x} + \frac{M_{x+1}}{D_x} + \frac{M_{x+2}}{D_x} \text{ etc., to end of table.}$$

That is, we have an insurance of 1 for life, to which is added the next year a deferred insurance of 1 for life and so on. If we sum the M values beginning with the oldest age and continuing to the youngest, recording the sums, we will have a column which is called R, which would have the quality of giving the value, single premium, for an increasing insurance beginning at the age taken and extending through life, when divided by the D for the corresponding initial age. Thus

we have the formula

$$IA_x = \frac{R_x}{D_x} \tag{40}$$

71. Suppose a policy has for its consideration 20 equal annual premiums of 37.70 which are to be returned in case of death during the premium-paying period. What is their present value or net single premium, the insured being 40 years of age, computing by Actuaries table and 4%? If we write the number $R_x - R_{x+n}$ as the numerator just as in case of an ordinary temporary insurance, it is clear that the result will not be correct because 20 insurances of 1 will still remain uncancelled and it will therefore be necessary to deduct $20M_{60}$ to cancel these. We therefore have the formula:

$$IA_{40\ \overline{20}|} = \frac{R_{40} - R_{60} - 20M_{60}}{D_{40}}$$

which is numerically $(133639.96 - 37833.97 - 20 \times 3189.47) \div 16382.56 = 1.9543$. The latter sum must be multiplied by 37.70 which gives the single premium sought, 73.68.

72. Let us now find the net single premium for the whole benefit of the insurance of Article 71, including the return of the premiums in case of death during the 20 years premium-paying period. We must have $1000\ (M_{40} \div D_{40}) +$ the value of the return premises as already found. This is

$$\frac{1000\ M_{40}}{D_{40}} + \frac{37.70(R_{40} - R_{60} - 20M_{60})}{D_{40}} \qquad (42)$$

or $[1000\ M_{40} + 37.70(R_{40} - R_{60} - 20M_{60})] \div D_{40}$. The values of M and R may be substituted from the table and the result obtained by the arithmetical processes indicated by the signs, will be $455.32.

73. The net annual premium for the policy will be found by dividing the last sum by a 20-year annuity due on the same life, 12.71 giving as the net annual premium, $35.82.

74. The symbol and formula for an insurance of a certain sum increasing (or decreasing) each year by another sum and thence remaining constant during the remainder of life are sometimes written as follows:

$$(I_{\overline{n}|}A_x) = \frac{kM_x \pm h(R_{x+1} - R_{x+n})}{D_x} \qquad (43)$$

In this formula, k represents the initial sum insured and h the annual increase. It differs from the preceding formula in that instead of cutting the insurance down to the initial sum after n years, only the future increase is cut off the amount of insurance remaining at the point reached at age x+n.

75. For an ordinary life policy with return of the net annual premium, along with the capital benefit, the formula may be written as follows:

$$\pi = \frac{M_x}{N_{x-1} - R_x} \qquad (44)$$

76. The benefit secured by the policy of the last article is A_x, being the value of the capital insurance and $\pi(IA_x)$, that is an increasing insurance of 1 multiplied by the net premium, π. The net annual premium would be found by dividing the sum of these values by $1 + a_x$. The equivalent of these is, in terms of the commutation columns,

$$\pi = \frac{M_x + \pi R_x}{D_x} \div \frac{N_{x-1}}{D_x} = \frac{M_x + \pi R_x}{N_{x-1}}$$

Multiplying by N_{x-1}, we have
$\pi N_{x-1} = M_x + \pi R_x$. Transposing, we have
$\pi(N_{x-1} - R_x) = M_x$ and dividing by the co-efficient of π we have finally,

$$\pi = \frac{M_x}{N_{x-1} - R_x},$$ which is the formula assumed in Article 75.

77. The more usual form is to return the gross annual premiums, but it is sometimes important to find the net annual premium of such a policy for the purpose of valuation. The formula for an ordinary life insurance with return of gross premiums may be written as follows:

$$\pi = \frac{M_x + P'R_x}{N_{x-1}} \tag{45}$$

In this formula, P' is the symbol used for a gross or contract premium.

78. The reasoning on the last form of policy is similar to all in that the benefit side and premium side of the contract must balance at the inception of the contract. That is
$\pi(1 + a_x) = A_x + P'(IA_x)$, which, in commutation column terms, is

$$\pi \frac{N_{x-1}}{D_x} = \frac{M_x}{D_x} + \frac{P'R_x}{D_x}$$ from which we have

$$\pi = \frac{M_x + P'R_x}{D_x} \div \frac{N_{x-1}}{D_x} = \frac{M_x + P'R_x}{N_{x-1}}$$

In applying formula 45, M_x must be given as a co-efficient, the sum on which the gross annual premium is based and the co-efficient of R_x is the gross premium.

79. As an example of the last formula, let it be required to find the net annual premium of an ordinary life policy covering both policy and premiums, the policy being for 1000, on the life of A, aged 40 years, the gross annual premium being 53.24 employing Actuaries table and 4% in the calculation. From the table and formula we get $(6242.42 \times 1000) + 133639.96 \times 53.24 \div 263643.62 = 50.67$. The latter is not a well balanced premium, the annual gross premium having been fixed arbitrarily.

90. Let us suppose the loading of the premium of the last policy to be 25% of the net premium plus a constant addition of 5 per 1000 or, .005 on 1. Representing the percentage by k, the constant by c, and the gross premium by $P' = P(1 + k) + c$ and for the special annual premium, the formula:

$$\pi = \frac{M_x + cR_x}{N_{x-1} - (1+k)R_x} \tag{46}$$

The solution is as follows: $(M_{40} = 6242.42 + R_{40} = 133639.96 \times .005) \div N_{x-1} = 263643.62 - 133639.96 \times 1.25 = 6910.61 \div 96589.10 = .07155$ which, multiplied by 1000 gives 71.55, the net annual premium sought. The gross premium would be found by multiplying 71.55 by 1.25 and adding 5. as indicated by the formula $P(1+k)+C$.

91. A more usual form of policy is one having limited premiums all or part of which are returned in case of death during the premium-paying period. An example and some explanation of one form of the problem presented by such insurances is given in Articles 71 to 74. In that case, an arbitrary gross premium was assumed. Suppose that in the last problem considered, we substitute n annual premiums, the same to be returned in case of death during the premium-paying period, the loadings to be as before. The value of the benefit is first A_x, the single premium for whole insurance, second $\pi(IA)x'\,\overline{n}|$ an increasing term insurance covering the gross annual premiums, which may be written $(\pi(1+k)+c)\,(IA)x'\,\overline{n}|$. To obtain the net annual premium, the value of the sum of these benefits will be divided by the value of an n year annuity due. $1 + a_{x\,n-1}|$. Writing these values in terms of the commutation columns and simplifying algebraically, the following formula is developed:

$$\pi = \frac{M_x + c(R_x - R_{x+n} - nM_{x+n})}{N_{x-1} - N_{x+n-1} - (1+k)\,(R_x - R_{x+n} - nM_{x+n})} \tag{47}$$

92. In case the conditions of the policy were the same as in Article 91, except that the premiums are to be repaid along with the capital benefit at death whenever it may occur, the formula assumes the following form:

$$\pi = \frac{M_x + c(R_x - R_{x+n})}{N_{x-1} - N_{x+n-1} - (1+k)\,(R_x - R_{x+n})} \tag{48}$$

NOTE: In the last two articles the gross or loaded premium would be $P' = \pi(1+k)+c$. It should be further noted that the difference in the two last equations is that only R_{x+n} is cut off of the increasing insurance in the latter, leaving the n premiums paid uncancelled. In the former equation, the whole premium benefit is cancelled at the end of n years by deducting $n(M_{x+n})$.

93. The principles illustrated in the preceding Articles will enable the reader to deal intelligently with the more usual forms of varying benefits. It may be remarked that the commutation columns are adapted as well to the decreasing insurances and annuities as to increasing ones, but there is in practice little occasion for such application.

94. A **Tontine Fund** represents the accumulations of annuities granted at some previous time, but not drawn, and is to be distributed among the survivors of the group. The annuities are in other words, forborne during the tontine period. The formula given for the share of a given annuitant, is:

$$-n|a_x = \frac{N_{x-n} - N_x}{D_x} \tag{49}$$

95. A similar principle is involved in the formula:

$$|-nA_x = \frac{M_{x-n} - M_x}{D_x} \tag{50}$$

which represents the value of the accumulated annual costs of insurance including interest during the term of n years, on the supposition that said annual costs are forborne or not paid.

96. All of the insurances discussed in the foregoing articles are payable at the end of the year in which the death of the insured shall occur. On the assumption that deaths at a given age are uniformly distributed throughout the year, the payments are due six months after death. As a matter of fact, deaths do not occur thus uniformly, but the increase in the force of mortality for so short a period is negligible and is generally disregarded in practice.

97. If it is desired to make the insurance payable immediately after death, the company should be compensated for the loss of six months interest which it would sustain by the earlier payment. That is, it should receive roughly, $A_x(_{1+\frac{i}{2}})$, or, more accurately, $A_x(1+i)^{\frac{1}{2}}$ and for such an insurance, also called a continuous insurance, we have the formula: $\bar{A}_x = A_x(1+i)^{\frac{1}{2}}$. For an insurance payable three months after death, the approximate value would be $A_x(_{1+\frac{i}{4}})$ and for one month, $A_x(_{1+\frac{i}{12}})$ The value of A_x may also be found by the formula, $\bar{A}_x = 1 - d'\bar{a}_x$. Also, conversely, $A_x = \bar{A}_x(1+i)^{-\frac{1}{2}}$. Here d' represents the force of discount.

98. **Joint Life Insurances** are governed by principles similar in most respects to those controlling insurances on single lives, and most of the formulas considered in this chapter may be employed to compute premiums for such insurances by merely substituting the symbols for joint lives in the formula and their values in the equations, thus:

99. $A_{xy} = v(1 + a_{xy}) - a_{xy}.$ (51)

100. $A_{xy} = 1 - d(1 + a_{xy}).$ (52)

101. $P_{xy} = \frac{A_{xy}}{1 + a_{xy}}$ (53)

102. $P_{xy} = \frac{dA_{xy}}{1 - A_{xy}}$ (54)

103. $$A_{xy} = \frac{M_{xy}}{D_{xy}} \qquad (55)$$

104. $$P_{xy} = \frac{M_{xy}}{N_{x-1:y-1}} \qquad (56)$$

105. The computation of joint life commutation tables for all combinations of three or even two lives is an immense undertaking and recourse must be had to methods of approximation. For all kinds of life tables, Lubbock's formula is probably the best since it does not involve as one of its factors the force of mortality. Equidistant values of single year insurances for use in the formula may be computed by the formula:

$$\frac{v^n(1_{x+n:y+n} - 1_{x+n+1:y+n+1})}{1_{xy}}$$

in which n is general representing any number of years. For insurances, the form of the formula is:

$$A_{xy} = n(u_0 + u_n + u_{2n} + u_{3n} +) - \frac{n-1}{2} u_0 + \frac{n-1}{12n} \Delta u_0 - \text{etc.}$$

See Articles 54 and 55, Chapter 4.

But it is obviously less laborious to use either formula 51 or 52 in the calculation of the single premium which requires the approximate value of the annuity only and the same annuity may be employed to find the annual premium, thus, suppose the value of a joint life annuity on three lives aged 30, 36 and 41 is 11.375. Now, by formula 51 $A_{xyz} = v(1 + a_{xyz}) - a_{xyz} = (.966184 \times 12.375) - 11.375 = 11.957 - 11.375 = .572$, which is the single premium. Also,

$$P_{xyz} = \frac{A_{xyz}}{1 + a_{xyz}} \quad \text{and}$$

therefore, $P_{xyz} = .572 \div 12.375 = .04703$, which is the annual premium. For 1000, it, of course, would be 47.03.

106. Illustrating the computation of single and annual premiums by means of Lubbock's Formula, let it be required to find the single and annual net premiums for an insurance of $1000 on the life of A, aged 35, and B aged 45, using American Experience and 4%. We may use formula 51 and here the value of a joint life annuity is the first requisite. First compute the present value of six payments in 10, 20, 30, 40, 50 and 60 years which will carry us to the end of the table. To do this, four columns may be prepared. In the first, write the logarithms of the present value of 1 at four per cent in 0, 10, 20, 30, 40, 50 and 60 years which will carry the younger of the two lives to the end of the table. In the second column, write the logs of l_{35}, l_{45}, l_{55}, l_{65}, l_{75}, l_{85} and l_{95} which carries that life to the end of the table. In the third column, write logs of l_{45}, l_{55} and etc., to l_{95}. Add the logs of l_{35}

and l_{45} and deduct the sum from 10 − 10, that is, take the co-log or arithmetical complement of the sum and place it in the fourth column. Add the four columns across the page forming the 5th column which will be the logs of the payments. Take out the natural numbers, and these will be the payments for the 1st. 10th, 20th, &c. years. Arrange these in a column for differencing. Take out to third differences and sum by the formula. The work described is set out below:

(1)	(2)	(3)	(4)	(5)
0.000000	4.912870	4.870246	.216884	.000000
9.829667	4.870246	4.809984	.216884	9.726781 − 10
9.659333	4.809984	4.693208	.216884	9.379409 − 10
9.489000	4.693208	4.418914	.216884	8.818006 − 10
9.318666	4.418914	3.739177	.216884	7.693641 − 10
9.148333	3.739177	.477121	.216884	3.581515 − 10
8.978000	0.477121	.000000	.216884	9.667005 − 10

	(6)	△	△2	△3
(0)	1.000000	− .466934	+ .173425	− .053706
(10)	.533066	− .293509	+ .119719	
(20)	.239557	− .173790		
(30)	.065767	−		
(40)	.004939			
(50)	.000001			
(60)	.000000			
	1.843330 × 10			18.433300

$$\frac{-n+1}{2}\times 1 = -5.500000$$

$$\frac{n^2-1}{12n}\times -.466934 = -.385221$$

$$\frac{-n^2-1}{24n}\times +.173425 = -.071538$$

$$\frac{(n^2-1)(19n^2-1)}{720n}\times -.053706 = -.014017 - 5.970776$$

$$a35{:}45 \qquad 12.462524$$

Having now the joint life annuity on the two lives, for the single premium according to the formula selected, we have (13.462524 ÷ 1.04) − 12.462524 = .4822, and for the annual premium by the familiar formula, we have: .4822 ÷ 13.4625 = .03582. The latter sum multiplied by 1000, gives the net annual premium for 1000, i. e., 35.82.

107. Columns 1, 2 and 3 above will be recognized as v^n, and the logs of l_{x+n} and l_{y+n}, respectively, and the effect of adding their logs is to take their product, that is, to multiply them together. Adding on the co-log of l_x and l_y is equivalent to dividing the products of 1, 2 and 3 by the product of l_x and l_y. The work may therefore be performed, but with more labor, arithmetically and without resorting to logarithms. By means of the process just illustrated, the premiums may be readily computed with no other data than a life table and a table of v^n.

108. The insurance considered in the last article pays 1000 to the survivor at the end of the year of the death of either A or B. Had the policy provided that the benefit should be paid only to B, that is, that it should be paid only upon the contingency that B should survive A, it would be called **Contingent Insurance** or **Survivorship Insurance.** The symbol assigned it is, A^1_{xy}.

109. To compute such insurance without previously prepared commutation tables requires considerable labor. This book affords facilities for computing joint life insurances at the rate of $3^1/_2\%$ on the American Experience and at 4% on the Actuaries table, on two and three lives, by means of the joint life equal age Commutation and Annuity tables, used in connection with the force of mortality tables in the manner explained in connection with life annuities (Articles 36 - 39, Chapt. IX). By means of formulas already given contingent insurances may be readily derived from the joint insurances.

110. Let it first be required to find the single and annual premiums of a joint life insurance of $1000 on the lives of A, aged 28, and B, aged 40, by the American Experience table at $3^1/_2\%$.

M columns have not been furnished, but we may compute the insurance from the $3^1/_2\%$ annuity table. First, we find the force of mortality, $\mu_{28} = .00821$, $\mu_{40} = .00977$, the average is .00899. That is, greater than the force of age 35 which is .00888 and below 36 which is .00902. The difference of the last two is .00014, and the difference between the average force and the last is .00003. The difference in the joint life annuities at the same ages and rates is by the table, $14.57817 - 14.36078 = .21739$. $^3/_{14}$ of the latter sum is .04667, which must be added to the annuity for age 36:36, and we have the annuity 14.40745. Adding one to make the annuity immediate, then dividing by 1.035 and deducting 14.40745 according to the formula 51, we have for the single premium, .48284 for 1.00 or $482.84 for a policy of $1000 And by formula 53, $P_{xy} = \frac{A_{xy}}{1 + a_{xy}}$, and we have $.48284 \div 15.40745$, .031344 or $31.34, as the annual premium for a policy of $1000.00.

110. Now let the contract be such that the insurance shall be payable only to B and only in case he shall survive A. What then would be the premium?

111. For the latter problem, the following formula has been deduced:

$$A^1_{xy} = \frac{1}{2}\left(A_{xy} + \frac{a_{x-1:y}}{p_{x-1}} - \frac{a_{x:y-1}}{p_{y-1}} \right).$$

The value of p_{27} and p_{39} we get from the mortality table (Table No. X) under the head "Yearly probability of living," and are respectively .99180 and .99041. The annuities for ages 27:40 and 28:39 may be computed from the equal age annuity table as illustrated in article 109, and are respectively 14.57817 and 14.45416. The quotients are 14.69782 and 14.59944. Inserting these values in the formula with the value of $A_{28:40}$ and solving and then dividing the result by two, we have $290.61 as the required single premium. The annual premium would be found by dividing by $1+a_{xy}$ as before.

112. To find the value of a contingent insurance payable to A in case he shall survive B, we have the formula:

$$A^1_{x\overline{y}} = A_{xy} - A^1_{xy}. \qquad (57)$$

It follows that the single premium in this case would be $482.84 − $290.61 = $192.23. The value may be found by a slight modification of the formula of Article 110 in that x and y are transposed so that the formula becomes:

$$A^1_{x\overline{y}} = \left(A_{xy} + \frac{a_{x:y-1}}{p_{y-1}} - \frac{a_{x-1:y}}{p_{x-1}} \right) \div 2$$

In the last two articles, the annuities were taken at the nearest approximate equal age without interpolating for fractional parts of the years and in that respect, are only approximately correct.

113. Suppose an estate is left to an institution contingent upon the death of both A and B, the death of B preceding that of A. That is it goes to B if A dies first, but goes to the institution on the death of A if he survives B. What is the value of the interest of the institution assuming the estate is valued at $100000? The value is that of an insurance on the life of A, less the value of an insurance contingent on his dying before B. Its symbol is:

$$A^2_{x\overline{y}} = A_x - A^1_{xy} \qquad (58)$$

114. To solve the problem of Article 113, we require the single premium on the life of A, aged 28. It is .32512. In Article 111, we computed an approximate value of A^1_{xy}. It is .29062. And by formula 58, $A^2_{x\overline{y}} = .32512 - .29062 = .03450$. Multiplying by 100000, we have as the present value of the interest sought, $3450.00.

115. The force of mortality and regraduated annuity tables may be used to compute contingent insurances on two and three lives directly from the joint life annuity tables. We will propose three problems and state and apply the formulas for their solution as a con-

clusion to this chapter. Three lives will be considered, A aged 20, B aged 30, and C aged 40. The American Experience table of Annuities, No. 28, at 6% will be employed.

116. First, what will it cost A to buy an insurance on the life of B for $25000.00, contingent upon A being alive at the death of B to receive it?

This is a survivorship insurance A_{xy}^{1} and may be solved by the formula:

$$\overline{A}_{xy}^{1}=\mu_y(a_{xy}+{}^1/_2)+{}^1/_2(ax:y_{-1}-ax:y_{+1}) \qquad (59)$$

$\mu_{30}=.00835$ and $a_{xy}=11.973$. Adding 50c it becomes 12.473. $a_{20:29}$ computed by the equal age method is 12.004 and $a_{20:31}$ by the same method is 11.895, and we have $\overline{A}_{xy}^{1}=.00835\times12.473+{}^1/_2(12.004-11.895)=.15865$. Multiplying by 25000, we get $3966.25. If we substitute z for y in the last problem, we would have $\overline{A}_{xz}^{1}=.00977\times11.689+{}^1/_2(11.286-11.082)=.21519$. The insurance is payable immediately after death.

117. Second, What is the value of an insurance payable to A upon the death of B, provided it occurs first of the three lives, A, B and C? For this value, the following formula has been demonstrated:

$$A_{xyz}^{1}=\mu_y a_{xyz}+{}^1/_2+{}^1/_2(a_x:_{y-1}:_z-a_x:_{y+1}:_z) \qquad (60)$$

$\mu_y=.00835$, a_{xyz} we have found, is 10.093. Adding 50c, we have 10.593. We may compute $a_{20:29:40}$ and $a_{20:31:40}$ by the equal age method discussed in Chapter IX, and we find them to be 10.120 and 10.065, respectively. We now have $A_{xyz}=.00835\times10.593+{}^1/_2(10.120-10.065)=.11595$. Multiplying by 25000, we have $2898.75. If we substitute z in the place of y in the last problem, we will have $A_{xyz}^{1}=00977\times10.593+{}^1/_2(10.081-10.010)=.13919$.

118. Finally, let it be required to find the present value of the insurance payable to A upon the contingency that he shall survive both B and C.

For this value, we may use the symbol and formula:

$$A_{\underset{3}{x}:\overline{yz}}=A_{xz}^{1}+A_{xy}^{1}-A_{xyz}^{1}-A_{xyz}^{1}. \qquad (61)$$

We may employ the values found in the two preceding articles and we thus have:

$$A_{\underset{3}{x}:\overline{yz}}=.21519+.15865-.15195-.13919=.11870.$$

This multiplied by 25000, will give the value of the reversion or insurance to A . It is $2965.00.

119. A formula for finding the single premium for an insurance of the form given in the last article is demonstrated in the Text Book of the Institute and is as follows:

$$A_{\underset{3}{xy}:\overline{z}}=\mu_x(a_{zs}-a_{xyz}+\mu y(a_{yz}-a_{xyz})$$
$$+{}^1/_2(a_{x-1:z}-a_{x+1:s}+a_{y-1:s}-a_{y+1:s})$$
$$-{}^1/_2(a_{x-1:y-1:z}-a_{x+1:y+1:z})$$

Contingent insurances involving three lives are not of frequent occurrence and no more space need be devoted to them here.

CHAPTER XI

Of Valuation

Insurance policies as a rule are based upon values derived from some reliable mortality table and computed at a rate of interest which it may be safely expected will be realized from the investment of insurance funds. In this country, $3^1/_2$ per cent is now the most usual rate, while it is not unusual for companies to earn $4^1/_2$, or 5 or even 6 per cent. This leaves a surplus unless taken up by expenses, unfavorable mortality or unfortunate investments. Upon policies so based, in case none of the latter conditions arise, there might be little occasion for frequent valuations so far as the interests of the companies are concerned. But the laws of some of the states require that annual distributions of surplus shall be made, and in many cases the companies issue policies contracting to pay annual dividends. Again, the laws of many, perhaps most, of the states, require companies doing business in the state to make reports to the insurance department showing solvency as a condition precedent to beginning or continuing to do business in the state. These circumstances require the valuation of the contracts of the companies outstanding for the purpose of ascertaining the liabilities of the companies arising on the obligations of said contracts in order that these liabilities may be compared with the resources of the company and its solvency ascertained. The process of ascertaining this liability is called **valuation.** The liability of an insurance company arising upon its contracts has strangely been assigned the name, **reserve,** and this designation taken in connection with one of the methods of **net valuation** sometimes employed, has often been misleading and in some instances, has led the courts into an erroneous and unscientific statement of the laws relating to valuation. The liability disclosed by a valuation of the outstanding contracts of an insurance company is not a reserve in any sense other than the liabilities of a commercial house upon its accounts or bills payable as disclosed by a balance sheet is a reserve. The word, however, is thoroughly inbedded in the nomenclature of the Actuarial profession and when properly understood, will answer as well as another. The text writers on the subject, as a rule, are careful enough to say that the reserve represents the sum which the company *ought* to have on hands as an offset to the obligations arising upon its policies. The Text Book of the Institute uses this form to which there can be no objection, when properly understood.

"**The value of the policy** is thus seen to be the sum which the Assurance Office must have in hand to provide for that portion of the liability under the contract which the future premiums will not cover, and the only source from which it can be derived is the accumulations of the balance of past premiums not absorbed by the risk already incurred."

In some instances, the last clause has been erroneously interpreted to mean that unless the sum required has been collected or accumulated from the balance of past premiums not absorbed, the liability (reserve) does not exist, a palpable non sequitur. Occasion for valuation also frequently arises on settlements on policies granting surrender values and also on lapsed policies under non-forfeiture laws, in force in several states. The theory of valuation will be readily understood from examples and explanations which will now be given.

1. As a standard for valuation, let us adopt for the present the Actuaries table and four per cent interest. Also as an example, let us take a person aged 30 years and $1000.00 as the amount insured, the valuation to be made five years after the date of the policy. For an ordinary life policy what is the net value? The symbol of value is V or nV, the n indicating the age of the policy at the time of valuation.

2. At the inception of the contract, as we have seen, the value of the annual net premiums and the value of the benefit exactly balance. That is, the net single premium is the mathematical equivalent of a life annuity of the annual net premiums. That is, the net single premium, $306.17, is of the same value as a life annuity of $16.97, the net annual premium. At age 35, the value of the insurance has increased to $340.60 the policy now being five years nearer maturity but the annual premiums to be received are now worth only $290.93, because it also has approached nearer to its end. The value of a life annuity due of 1 at age 35 is 17.144 and the value of an annuity due of 16.97 is $16.97 \times 17.144 = 290.93$. It now appears that the value of two sides of the policy no longer balance but the company will in the future receive $49.67 less than the present value of the benefit it is obligated by the contract to provide. The latter sum is the net value of the policy. From this, we derive the formula:

$$nV_x = A_{x+n} - P(1 + a_{x+n}) \qquad (1)$$

3. We may suppose that our subject at age 35 will apply to an insurance company for another insurance in the same amount issued at his then age. The net annual premium would then be 19.87 or 2.90 more than it was when he first applied 5 years earlier. Then the net annual premium, as we have seen, was 16.97. The value of 2.90 per year for life at age 35 is that of an annuity of that sum or $2.90 \times 17.144 = 49.72$. This is the amount of the extra liability on the first policy because of the difference in the premiums. The formula is therefore

$$nV_x = (P_{x+n} - P_x) \quad 1 + a_{x+n} \qquad (2)$$

4. By substituting in formula 1, the values of A_{x+n} and P_x in terms of annuities and transforming the result algebraically, a formula is developed for finding nV_x from the annuities alone. This formula is usually stated as follows:

$$nV_x = \frac{a_x - a_{x+n}}{1 + a_x} \qquad (3)$$

5. Solving the problem of Article 1 by formula 3, we have the following factors and result:
$[(17.040-16.144)\div 18.04]\times 1000=49.72.$

6. When only a table of net annual premiums is at hand, the following formula may be employed.

$$nV_x=\frac{P_{x+n}-P_x}{P_{x+n+d}} \qquad (4)$$

7. Solving our problem by this formula, we have the following factors and result:
$[(.01987-.01697)+(.01987+.03846)]\times 1000=49.72.$

8. The value may be found, using a table of single premiums only by the following formula:

$$nV_x=\frac{A_{x+n}-A_x}{1-A_x} \qquad (5)$$

9. Solving the same problem, taking the single premium for 1 at the two ages the figures are as follows:
$[(.34060-.30617)\div(1-.30617)]\times 1000=49.63.$

NOTE: The five preceding formulas are examples of what is termed the **prospective method of valuation.**

10. If it be assumed that net annual premiums fixed at age 30 have been exacted and accumulated during the years from age 30 to 35 the net value may be computed by a method which well illustrates some of the principles of life insurance. Having the net annual premium, 16.97, paid down in advance the first year we place it at interest at 4 per cent and at the end of the year we have $16.97\times 1.04=17.649$. From the q_x column of the fundamental table at age 30, we get the figures .008425, which multiplied by 1000, gives us 8.425 as the cost of the insurance for one year on the life of our subject. This deducted from 17.649 gives 9.224 as the terminal net value at the end of the first year. This 9.22 is now available to pay death losses and only $1000-9.22=990.78$ is now to be insured. We find q_{31} to be .008578. This multiplied by 990.78 gives 8.49 as the **cost of the insurance** the second year. Add 9.22 to the annual premium 16.97 and increase it by 4%, making 27.24, from which deduct cost as above found and we have 18.75 as the terminal net value at the end of the second year. This course pursued successively will give the net values for each year. The value of one being computed from the previous one. The work is much shortened when the costs have been worked out and tabulated. We may formulate an equation representing this process, using S to represent the sum insured and nV_x the value of a policy issued at age x after n years.

$$n+_1V_x=(1+i)\ (P+nV_x)-q_{x+n+1}\ (S-nV_x) \qquad (6)$$

11. On principles elucidated in Articles 94 and 95 of the preceding chapter, the following retrospective formula may be developed:

$$nV_x = \frac{P(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}} \qquad (7)$$

12. Solving our familiar problem by the last formula, it works out as follows:

$$\frac{.01697(479951.73 - 358785.45) - (8145.75 - 7127.86)}{20927.30} = .04959.$$

Multiplying by 1000, we obtain the net value 49.59, substantially as before. In this problem, we have assumed that both the premiums and the insurance costs have been forborne, that is allowed to accumulate for five years. The excess of the value of the premiums over the value of the accumulated annual costs is, of course, the value of the policy. By this process it is not necessary to have or compute the values of the intermediate years.

13. Formula (7) may be reduced by canceling M_x by $P_x\ (N_{x-1})$, which is its equivalent so that it may be stated in a simpler form, thus:

$$nV_x = \frac{M_{x+n} - P_x(N_{x+n-1})}{D_{x+n}} \qquad (8)$$

14. The work required by the last formula is shorter, requiring but three values from the table and the net annual premium.
$7127.86 - .01697 = 58785.45 \div 20927.30 = (7127.86 - 6088.59$
$1039.27) \div 20927. = .049661$, which gives 49.66 for the 1000 policy.

15. In the case of limited premium whole life policies exactly the same principles govern. If we suppose the insurance we have been discussing is to be paid for by, say 20 annual premiums, we have seen that the annual premium is $A_x \div 1 + a_{x\,n-1}|$, in this case, $306.17 \div 13.087 = 23.39$. After five years, the insurance is worth 340.60, but the fifteen annual premiums which the company is still to receive are only worth 23.39 into a 15-year annuity due at age 35, that is, $23.39 \times 10.848 = 253.72$, which is 86.88 less than the then value of the insurance.

16. The process employed in Article 15 may be expressed by the following formula representing the number of premiums contracted by r.

$$nV_x = A_{x+n} - rP_x(1 + |\overline{r-n-1}\, a_{x+n}) \qquad (9)$$

17. Formula 7 may be employed to solve the problem of Article 15 merely substituting the annual premium for limited payments in place of P_x. This is true because by that formula the annual premiums and mortality costs are simply accumulated and compared to find the value. Obviously, formula 8 can not be so employed because $rP(N_{x-1})$ is not the equivalent of $P_x(N_{x-1})$, and hence, not of M_x.

18. The value of a limited payment policy may also be found by formula 2, since the value of the policy at any year subsequent to its

date is the value of an annuity (in this case a term annuity) of the difference in the premiums for the initial and later ages. Thus $rP_x = 23.39$, $\overline{r-n}P_{x+n} = 31.40$. The annual saving on the last 15 years is therefore 8.01. This sum into a 15-year annuity at age 35, that is 10.848, gives us 86.89 as the policy value.

19. We have seen that an endowment insurance combines a term insurance with an endowment payable in case the term is survived. Let us substitute a twenty-year endowment insurance in place of the contracts just considered. Its value at the beginning is 496.65. The annual premium payable during the term is 37.95. The value of the benefit at age 35 is 582.77. At that time there are 15 payments to be received, the present value of which is $37.95 \times 10.848 = 411.68$. The value of the benefit therefore exceeds the future premiums payable under the contract by $582.77 - 411.68 = 171.04$.

20. The process of the last solution may be represented in a formula as follows:

$$rVP_{x\,\overline{n}|} = A_{x+r:\overline{n-r}|} - P_{x+n}(1 + |n-r-_1a_{x+r}) \qquad (10)$$

$(n = 20,\ r = 5$ in the last problem).

21. Formula 2 may likewise be adapted to the solution of the problem of Article 19, thus:

$$rVP_{x\,\overline{n}|} = (P_{x+\overline{n-r}|} - P_{xn})\ 1 + |n-r-_1a_{x+r}) \qquad (11)$$

22. Let us now suppose that our subject has contracted a twenty-year temporary insurance. What is its value at the end of five years?

Solution: The single premium for the insurance is 129.04 and is equivalent to an annual premium of $129.04 \div 13.087 = 9.86$. The single premium to purchase a 15-year term insurance at age 35 is 115.31. But the value of 15 premiums of 9.86 at age 35, is $9.86 \times 10.848 = 106.96$. The balance of the account then stands in favor of the insured, $115.31 - 106.96 = 8.35$.

23. From the foregoing, we deduce the formula:

$$rV|nP_x = |n-rA_{x+r} - |nP_x(1 + |\overline{n-r-1}a_{x+r} \qquad (12)$$

24. The problem of Article 22 may be solved by the method of formula 2, that is, by finding the present value of the difference in the net annual premiums at the initial age and the age when the valuation is made.

25. For the computation of net values by a continuous retrospective method, a formula was devised by Elizur Wright, a famous American actuary, as follows:

$$\overline{n+1}V_x = u_{x+n}(nV_x + P_x - c_{x+n}) \qquad (13)$$

26. For convenience in applying the foregoing formula, tables of u_x and c_x have been computed u_x being equivalent to $\frac{1+i}{p_x} = \frac{D_x}{D_{x+1}}$ and c_x being equivalent to q_x discounted one year, that is, $\frac{q_x}{1+i}$.

The first year the net value would be $u(P_x - c_x)$. The second year, $1V_x$ would be added to the premium and c_{x+1} deducted and this result multiplied by u_{x+1} and so on. (13) is the general expression of the formula.

27. The problems considered in the foregoing articles relate to the simpler and more usual forms of insurance, but any policy whatever, having fixed premiums and fixed benefits, may be valued by comparing the value of the benefit promised with the value of the premiums. promised. Both values being computed by the same standard of mortality and rate of interest. Conversely, no policy can be valued, however much may have been *paid* upon it in the past in the way of premiums if either the future premiums or the benefit is not fixed and certain. Like an account, both sides must be known before a balance can be struck. The valuation formulas considered all relate to net premiums. That is, the pure premiums derived from the table of mortality and rate of interest employed, without taking into consideration the expenses of transacting the business or profits to the insurer. It may be worth while to add that the processes of net valuation are purely mathematical and do not involve metaphysical or ethical considerations at all. There can no more be an "equitable theory" of net valuation, as has sometimes been supposed by courts dealing with the subject, than there can be of the Binominal theorem or Taylor's Formula

28. The remainder of this chapter will be devoted to the discussion of various problems involving questions of policy values, surrender values and non-forfeiture contracts, first for convenience giving a few definitions.

29. **Net value,** we already understand to be the difference between the present value at the date of valuation, of the benefit insured and the present value at the same date of the future net premiums indicated by the form of the policy. In net valuation *net* premiums are employed and the gross premium otherwise called the office or contract premium is not considered at all except to determine the form of the policy unless it should happen to be less than the net premium. If the gross premium indicates an ordinary life, limited pay life, term, step rate or endowment insurance, the net premiums will be computed on that form of policy, but the amount of the gross premium, if adequate, will not affect the valuation otherwise in any respect.

30. **Surrender Value** is defined in Principles and Practice of Life Insurance as being the equivalent given by an insurance company

to a policy holder on giving up his rights under the policy. Such surrender values are usually payable in one of three forms: (1) Cash; (2) Paid-up Insurance; or (3) Extended Insurance.

31. **Non-Forfeiture** laws or contracts relate to statutory provisions or to clauses in insurance policies which provide that all or some portion of the net value shall not be forfeited to the company in case of the lapse of the policy for failure to pay premiums when due. It was long the custom of insurance companies to provide in their policies that all premiums paid should be forfeited in case of breach of any of the numerous conditions of the policy and by this means the liability of the company, which was in some cases large, was cancelled without consideration to the policy holder, because of his inability or unwillingness to perform some condition or make further payments. It was largely through the initiative of Elizur Wright, that the injustice of this practice was exposed and the legislature of Massachusetts induced to pass a law requiring the company to apply a portion of the net value of the policy to the purchase of extended insurance. Many of the states have adopted the principle of such legislation and either require the companies to incorporate such provisions in their policies as a condition on which they are permitted to do business in the state or else the provision is incorporated in the policy by the force of the statute itself. At the present time, the companies, as a general thing, contract for surrender values in some form, and in many instances these are more liberal than those required by statute.

32. **Paid-up Insurance** is usually understood to mean whole life insurance for which no further premiums are to be received.

33. **Extended Insurance** contemplates the continuation of the insurance written in the policy for such a term as the surrender value applied as a single premium will purchase.

34. A simple method of computing the amount of paid-up insurance which a single premium will purchase is to solve by proportion. Thus, suppose the policy of Article 15 is to be settled by applying three-fourths of the net value to the purchase of paid-up insurance. What would be the amount of paid-up insurance granted? The net value being 86.88, three-fourths would provide a net single premium of 65.16. The net single premium for 1000 at age 35, is as we saw, 340.60 and we have the proportion $340.60 : 65.16 :: 1000 : (x)$. Solving for x we have 191.40 as the face of the paid-up policy.

35. There are various methods of arriving at the term of extended insurance which a given net premium will purchase, one or two which may be used without tables other than those given in this book, will be elucidated.

36. If the single premium is small, that is, if by inspection it appears that the term will be short, a practical method will be to improve the single premium at interest one year, then deduct the cost of one year's insurance which may be easily found by using the q_x

table, then improve the balance and deduct as before until the whole is absorbed. In case a fractional premium remains for the last year, it will purchase such a proportion of a year's insurance as the fraction bears to the cost of a full year's insurance for that year, it being the practice to assume that the death rate is uniform throughout the year. The difference between the age of valuation and the age at the end of the term is the term sought.

37. We have seen that $|nA_x = \frac{M_x - M_{x+n}}{D_x}$. Then $|nA_xD_x = M_x - M_{x+n}$, 1 being assumed as the capital sum insured. If A_x be changed to A'_x one of two results must follow, either the sum insured must be changed or the term must be changed. Let us suppose that the term n is changed to n′, an unknown quantity. We then have $|n'A_xD_x = M_x - M_{x+n'}$. $|n'A_x$ is known, being the given single premium. By transposing the terms of the last equation, we have the equation $M_x - D_x(|nA_x) = M_{x+n'}$ in which the unknown quantity $M_{x+n'}$ stands alone and we may now solve the problem.

38. Let us take the single premium of Article 34 and apply it to purchase extended insurance. First we divide the premium 65.16 by 1000 to reduce it to the basis of unity. Substituting this and the values D_x and M_x in the above equation, we have:
$7127.86 - (.06516 \times 20927.30) = (7127.86 - 1363.62) = 5764.24 = M_{x+n'}$.
Looking now at the table of M, we find 5764.77 opposite age 43 from which we learn that the term is 43−35 or eight years—approximately. It rarely happens that $M_{x+n'}$ falls upon a complete year as in the above case. When it does not, $M_{x+n'}$ is taken to the nearest complete year and we interpolate for the months and days beyond the complete years.

39. As another example, let it be required to find the term of the extended insurance which the net value found in Article 2 on the life considered in that article will purchase. On an insurance of 1, the single premium would be .04967. Employing this in the formula, we have: $7127.86 - (.04967 \times 20927.3) = 7127.86 - 1039.46 = 6088.40$. Looking down the M_x column, we find 6079.19 as the largest number within the last number. This is opposite age 41. Taking the difference of these numbers, we have 9.21 excess to be considered. The next number above M_{41} is 5920.13. Deducting M_{42} from M_{41}, we have for the whole year, $6079.19 - 5920.13 = 159.06$. Now, 159.06:9.21::365:(x days). Solving for x, we have 21.7 days which in practice is called 22. The term of extended insurance is therefore 6 years 22 days.

40. In Mr. Dawson's book, Practical Lessons in Actuarial Science, the time represented by the surplus after deducting $M_{x+n'}$ from M_{x+n} (9.21 in the above example) is called w and for finding its value he gives the following formula:

$$w = \frac{365\ (M_{x+n} - M_{x+n'})}{M_{x+n} - M_{x+n+1}} \qquad (14)$$

41. Let it be required to find the net value and term of extended insurance of a policy for 3000 insured at age 35 on the 20-pay life plan, the annual premium being 113.10, one-half the annual premiums to be returned in case of death during the premium-paying period, the valuation to be made at age 45 on Actuaries table and 4%. Let us employ the prospective method of valuation. The first thing required is the net annual premium. The benefit insured consists of two parts, the capital sum, 3000, and a temporary increasing insurance of 56.55 per year for 20 years. The net annual premium of the first is

$$\frac{M_{35}}{N_{34}-N_{54}} \times 3000 = 7127.86 \div (385785.34 - 87924.72) = .026315 \times 3000 = 78.95.$$

For the second item the net annual premium is:

$$\frac{R_{35}-R_{55}-20\,(M_{55})}{N_{34}-N_{54}} \times 56.55$$

$167460.34 - 56143.92 - 20 \times 3958.84 \div 385785.34 - 87924.72 = .118695 \times 56.55 = 6.712$. The total net annual premium is the sum of 78.95 and 6.71 = 85.66. The two branches of the solution might have been combined as shown in treating of annual premiums by uniting the numerators, thus saving one division since both denominators are the same, but the solution may be better understood by the method pursued. We next are to find the value of the insurance at age 45 at which the valuation is to be made. The value of the capital sum insured at age 45 is $\frac{M_{45}}{D_{45}} \times 3000 = 1285.71$. The value of the insurance on premiums consists of ten year term insurance for 565.50 arising on the ten premiums already paid and an increasing temporary insurance for the remaining ten premiums. Combining these, the single premiums for this part of the benefit may be expressed as follows:

$$\frac{(M_{45}-M_{55})565.50+(R_{45}-R_{55}-10(M_{55})56.55}{D_{45}} = 107.898.$$

Combining the values found, the present value of the insurance is 1285.71 + 107.90 = 1393.61. The present value of the future premiums is $P(1+9a_{45}) = 85.66 \times 7.957 = 681.59$. Deducting the value of the future premiums from the value of the insurance, we have as the net value of the policy, 712.02. If the 3000 only is to be carried forward as extended insurance the term would be found as in Articles 38 and 39 and the problem presents no difficulties to one familiar with those articles. The problem, however, contemplates the return of one-half the gross premiums paid in case of death during the premium paying period. It would be proper then to carry the 565.50 along with the 3000 for ten years and continue the 3000 thereafter until the single premium is exhausted. A

simple plan to pursue in case it is apparent that the term will extend beyond the term of the temporary insurance would be to find at once the present value of the term insurance and deduct it from the single premium and apply the balance to extend the face of the policy. Proceeding in this way, we have:

$$\frac{M_{45}-M_{55}}{D_{45}}\times 565.50=(5461.36-3958.84)\div 12743.15=.11791\times 565.50$$

$=67.68$. Deducting this sum from 712.02, we have as a single premium for 3000 term insurance, 644.34 or .21478 for an insurance of 1. $D_{45}\times .21478=2737.23$ and $5461.36=M_{45}-2737.23=2724.13$. $M_{63}=2722.87$ and the term is approximately $63-45=18$ years. Solving for the parts of a year according to formula 14 gives three days, nearly, additional, so that the surrender value of the policy is 3565.50 insurance for ten years and 3000 insurance for 8 years, 3 days, additional.

42. Let us suppose that a 20-pay life policy issued at age 35 for 1000 is wrongfully cancelled by the company after ten years, what value would be a proper measure of damages?

The courts have taken different views of this question and without undertaking to determine the proper one, a few answers will be suggested, on the supposition that the object of the law is to compensate the policy-holder for his pecuniary loss.

1. If only the facts stated in the problem are submitted, the net value of the policy computed upon the mortality table and at the rate of interest prescribed for valuations in the state where the cause arose would be a proper measure. If the standard of valuation were American Experience and $3\frac{1}{2}\%$, this value, computed by formula (9), would be 291.42.

43. 2. If it were admitted or proven that the office premium of regular and solvent insurance companies as generally charged, is, say, 38.35 for a 20-pay life policy at age 35 and 74.79 for a ten-payment life policy at age 45 and that the policy-holder was still insurable and desired insurance paid up at age 55, a fair compensation, and perhaps the most fair, that could be awarded would be to allow him the present value of the difference in the two premiums, that is, $(74.79-38.35)\ 1+|9a_{45}$, or $36.44\times 8.17=\$297.71$. The reason why the latter value is regarded as proper is that policy-holders do not do business in terms of net premiums. On the other hand gross premiums in reliable companies do not materially differ for the same risks and such premiums bear great analogy to the market value of the commodities dealt with in the business world, and indeed are less variable than the value of most commodities. The market value is an accepted standard in measuring damages in the commercial world and is a fair criterion in the case supposed in this Article.

44. Suppose the policy-holder by reason of age or infirmity is no longer insurable, what is the measure of his damages?

The net value of the policy, of course, would be the minimum, for the reason that that amount of liability is cancelled on the company's books by the termination of the policy and the company could not expect to profit by its fault. If the physical condition of the insured should be proven with certainty to be such that his life could not be prolonged beyond a certain brief period, it is possible that a larger recovery would be permitted. For instance, if the insured was afflicted with a cancer, tuberculosis or some other disease upon which medical witnesses could base a positive opinion fixing a maximum brief term of life, the damages measured by the interests of the beneficiary as well as relief upon the resources of the company, would justify a larger compensation, corresponding to the present value of an endowment insurance for the maximum period of the life, reduced of course, by the present value of premiums which would accrue during the term. It might be shown that the risk might be insured as an impaired life at a higher rate, in which case, the damages would be the present value of the difference in the two rates, computed on the principle of formula (2).

45. If the insurance did not attach and was not in force from the date of the policy, the total premiums paid, with interest at the legal rate in force in the jurisdiction of the contract, would be a proper measure of damages, and cases may be easily supposed where further damages might be sustained growing out of the failure of the company to furnish insurance for the future as contemplated in the original negotiations, but it is improper, at least from an actuarial standpoint, to allow the recovery of back premiums when the risk has attached and the policy was in force, though this has been permitted in a few instances.

46. The **retrospective method** of computing the damages, that is, the method of improving the annual premiums at interest and deducting the mortality cost, is wholly unsuitable as a means of ascertaining the damages in any case unless *net* premiums are used in the computation or unless the tabular mortality cost and expenses are both deducted, it being supposed that the company has earned both the expenses and the current mortality costs while the policy was in force.

47. The expectation of life treated as a term certain either for life insurance or for annuities is not a proper criterion for use in any case in estimating damages. Its employment for that purpose is unscientific and leads to incorrect results.

48. In case dividends or bonuses of fixed amount are guaranteed by the contract, they constitute a part of the benefit insured and are to be considered along with the face of the policy in valuing the policy. When these dividends are made payable annually and vary in amount as they frequently do, they present some difficulty because of the arithmetical labor involved in the calculations. When the value of the dividends is represented in an agreement to increase the benefit at the end of the premium paying period by a sum certain in consideration of the forbearance and cancellation of the dividends; or, to pay a gross

sum in cash in discharge of them; or, when the premium-paying period is shortened because of forbearance to collect such dividends, the work is much simplified.

49. Suppose, for example, a 20-payment life policy, issued at age 35 for $1000, with guaranteed annual dividends during the premium-paying period, with the option, to accept in lieu of dividends the sum of $260.00 in cash, 459.00 in increased paid-up insurance at the end of 20 years, or that the premium paying period should be reduced to 16 years, if the dividends were allowed to accumulate during that period, so that the policy would become paid up for $1000 in 16 years. In this case, we may regard the 260 as an endowment payable in 20 years. The formula for a pure endowment we have seen is: ${}_nE_x = \frac{D_{x+n}}{D_x}$. In this case, employing American Experience and $3\frac{1}{2}\%$ the figures are $(9733.40 \div 24544.70) \times 260 = 103.11$. The latter sum represents the present value of the annual dividends as reflected in 260 cash settlement offered for them in the policy and they for the purposes of valuation may be treated as single premium for insurance. Now the single premium for 1000 insurance at age 35, is 370.55. Adding 103.11 the new single premium 473.66 would represent a policy for 1278.20. This policy is the mathematical equivalent of the original policy and may be used as the basis for computing the net premiums, the net value and term of extended insurance upon said policy.

Again, $459.00 paid up insurance at age 55 has a present value of 259.86. The present value of this sum 20 years earlier is $260v^{20}\ {}_{20}p_{35} = \$103.11$ as before. From this, we may find the net single premium and equivalent whole life policy $1278.20, which may be valued according to formulas (1), (2), (3) or others.

Finally, a policy paid up at age $35+16$ or 51, is purchased by an annual premium of $370.55 \div 1+a_{\overline{15}|}$ and on the theory that the amount of the gross or office premium, if adequate, does not affect the net value of the policy, the policy supposed may be valued directly as a 16-payment whole life policy.

50. Let us now consider the effect on the value of a policy, which will arise in case the gross or office premium is less than the net annual premium. The process of valuation as already shown is intended to show the *liability* of the company upon its contract. This liability we have seen, may always be found by equating against the value of the insurance promised in the policy, the value of the future premiums to be received by the company under the contract. Let us suppose that the gross annual premium of the policy of the last previous article was $25.00, what would be the net value of the policy at the end of ten years?

The present value of the benefit, $1000, payable at the death of a person aged 35, by the American Experience table and $3\frac{1}{2}\%$ is $370.55. The net annual premium for this benefit is $27.40. That is a twenty-

year annuity due of $27.40 is the mathematical equivalent of $370.55 in cash. But by the conditions of the contract the company, while assuming a liability of $370.55, has done so for a consideration of $2.40 less each year than a sum sufficient to offset this liability. The value of this annual shortage is $(1+|n-1a_x)\ 2.40$ or $13.889 \times 2.40 = 33.334$. So that the policy is worth $33.33 more than cost at its inception. The company by its contract, has assumed this liability. At the end of ten years, the period of valuation supposed, the value of the benefit has increased to $456.00 and against this cash liability, the company has as an offset a ten-year annuity of $25.00, the present value of which is at age forty-five, $25 \times 8.170 = \$204.52$. The liability of the company therefore, is now $\$456.00 - 204.25 = 251.75$, and the net value of the policy is therefore $251.75. Had the gross or contract premium been in excess of the net annual premium, the net value of the policy at the end of the tenth year would have been $232.19 or $19.56 less. In order, therefore to compute the net value of a policy having a gross premium which is less than the net premium it is necessary to add to the ordinary net value the present value of an annuity due of the difference between the net and the gross premium.

51. The net value hereinbefore considered are what are known as **terminal net values** or terminal reserves. These values are appropriate to individual policies. In the case of valuations for insurance companies, for the purpose of ascertaining their financial condition it is more practical to group the policies with reference to age and date of issue and it is usual to assume that all of the policies issued during a given insurance year were issued at the middle of the year on the theory that the shortage in interest on the premiums on the later policies will be offset by the excess on the earlier policies. The values thus found are called **mean reserves.** Such reserves may be found approximately from the terminal reserves by dividing the sum of the reserve for the $n-1$th year, the net annual premium and the reserve for the nth year by two. This gives the mean reserve for the nth year. Thus, the mean reserve for the problem of Article 1 may be found as follows: $4V_{30} = 39.06, P_{30} = 16.97$ and $5V_{30} = 49.63$ and $(39.06 + 16.97 + 49.63) \div 2 = 52.82$. For the mean reserve for the 6th year $5V_x$ and $6V_x$ will be substituted. As a formula the process may be stated as follows:

$$nV_x = \frac{\overline{n-1}V_x + P_x + nV_x}{2} \qquad (15)$$

52. In actions brought under the **Federal Employers' Liability Acts**, for deaths or personal injuries, the probabilities of life of the injured person in case of injuries not resulting in death and of both the deceased at the time of the injury and of the beneficiary claimant in cases of injuries resulting in death, enter into the estimation of the damages, and the apportionment of the same among the dependents in case of death. The courts have so far not gone further into the question of life contingencies than to allow the introduction in evidence of the tables showing the expectation of life of the injured person and

in case of his death, the expectation of life of the beneficiary also. Probably any of the tables published in this book (Table No. XXXI) would be regarded as standard and competent for such purposes. Certainly the one recognized in the state of the forum by statute for insurance valuations or for the valuation of life estates would be regarded as competent. The value of the shares of plural dependents, such as a widow and minor children in the damages found, presents a question which could be solved with accuracy on the principles discussed in the chapters on life contingencies and annuities; but such nicety of computation has not been undertaken in the cases so far decided. Indeed this could hardly be done by a jury and even the court would probably require the assistance of the testimony of Actuaries. It has therefore been considered sufficient to resort to approximations based upon the tables of the expectation of life. Familiarity with the principles illustrated in the foregoing pages will, nevertheless, greatly aid counsel in the intelligent presentation of such cases to courts and juries.

CHAPTER XII

Of Surplus and Dividends

The annual premium of an insurance policy usually consists of the net annual premium as a basis to which is added a **"loading"** also sometimes called **"Margin"** intended to cover the expenses incident to the business of writing the insurance and managing the business and also for the profit of the stockholders, if it be a stock company. This loading is ordinarily a certain per cent of the net premium, but it sometimes includes also a constant quantity bearing no relation to the net premium.

The net premium is usually based upon a rate of interest somewhat less than the company might reasonably expect to earn on the funds of the company. The gross premium should always be made large enough to meet all the conditions necessary to assure the payment of every claim arising on the contracts of the company. It often happens that owing to favorable mortality, good management and other causes, the earnings of the company on its investments and savings on expenses, the company will have a sum on hands in excess of the requirements for reserve and expenses. This excess is called **surplus.** It is about the same thing that is called "profits" in some other financial institutions. In mutual companies this surplus equitably belongs to the policy holders. In stock companies, in the absence of some legal or contractual obligation to the contrary, it belongs to the stockholders. But in both classes of companies, it is usual to set apart at least a portion of this surplus, for distribution among policy holders as dividends on their policies. Policies which are entitled to share in such dividends are called **participating policies.** Those not so entitled are called **non-participating policies.** It was usual in earlier years to place the distribution period at some distant future period, say ten, fifteen or twenty years, a practice which resulted in disappointment to the policy holder, so frequently, that by legislation in some cases and by force of public opinion and to meet competition in others, it has generally become the practice to declare annual dividends after the policy has been force a few, usually two years. These dividends are, in this country, usually apportioned according to what is known as the "Contribution Plan," recommended by Sheppard Homans, an eminent actuary who also has the distinction of being the author of the American Experience table of mortality. The principle upon which this plan is based, is that each policy should participate in the distribution of surplus in the same proportion which it has contributed to produce the surplus. The modus operandi of the plan roughly stated is to charge against the policy its proportion of the expenses of management, the reserve required on the policy at the time and its proportion of the mortality cost of insurance according to the experience of the company, and crediting the policy

with the gross premiums paid under the policy and net interest earned. The balance if in favor of the policy is the surplus. If at the time of computing the dividend there is a reserve to the credit of the policy for the previous year, it should be added to the gross premium and the interest computed on the sum at the rate earned by the company after deducting expenses incident to the investment. There are other sources from which surplus may arise, such as rents, lapsed or forfeited policies and other miscellaneous sources. These funds are disposed of in different ways. Sometimes by using them to reduce the item "expense of management" or by apportioning them to the credit side of the account along with the "net interest earned" or other uses are made of it according to the ideas of those managing the company. Other methods of distributing surplus, no doubt, are employed in practice, but the subject need not receive further attention here It may be remarked that until dividends are declared, the surplus is the property of the company and the policy holder has at most a mere inchoate right therein, but after they are declared, they, if not paid, may under some circumstances, be used as an offset against liabilities to the company or to prevent forfeitures.

CHAPTER XIII

Of Fraternal Insurance

A few words upon the subject of fraternal or mutual benefit societies may be helpful. These societies have been an immeasurable source of good to thousands of homes in this country and they should be so fostered and improved as to increase their usefulness. Very few of them have been constructed upon the scientific principles applied in the preceding chapters and, perhaps, it would not be possible to so construct them without destroying to some extent their fraternal characteristics. There is one thing that should be kept in mind, however, by those promoting or managing such societies, seeking membership therein or legislating for their welfare. It is certain that insofar as the insurance feature of the society is concerned, there is no magic in the name "fraternal." A man holding a policy in an "old line" insurance company and a certificate of membership in a fraternal society for like sums and at the same time, holds contracts imposing indentically the same obligations upon the two institutions in case of his death. It takes the same amount of the same kind of money in the case of each. The money in both cases alike must be paid by the members. True, it is in the case of old line companies the death benefit is largely met by interest accumulations, but this is, after all, merely earnings of excess premiums advanced by the insured so that in the last analysis the insured pay the death losses, all of them. The company, whatever its character, pays none of them. No institution has yet been organized which has attained the philanthropic character of paying death benefits from its own resources. The membership which does not deny itself to create funds to meet the growing demands arising from increasing age and "keeps its reserve in its pockets" must accumulate in some other way the interest necessary to meet the increasing mortality costs that are bound to come. If the members do not each faithfully shoulder the burden of paying his own way, there must in the end remain a group of the faithful brethren, who while most worthy of all, will leave widows who will find their certificates of membership worthless paper containing promises which are delusions.

If those who enter a society which does not demand of its members that they shall pay every year the full cost of their insurance, understand the fact that those who live to pay the most premiums will lose. If they *understand* this and are willing to accept the fact of living as full compensation for paying for others and receiving nothing for themselves, well and good. They should be allowed to do it. If one is induced to enter such a society without appreciating this fact he has been subjected to a cruel fraud which length of days will certainly disclose. This country has been the greatest field for the promotion of fraternal societies that the world has seen. The sovereign lodges

have had state after state to add to "the juridiction" to take in new blood and keep down the average death rate but with all this, the inexorable law of human mortality has had its way and thousands of men who had supposed their lives were insured for the whole of life have found that their hopes were vain, their certificates worthless. This has so often happened that all who want to know should know it. The lesson then is, that a rate of contribution should be established from the first that will exact from each member a contribution which at least equals the cost of his insurance for the period covered by the payment. In this way, his increased age will not affect prejudicially the new member who enters ten, twenty or fifty years later, and the new member, therefore, will have no cause to shun his society. Such a rate may be made from the tables and on the principles discussed in the preceding chapters, but the wise course would of course be to have them prepared by a competent actuary. As a corrollary to this proposition, the man who seeks membership in any of the various forms of mutual benefit insurance societies should first ascertain that his company is exacting of each of its members each year the cost of his insurance. This is the only guarantee of permanence. If it is not so doing, he should keep out. Self respect should deter him from seeking to cheat his brethren, should he die early, and prudence, from being himself cheated should he be favored with long life.

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ERRATA

Page 8.	Art. 7.	The illustration given in the last paragraph article is entirely erroneous, reversing the cor principle stated in Articles 3 and 4, same chapt
Page 8-9.	Art. 9.	The rule here given is correct but the reason is erroneously stated. Different powers of the sa factor and not different numbers, should be c sidered.
Page 10.	Art. 18.	Insert the words "such number" after "and" in the 7th line.
Page 14.	Art. 8.	Substitute "$a_{\overline{30}|}$" for "$A_{\overline{30}|}$."
Page 36.	Art. 8.	In last line substitute "448732" for "551267."
Page 37.	Art. 19.	Substitute $\frac{(1+{}^{i}/_{p})^{np}-1}{m(1+{}^{i}/_{p})^{p/m}-1}$ in second member of formula (23).
Page 40.	Art. 31.	Substitute 3 per cent for 3½ in example.
Page 41.	Art. 35.	Substitute "a′" for "a" after the word annuity in first line.
Page 43.	Art. 44.	Erase symbol, $S\overline{n}|$
Page 44.	Art. 46.	Substitute the word "principal" for "rate" in third line of rule.
Page 52.	Table 6.	Put "multiplying" for "dividing."
Page 104.		"px" and "qx" should be transposed and heads changed to correspond.
Page 134.		10axxx 5%, should be written 12.90393.
Page 153.		M 57 should be written 2611.373.
Page 172.	Art. 34.	Change formula number to (17).
Page 206.	Art. 7.	Change + between parentheses to ÷. In several places, the annuity symbol "$a.$" is improperly written "a" and "member" of equations improperly written "number." To avoid mistakes, note these errors at the places indicated.

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